The Wireless Networking Workbook

A companion to CPE 462

Version 1.1
Fall 2018

Matthew C. Valenti, Ph.D., P.E., FIEEE
West Virginia University

Contents

Chapter 1

Radio Propagation and Path Loss

1.1 The Shannon Capacity

The main *resources* of a wireless system are *bandwidth* and *power*. By *bandwidth* we literally mean the amount of radio-frequency (RF) spectrum available for communications. Bandwidth B is measured in units of Hertz (Hz). This is in contrast with the notion of *bandwidth* in popular culture as a proxy for *data rate*, which is measured in bits per second (bps). Here, we prefer the use of C for *capacity*, which is the maximum transmission rate (in bps) that can be supported on a channel. The values of B and C are related, as will be shown below.

The quality of a received signal is a function of its *power* relative to the *noise* and/or *interference*. We will be spending a lot of time this semester tracking the ratio of signal-to-noise power; i.e., the SNR, as well as the signal-to-interference ratio (SIR) and signal-to-interference-and-noise ratio (SINR).

There is a relationship between the capacity "C" and the bandwidth "B" that comes from *information theory*. First postulated in 1948 by Claude Shannon (a researcher at Bell Labs), the *Shannon capacity* states that the capacity of a channel with a bandwidth of "B" Hz and signal-to-noise ratio SNR is given by:

$$C = B \log_2 (1 + \mathsf{SNR}) \qquad (1.1)$$

Capacity has units of bps, and is the maximum transmission rate that can be supported over the channel. Of course the equation doesn't say exactly how to design a system capable of achieving capacity: That is the job of communication engineers, and indeed it took nearly 50 years before engineers figured out how to design capacity-approaching systems (the key: to use the so-called *turbo* codes).

Equation (1.1) assumes that transmissions are limited only by Gaussian noise. If the system is interference limited, then SIR may be used in place of SNR. More generally, if the system is prone to noise and interference, then the SNR may be replaced by SINR.

Throughout this course, we will be measuring SINR and/or SIR, but remember from (1.1) that these values have a direct bearing on the data transmission rate supported by the system.

Example: Consider a receiver that operates over a bandwidth of $B = 10$ MHz. If the received signal-to-noise ratio is $\mathsf{SNR} = 20$ dB, then determine the maximum theoretical data rate that can be supported.

1.2 Overview of the Wireless Channel

Let's consider the communication channel between a transmitter and receiver separated by a distance d (assumed to be in meters). Let v be the relative velocity (in m/s) between the transmitter and receiver. Without loss of generality, we can assume that the transmitter is a base station and the receiver is a mobile, as illustrated in Fig. 1.1.

Figure 5.2. The mobile radio channel as a function of time and space.

Figure 1.1: From T.S. Rappaport, *Wireless Communications: Principles and Practice*

The wireless channel affects the transmitted signal in a variety of ways. When analyzing wireless networks, we are very concerned about the *strength* of the signal at various locations in the network. To be more precise, we will talk about signal *power* rather than *strength.* Let P_t be the transmitted power and P_r be the received power at distance d. An example measurement of received signal power as a function of distance might look like the plots shown in Fig. 1.2 on the next page.

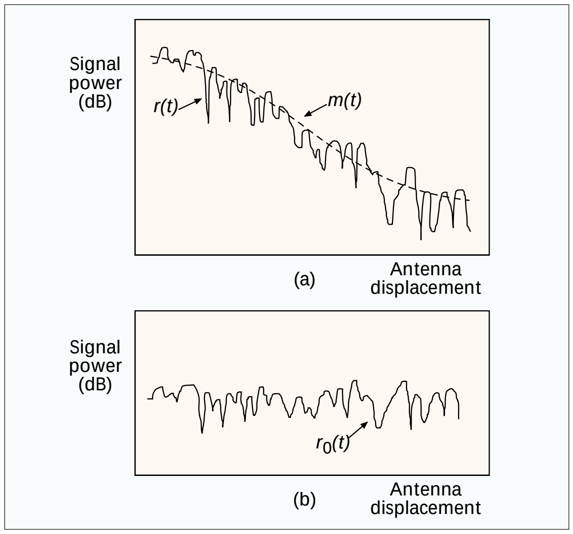


Figure 3. *Large-scale fading and small-scale fading.*

Figure 1.2: From B. Sklar, "Rayleigh fading channels in mobile digital communication systems"

Since we are assuming that the mobile moves at constant velocity v, the displacement d is proportional to the elapsed time t; i.e., $d = vt$. In the figure, we can see that the local mean signal power $m(t)$ changes slowly with distance and generally decays as a function of distance. However, there is an additional rapid fluctuation about the mean. In the figure $r(t)$ denotes the instantaneous received power, while $r_0(t)$ is the deviation of the instantaneous power from the mean $m(t)$. Based on this observation, the propagation mechanisms can be roughly divided into *large-scale* effects, which deal with the nature of $m(t)$, and *small-scale* effects, which deal with the nature of $r_0(t)$.

1.2.1 Large-Scale Effects

{Rappaport Chapter 4} As a signal propagates through space, its *local mean power* decreases with increasing distance. Here, the "local mean" is found by measuring the received power at many locations over a span of several wavelengths. This average power reduction is mostly a function of distance, but is also affected by obstructions between transmitter and receiver. The reduction in local mean power is sometimes called *large-scale fading* and can be decomposed into the following components:

Path Loss: The signal strength decays exponentially with distance d between transmitter and receiver. The *path loss* is proportional to d^α, where α is called the *path loss exponent* and typically assumes values $2 \leq \alpha \leq 4$ depending on the environment.[1] $\alpha = 2$ in free space, but is often greater than 2 in terrestrial environments. The average path loss is a deterministic quantity (i.e. not random).

Shadowing: Often there are millions of tiny obstructions in the channel that absorb signal energy, such as water droplets if it is raining, or the individual leaves of trees. Similarly there are

[1]Some references, most notably the Rappaport textbook, use lowercase n for the path-loss exponent.

large obstructions such as hills and highway overpasses that cause diffraction. Because it is too cumbersome to account for all of the obstructions in the channel, these effects are typically lumped together into a *random* power loss. This power loss is called *shadowing* or *shadow fading* and sometimes called *slow fading*. Unlike fast fading (discussed below), a small change in position will not change the amount of random power loss due to shadowing fading.

Large-scale effects determine how the mean signal power at the receiver P_r is related to the transmitted signal power P_t.

1.2.2 Small-Scale Effects

{Rappaport Chapter 5} The channel itself acts like a filter. While the filter is (mostly) linear, it is typically *not* time-invariant. The received signal (output of the channel) is the convolution of the transmitted signal (channel input) with the time-varying *impulse response* of the channel. The time-varying convolution integral is {Rappaport equation (5.3)}:

$$y(d,t) \quad = \quad x(t) * h(d,t) = \int x(\tau)h(d,t-\tau)d\tau \tag{1.2}$$

where in the above equation:

- $x(t)$ is the transmitted signal at time t.
- $y(d,t)$ is the signal received at time t at distance d from the transmitter.
- $h(d,t)$ is the impulse response of the channel as seen by the receiver located at distance d from the transmitter.

As the receiver and/or transmitter moves, d changes, and thus the impulse response $h(d,t)$ will generally change. If the transmitter and receiver are moving apart at constant velocity v, then $d = vt$ and $h(d,t) = h(vt,t)$ can be considered to be evolving in time rather than changing with distance.

Measurements of the channel impulse response $h(d, t)$ at different displacements might look like:

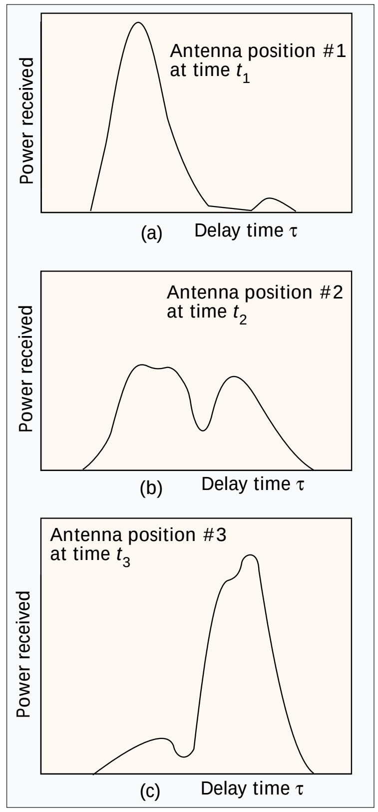


Figure 5. *Response of a multipath channel to a narrow pulse versus delay, as a function of antenna position.*

Figure 1.3: From B. Sklar, "Rayleigh fading channels in mobile digital communication systems"

As can be seen, the channel response is different at different locations. For a constantly moving mobile, this is equivalent to having a channel response that changes in time. Conceptually, the channel response can be shown in two dimensions, with one axis representing how the channel changes in time, and the other showing the response at a given time:

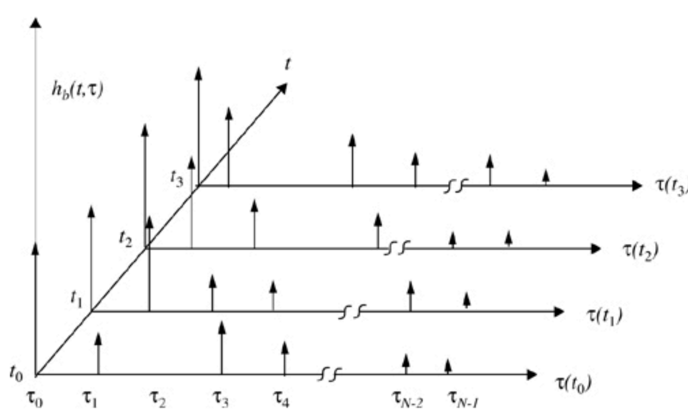


Figure 1.4: From T.S. Rappaport, *Wireless Communications: Principles and Practice* (Fig. 5.4)

The impulse response is determined by the following closely related mechanisms:

Reflections: Reflections occur when a signal bounces off an object with dimensions larger than a wavelength, such as the smooth wall of a building or the side of a distant mountain. If the object is far enough away, the reflected signal could arrive so late that it interferes with the next bit transmitted over the direct path. This results in *intersymbol interference* (ISI).

Scattering: Scattering occurs when the signal strikes an object with dimensions on-the-order of a wavelength (or smaller), such as a street sign, lamp-post, furniture, person, vehicle, etc. Even larger objects can scatter the received wave if it has rough surfaces. In cellular systems, scattering tends to be dominated by objects that are close to the mobile.

Multipath fading: Each reflected or scattered signal component travels over a slightly different path than the direct line-of-site (LOS) signal component. The receiver receives the superposition of all these components, hence the term *multipath.* Even if two paths have a similar length, their phases could be quite different. In some locations, the different signals have similar phase and add *coherently* or *constructively* interfere with one-another; in other locations the phases may be close to 180 degrees apart and thus the signals destructively interfere with one-another. The multipath structure is so delicate that moving just a few wavelengths could change destructive interference into constructive interference, and vice-versa. This will cause a corresponding fluctuation in received signal power. When the transmitter and/or receiver is in motion, it will rapidly move in and out of locations with destructive interference, and thus the received signal strength will rapidly change as a function of time. This rapid change in the instantaneous received power over short distances (on the order of a wavelength) is called *multipath fading* or *fast fading* or simply just *fading.*

Small-scale effects have a strong influence on the required minimum signal power P_r *needed for reliable signal reception. Other factors include: Noise power, interference level, modulation format, error control coding strategy, and receiver implementation (e.g. use of equalizers).*

1.3 Free Space Path Loss

If a signal of wavelength λ is transmitted with power P_t over a distance d and received by an antenna with effective area A_e, then what is the received power? Assume free space propagation, so there are no obstructions to block the LOS path, no objects to create reflections, and the transmission medium does not absorb energy.

- First, let's assume that the transmitting antenna is *isotropic.* That is, energy is radiated in all directions with equal intensity.
- The wavefront creates an expanding spherical bubble, much like a balloon. Conservation of energy applies here because the medium does not absorb any energy. This tells us that the power spread across the surface of the sphere must be P_t. But as the sphere expands (d increases) the *power density*, i.e. the amount of power divided by the surface area of the sphere, decreases.

- The surface area of a sphere of radius d is $A = 4\pi d^2$. Thus, what is the power density on the surface of the sphere (in W/m^2)?

- The received power is the power density times the *effective antenna area* A_e,

Let G denote the *antenna gain*. We will talk more about antenna gain later, but for now all we need to know is that the antenna gain and effective area are related by {Rappaport Equation 4.2}:

$$G = \frac{4\pi A_e}{\lambda^2}. \tag{1.3}$$

We will use G_r to denote the receive antenna gain and G_t for the transmit antenna gain.

Now, use the above expression to remove the dependency on A_e in our expression for received power P_r:

In the above derivation, we have assumed an isotropic transmit antenna, which has a gain of $G_t = 1$. Now assume that the transmit gain is G_t, which means that the signal power is boosted (multiplied) by amount G_t in the direction of the receiver. This results in:

Finally, there may be additional system losses due to electronics, transmission lines, etc. Let $L \geq 1$ denote the *system loss factor*. The received power is then given by {Rappaport Equation (4.1)}

This is called the *Friis free space equation*. If not specified, assume that $L = 1$.

1.4 Decibels

The decibel (dB) is a logarithmic measure of power gain. The convenience of dB is due to the following basic properties of logarithms:

$$\log\left(\frac{1}{X}\right) = -\log(X) \tag{1.4}$$

$$\log(XY) = \log(X) + \log(Y) \tag{1.5}$$

$$\log\left(X^Y\right) = y\log(X) \tag{1.6}$$

$$\log\left(\frac{X}{Y}\right) = \log(X) - \log(Y) \tag{1.7}$$

The *dB gain* of a system can be found from the input and output power as:

$$G^{(\mathsf{dB})} = 10\log\left(\frac{P_{\mathsf{out}}}{P_{\mathsf{in}}}\right), \tag{1.8}$$

where the base of the logarithm is 10. A *loss* is the reciprocal of a gain, thus:

$$\begin{aligned} L^{(\mathsf{dB})} &= 10\log\left(\frac{P_{\mathsf{in}}}{P_{\mathsf{out}}}\right) \\ &= -G^{(\mathsf{dB})}. \end{aligned} \tag{1.9}$$

For a radio channel, the free-space dB path loss at distance d (assuming the system loss $L = 1$) is

$$\begin{aligned} L_d^{(\mathsf{dB})} &= 10\log\left(\frac{P_t}{P_r}\right) \\ &= 10\log\left(\frac{(4\pi d)^2}{\lambda^2 G_t G_r}\right) \\ &= 10\log\left(\frac{(4\pi)^2}{\lambda^2 G_t G_r}\right) + 10\log(d^2) \\ &= L_o^{(\mathsf{dB})} + 20\log(d), \end{aligned} \tag{1.10}$$

where

$$\begin{aligned} L_o^{(\mathsf{dB})} &= 10\log\left(\frac{(4\pi)^2}{\lambda^2 G_t G_r}\right) \\ &= 10\log\left(\frac{4\pi}{\lambda}\right)^2 - 10\log(G_t) - 10\log(G_r) \\ &= 20\log\left(\frac{4\pi}{\lambda}\right) - G_t^{(\mathsf{dBi})} - G_r^{(\mathsf{dBi})}. \end{aligned} \tag{1.11}$$

is the dB path loss at a reference distance of $d_0 = 1$ meter without taking into account the antenna gains and $G^{(\mathsf{dBi})} = 10\log(G)$ is the dB gain of an antenna with respect to an *isotropic* antenna[2].

In the above, dB is used to describe the gain or loss of a system. Sometimes we just want to express the power of a signal in terms of dB. To do this we need a reference power. If the reference power is 1 W, then we can find the power of our signal relative to this reference:

$$P^{(\mathsf{dBW})} = 10\log\left(\frac{\text{P in W}}{1\text{ W}}\right). \tag{1.12}$$

[2]An isotropic antenna radiates equally in all directions, and therefore has unity gain ($G = 1$).

If instead the reference power is 1 mW, then

$$\begin{aligned} P^{(\mathsf{dBm})} &= 10\log\left(\frac{\text{P in W}}{1\text{ mW}}\right) \\ &= 10\log\left(\text{P in W}\right) - 10\log\left(10^{-3}\right) \\ &= P^{(\mathsf{dBW})} + 30. \end{aligned} \tag{1.13}$$

Putting things together, we find that when using dB units, the received power is simply the transmitted power minus the dB path loss (including antenna gains)

$$\begin{aligned} P_r^{(\mathsf{dBW})} &= 10\log\left(\frac{\lambda^2 G_t G_r P_t}{(4\pi d)^2}\right) \\ &= 10\log P_t + 10\log\left(\frac{\lambda^2 G_t G_r}{(4\pi d)^2}\right) \\ &= 10\log P_t - 10\log\left(\frac{(4\pi d)^2}{\lambda^2 G_t G_r}\right) \\ &= P_t^{(\mathsf{dBW})} - L_d^{(\mathsf{dB})} \\ &= P_t^{(\mathsf{dBW})} - \left\{L_o^{(\mathsf{dB})} + 20\log(d)\right\} \\ &= P_t^{(\mathsf{dBW})} - L_o^{(\mathsf{dB})} - 20\log(d). \end{aligned} \tag{1.14}$$

Likewise, if the transmitted and received powers are both in units of dBm:

$$\begin{aligned} P_r^{(\mathsf{dBm})} &= P_t^{(\mathsf{dBm})} - L_d^{(\mathsf{dB})} \\ &= P_t^{(\mathsf{dBm})} - \left\{L_o^{(\mathsf{dB})} + 20\log(d)\right\} \\ &= P_t^{(\mathsf{dBm})} - L_o^{(\mathsf{dB})} - 20\log(d). \end{aligned} \tag{1.15}$$

Example: Let $f_c = 2.4$ GHz, the transmitter-receiver separation $d = 1.6$ km, and transmit power $P_t = 2$ W. The transmit antenna has gain 4 dBi and the receive antenna is isotropic. Determine the path loss in dB and the received power P_r in dBm. Assume free-space propagation.

1.5 Signal-to-Noise Ratio

The SNR is the ratio of received signal power to noise power. The noise is primarily due to the thermal excitation of electrons in the receiver circuitry and is thus temperature dependent (see Rappaport appendix B for details). Let P_n denote the noise power. In linear units, the SNR is:

$$\mathsf{SNR} \quad = \quad \frac{P_r}{P_n}. \tag{1.16}$$

In dB units this becomes

$$\begin{aligned} \mathsf{SNR}^{(\mathsf{dB})} &= P_r^{(\mathsf{dBW})} - P_n^{(\mathsf{dBW})} \\ &= P_r^{(\mathsf{dBm})} - P_n^{(\mathsf{dBm})}. \end{aligned} \tag{1.17}$$

Example: Suppose that the received power is as calculated in the previous example and the background noise has a level of -110 dBm. Determine the signal-to-noise ratio (SNR) at the receiver (where SNR is the ratio of the signal power to noise power).

The noise power itself is a function of the signal bandwidth B. In an ideal receiver, the noise power is {Rappaport equation (B.3)}:

$$P_n \quad = \quad kT_0B, \tag{1.18}$$

where:

- $k = 1.38 \times 10^{-23}$ J/K is the *Boltzmann* constant.
- T_0 is the room temperature in K (usually 290 or 300 K).
- B is the bandwidth in Hz.
- P_n is the noise power in W.

Nonideal systems are described by a *noise figure* F, where $F \geq 1$, such that the noise power is {Rappaport equation (B.5)}:

$$P_n \quad = \quad FkT_0B. \tag{1.19}$$

Alternatively, the nonideal system can be described by its *effective noise temperature* T_e, which is related to F by:

$$F \quad = \quad \left(1 + \frac{T_e}{T_0}\right). \tag{1.20}$$

Example: A 1 MHz signal is received with a noise power of -110 dBm. Determine the receiver's effective noise temperature. Use $T_0 = 300$ as the reference.

1.6 EIRP and ERP

The amount of RF energy directed from a transmitter to a receiver is determined not only by the transmit power P_t, but also by the transmit antenna gain G_t. For instance, a low power transmitter can direct just as much power towards a receiver as a high power transmitter if it simply uses a higher gain antenna. To account for both of these parameters, the *effective isotropic radiated power* (EIRP) is often used {Rappaport equation (4.4)}:

$$\mathsf{EIRP} = P_t G_t. \tag{1.21}$$

Note that this equation is in linear units. In dB, it would be $\mathsf{EIRP}^{(\mathsf{dBm})} = P_t^{(\mathsf{dBm})} + G_t^{(\mathsf{dBi})}$. The EIRP is what the transmitted power would have to be if an isotropic antenna were used (i.e. if $G_t = 1$ in linear units or 0 in dB) instead of the given directional antenna.

An alternative to EIRP is the *effective radiated power* (ERP). This is the power that would have been transmitted had a half-wave dipole antenna been used (which has a gain of $G_t = 1.64$ in linear units or 2.15 dBi). The ERP is related to the EIRP by:

$$\mathsf{ERP} = \frac{\mathsf{EIRP}}{1.64} \tag{1.22}$$

in linear units, and

$$\mathsf{ERP}^{(\mathsf{dBm})} = \mathsf{EIRP}^{(\mathsf{dBm})} - 2.15 \tag{1.23}$$

in dB units.

Example: If the transmit power $P_t = 2$ W and the transmit antenna has gain 4 dBi, determine the EIRP and ERP in dBm.

1.7 Two-Ray Propagation

Now suppose that the transmitter is at height h_t and the receiver is at height h_r. Assume that the ground serves as a perfect reflector. In addition to the direct LOS path, there is a reflected signal that arrives a short time later. While the power of the LOS and reflected paths are virtually identical, the phases of the two received signal components may be different. If we just assume that the transmitted signal is a single-tone,

$$s(t) = \cos(2\pi f_c t), \tag{1.24}$$

then the received signal is of the form:

$$r(t) \propto \cos(2\pi f_c t + \theta_{\mathsf{LOS}}) + \cos(2\pi f_c t + \theta_{\mathsf{reflected}}). \tag{1.25}$$

For a given position, we can use geometry to determine the values of θ_{LOS} and $\theta_{\mathsf{reflected}}$ which will tell us if the two components constructively or destructively interfere with one-another. If we move the relative position and average over several wavelengths we find that the average received power is {Rappaport Equation (4.52)}:

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}. \tag{1.26}$$

Note that now power falls off according to the fourth-power of distance.

1.8 Exponential Path Loss

From the last two sections, we see that in free space the path loss is proportional to the square of distance, while with a perfect ground reflection it is proportional to the fourth-power of distance. Real terrestrial channels are neither in free-space or have a perfect ground reflection. So the actual relationship is somewhere in between. In dB, we can express the path loss as:

$$\begin{aligned} L_d^{(\mathsf{dB})} &= \left[10\log\left(\frac{(4\pi)^2}{\lambda^2}\right) - 10\log(G_t) - 10\log(G_r) \right] + 10\log(d^\alpha) \\ &= L_o^{(\mathsf{dB})} + 10\alpha\log(d), \end{aligned} \tag{1.27}$$

where α is called the *path-loss exponent*. For free-space, $\alpha = 2$ and with perfect ground reflection $\alpha = 4$. In an actual radio environment, this value can be measured. Measurements usually show that $2 < \alpha < 4$ as we would expect, with $\alpha = 3$ being a widely accepted value.

The L_o term represents the path loss at 1 meter, under the assumption that propagation up to one meter obeys free-space path loss. A reference distance d_0 other than one meter could be used, in which case the path loss becomes {Rappaport Equation 4.68}:

$$L_d^{(\mathsf{dB})} = 20\log\left(\frac{4\pi d_0}{\lambda}\right) + 10\alpha\log\left(\frac{d}{d_o}\right) - G_t^{(\mathsf{dBi})} - G_r^{(\mathsf{dBi})}.$$

In practice the path loss at distance d_0 can be obtained by measurement.

Putting everything together, we find that the received power at distance d is:

$$P_r^{(\mathsf{dBW})} = P_t^{(\mathsf{dBW})} - L_0^{(\mathsf{dB})} - 10\alpha\log\left(\frac{d}{d_o}\right) \tag{1.28}$$

where:

$$L_0^{(\mathsf{dB})} = 20\log\left(\frac{4\pi d_0}{\lambda}\right) - G_t^{(\mathsf{dBi})} - G_r^{(\mathsf{dBi})}. \tag{1.29}$$

The value of the exponent α is usually found empirically through a statistical analysis of measured data. The goal of the statistical analysis is to estimate the parameters $L_0^{(\mathsf{dB})}$, d_o, and α that minimize the error between the measurements and the values predicted by the model. The actual curve fitting typically requires the use of a computer.

Example: Measurements taken in Germany (see the figure below) have found that $\alpha = 2.7$ in a particular city. Let $f_c = 900$ MHz and $d_0 = 10m$. Determine the path loss at 10 km. If a signal is transmitted with a power of 4 Watts over this distance, and if the antennas on both ends of the link are dipoles (with 2.15 dBi gain), determine the received signal power in dBm.

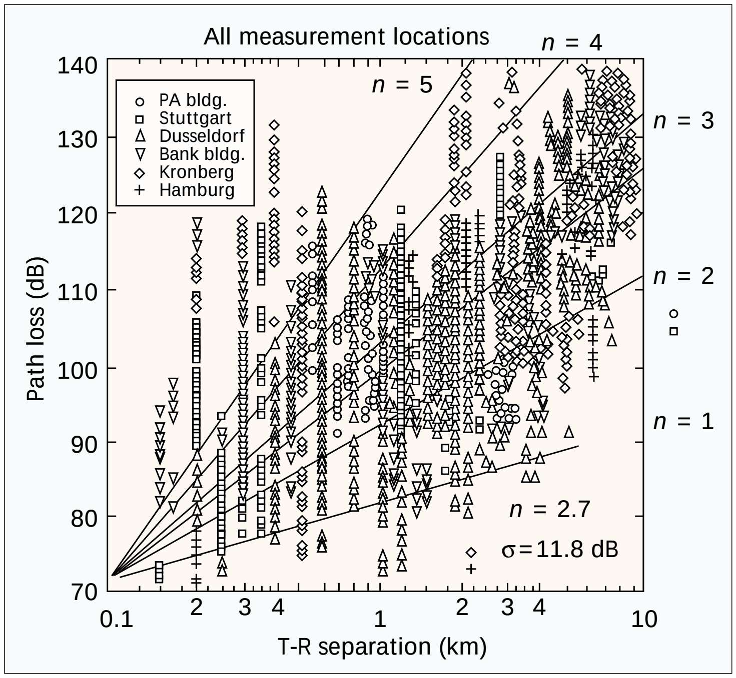


Figure 4. *Path loss versus distance measured in serveral German cities.*

Figure 1.5: From S.Y. Seidel, T.S. Rappaport, et al., "Path loss, scattering, and multipath delay statistics in four European cities for digital cellular and microcellular radiotelephones," 1991. Note that the references uses n for the path-loss exponent rather than α.

1.9 Interference

Cochannel interference (CCI) occurs when there are two or more signals being transmitted in the same radio channel at the same time. At the receiver, we must distinguish the power of the desired signal, S, from the power of the interfering signal(s), I. The *signal-to-interference* ratio (SIR) is thus:

$$\mathsf{SIR} = \frac{\mathsf{S}}{\mathsf{I}} \tag{1.30}$$

Similarly, the *signal-to-interference-and-noise* ratio (SINR) is

$$\mathsf{SINR} = \frac{\mathsf{S}}{\mathsf{I} + \mathsf{N}} \tag{1.31}$$

where N is the noise power (the same as P_n in section 1.4).

When expressed in dB, the SIR is

$$\mathsf{SIR}(dB) = \mathsf{S}(dBm) - \mathsf{I}(dBm) \tag{1.32}$$

Chapter 2

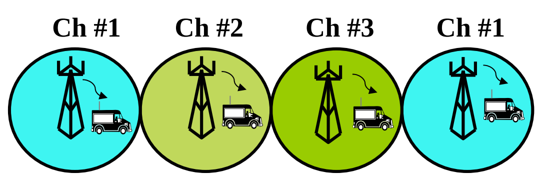
Cellular Networks

2.1 The Cellular Concept

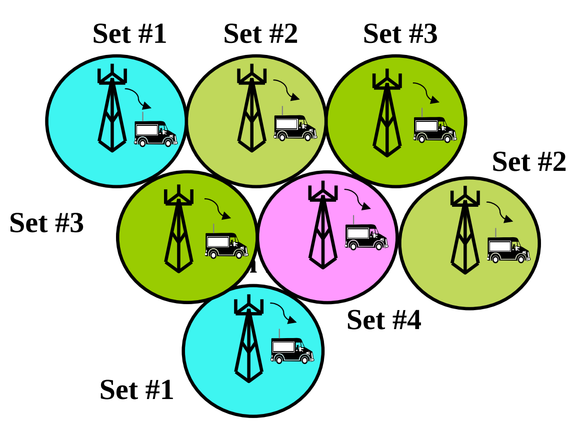
[Rappaport Chapter 3] The idea behind cellular wireless systems is:

- Instead of using a single high-powered transmitter, break the coverage area into many small cells with lower-powered transmitters or *base stations*.
- Each cell gets just a fraction of the Radio Frequency (RF) spectrum.
- Cells located far apart can use the same spectrum ("frequency reuse").

In one dimension (e.g. going down a highway), this is visualized as:



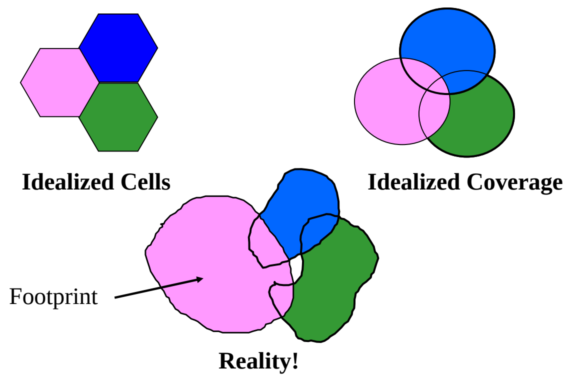
In reality, frequencies are reused in two dimensions:



In some cases, such as cities with large buildings, frequencies need to also be reused in three dimensions.

2.2 Cell footprints

- Each cell has a "footprint", which is the area covered by the base station that is near the center of the cell.
- With ideal isotropic propagation and in the absence of any interference, the footprint of a cell is circular.
- To account for interference, cells are assumed to take on a hexagonal shape.
- In reality, cell footprints are irregular.

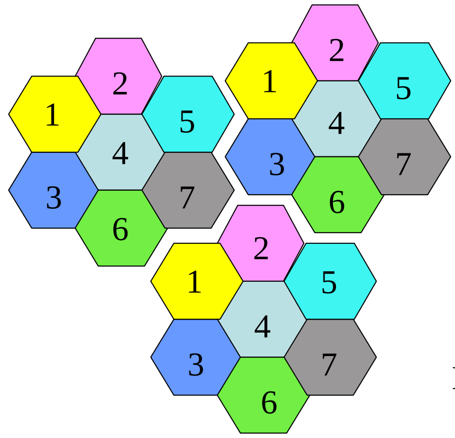


2.3 Clusters

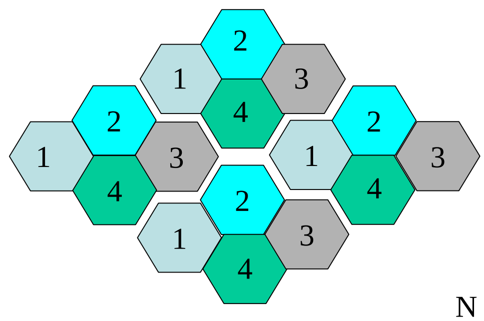
- Ideally, the coverage area is arranged into a honeycomb of contiguous hexagons.
- A *cluster* is a group of N cells that together use all the spectrum.
- Let C_k be the number of channels assigned to the k^{th} cell of the cluster, and C be the total number of channels allocated to the cluster (usually the entire available spectrum). These variables are related by:

$$C = \sum_{k=1}^{N} C_k \tag{2.1}$$

- Usually, but not always, C_k is the same for each cell in the cluster.
- To be a valid size, $N = i^2 + ij + j^2$ for integers i and j [Rappaport (3.3)].
- Cluster sizes are typically $N = 3, 4, 7, 12$ or 19.



N = 7



N = 4

Example: [Rappaport Example 3.1] A system has 33 MHz of available spectrum and uses a transmission technology that requires 25 kHz channels in each direction (uplink and downlink). Determine the number of full-duplex channels per cell when $N = 4$. Repeat for $N = 7$.

- What is the advantage of using <u>large</u> clusters?

- What is the advantage of using <u>small</u> clusters?

2.4 Signal-to-Interference Ratio

Co-channel cells must be spaced far enough apart in order for the signal-to-interference ratio (SIR) to be sufficiently high. Consider the SIR at the handset. Let:

- d be the distance from the handset to the base station it is connected to.
- S be the received power (in mW, **not** dBm) at the handset from the connected base station.
- M be the number of interfering base stations.
- d_k be the distance from the handset to the k^{th} interfering base station, where $k = 1, ..., M$.
- I_k be the received power (also in mW) at the handset from the k^{th} interfering base station.
- I be the total interference power (also in mW) at the handset.

Interference power is additive. It follows that:

$$I = \sum_{k=1}^{M} I_k \tag{2.2}$$

and the SIR is [Rappaport (3.5)]:

$$\mathsf{SIR} = \frac{S}{I} = \frac{S}{\sum_{k=1}^{M} I_k} \tag{2.3}$$

The above equation requires the powers to be in W, not dBm. Assume an exponential path loss model. Again assume an exponential path loss model. More specifically, let P_0 be the power at reference distance d_0. Assume that the signal power decays according to the square of distance out to distance d_0, and beyond that, it decays according to the α^{th} power of distance. The received power at distance d is then [Rappaport (3.6)]:

$$P_r(d) = P_0 \left(\frac{d}{d_0} \right)^{-\alpha}, \tag{2.4}$$

where, from the Friis free-space equation [Rappaport (4.1), derived in Chapter #1]:

$$P_0 = \left(\frac{\lambda}{4\pi d_0} \right)^2 G_t G_r P_t. \tag{2.5}$$

with G_t and G_r being the antenna gains of the transmitter and receiver (absolute gains, not dB gains), and P_t is the transmit power (in mW or W, not dB units). Note that (2.4) and (2.5) can be easily found by converting the dB-versions of the equations, i.e. equations (23-24) from lecture note set #1, back to linear units.

Use (2.4) to find $S = P_r(d)$ and $I_k = P_r(d_k)$ and then substitute into (2.3) to give:

$$\mathsf{SIR} = \frac{P_0 \left(\frac{d}{d_0} \right)^{-\alpha}}{\sum_{k=1}^{M} P_0 \left(\frac{d_k}{d_0} \right)^{-\alpha}}, \tag{2.6}$$

where d is the distance from the mobile to the serving base station and d_k is the distance from the mobile to the k^{th} interfering base station. Notice that d_o cancels out, and since we are concerned with co-channel interference, λ is the same for the desired transmission as well as all the interfering transmissions. In addition, if all base stations use the same transmit power P_t and antenna gain G_t, then P_0 is common to all transmissions and the SIR simplifies to:

$$\mathsf{SIR} = \frac{d^{-\alpha}}{\sum_{k=1}^{M} d_k^{-\alpha}} = \frac{1}{\sum_{k=1}^{M} \left(\frac{d}{d_k}\right)^{\alpha}}. \quad (2.7)$$

On the other hand, if the transmit powers are not all the same (but antenna gains are the same), then:

$$\mathsf{SIR} = \frac{P d^{-\alpha}}{\sum_{k=1}^{M} P_k d_k^{-\alpha}} = \frac{1}{\sum_{k=1}^{M} \left(\frac{P_k}{P}\right)\left(\frac{d}{d_k}\right)^{\alpha}}, \quad (2.8)$$

where P is the power of the serving base station and P_k is the power of the k^{th} interfering base station.

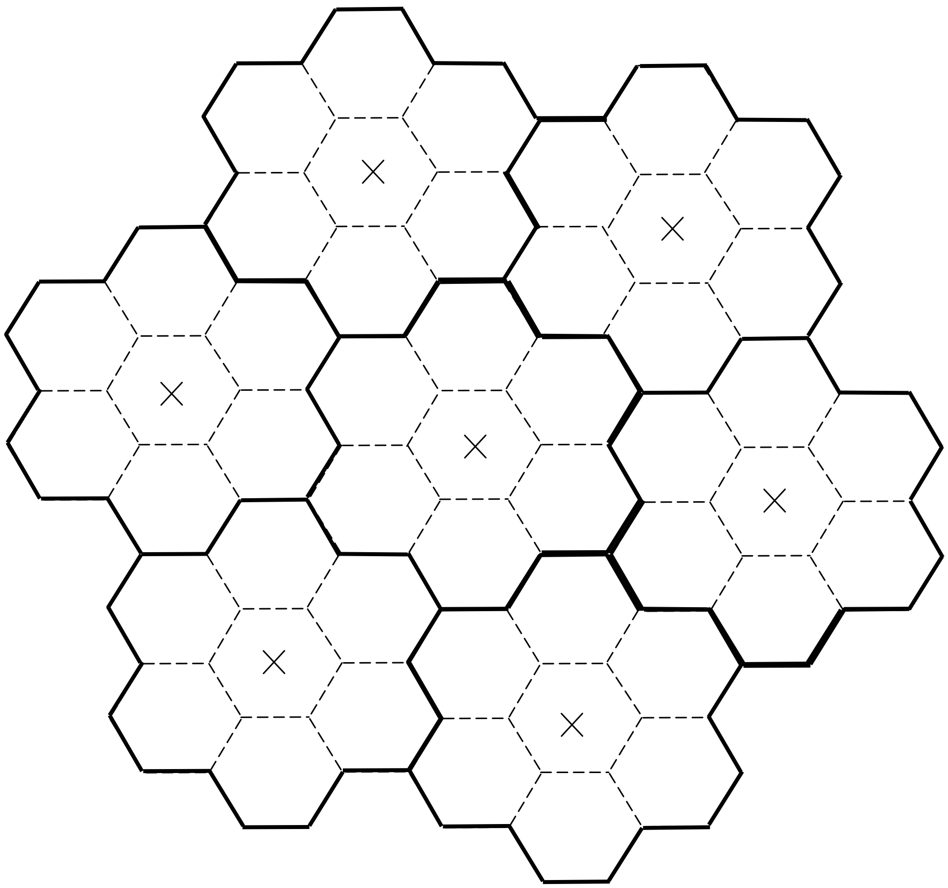
If the gains of the transmit antennas are different, replace each transmit power P_t with its corresponding EIRP $P_t G_t$. More specifically, replace P with PG where G is the absolute (i.e. not dBi) antenna gain of the serving base station, and replace P_k with $P_k G_k$ where G_k is the absolute gain of the k^{th} interfering base station's antenna.

$$\mathsf{SIR} = \frac{1}{\sum_{k=1}^{M} \left(\frac{P_k G_k}{PG}\right)\left(\frac{d}{d_k}\right)^{\alpha}}. \quad (2.9)$$

Since all signals are received by the same antenna, the receive antenna gain G_r is usually the same for all transmissions. An exception would be if a highly directional antenna (e.g. parabolic dish) were used by the receiver, in which case the receive gain could be significantly higher in one direction than another. However, parabolic dishes are typically not used by mobile handsets.

2.5 First-Tier SIR

Since a hexagon has six sides, each cluster will have six neighboring clusters. For a given cell in the center cluster, there will be a corresponding co-channel cell in each of the adjacent cluster. These six co-channel cells located in adjacent clusters constitute the *first tier* of interferers.



Let's get an approximate expression for SIR, which we will follow later with an exact expression. We need to consider the worst case, which is when the mobile is located in one of the six corners of its cell. When it is in the corner, it is furthest from its base station and closest to two of the co-channel cells in the first tier.

Let:

R be the cell *radius*, which is the distance from the center of the cell (the location of the base station) to the corner of the cell (the assumed location of the mobile).

D be the distance between the serving base station and each of the co-channel interfering base stations in the first tier.

$Q = D/R$ be the *co-channel reuse ratio*, which will be a convenient variable to work with.

2.5.1 Rough Approximation

As a rough approximation, determine the downlink SIR assuming that the mobile is distance R from its serving base station and distance D to the first-tier base stations. *Is this a good approximation?*

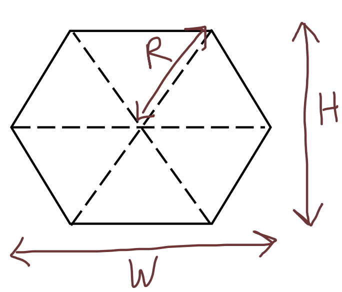
For hexagonal cells [Rappaport (3.4)],

$$Q = \frac{D}{R} = \sqrt{3N} \tag{2.10}$$

Determine the first-tier SIR for each of $N = \{3, 4, 7, 12, 19\}$ using this approximation. Consider path loss coefficients $\alpha = 3$ and $\alpha = 4$. Express your answer in dB.

2.5.2 Exact Expression

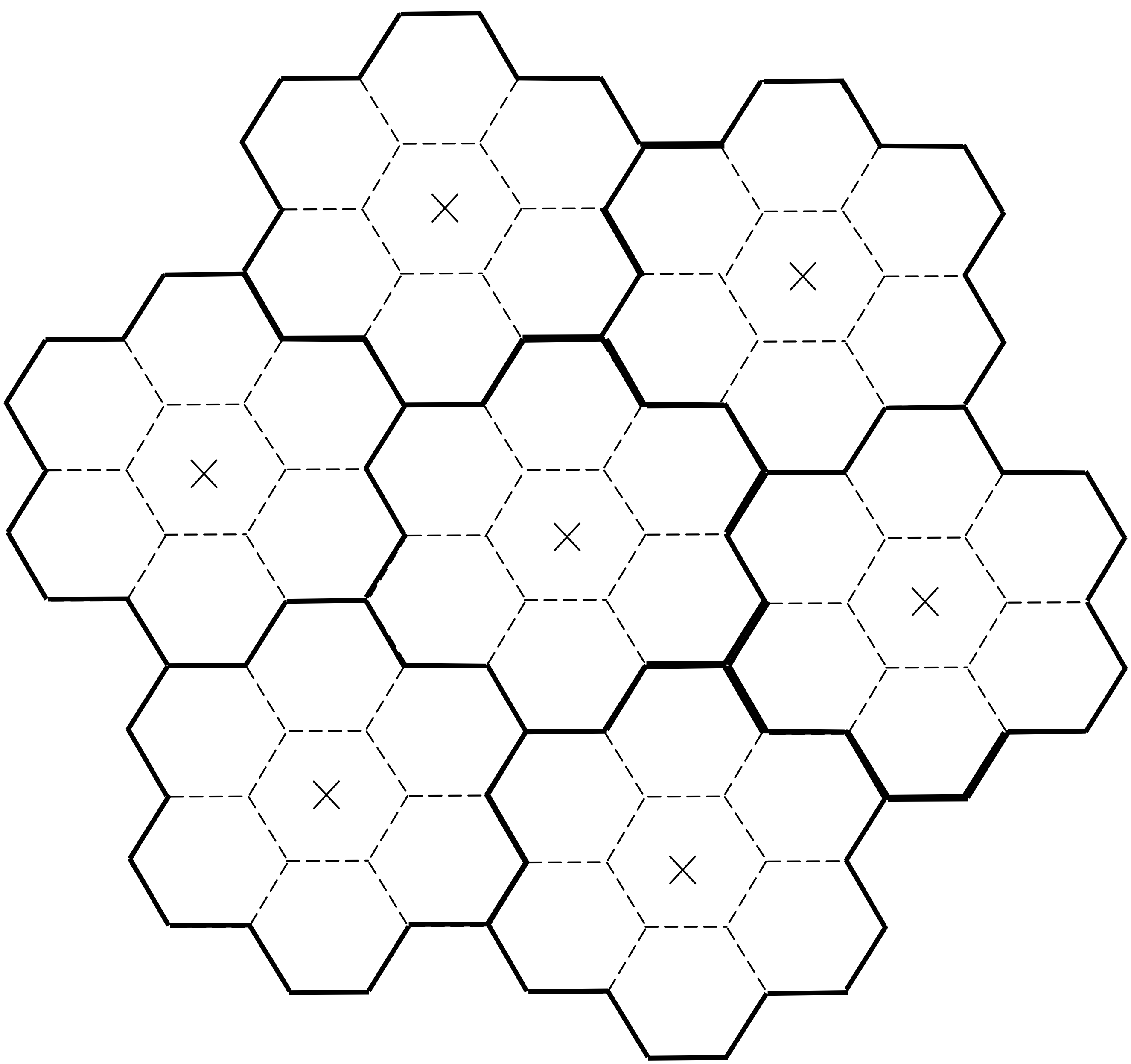
Now let's work out an exact expression for the SIR. To do this, we need to first contemplate the geometry of a hexagon. Consider the following hexagon:



If R is the *radius* of the hexagon, what is its height? Its width?

To derive the exact SIR, place the mobile at a corner of its serving cell, and then use the geometry of the hexagon to compute the distance to each of the six interfering base stations in the first tier. In general, the exact expression will depend on the value of N (number of cells per cluster).

Example: For $N = 7$, determine the *exact* SIR by using the actual geometry:



2.5.3 Comparison

Use the exact expression and the approximation to compute SIR for $N = 7$:

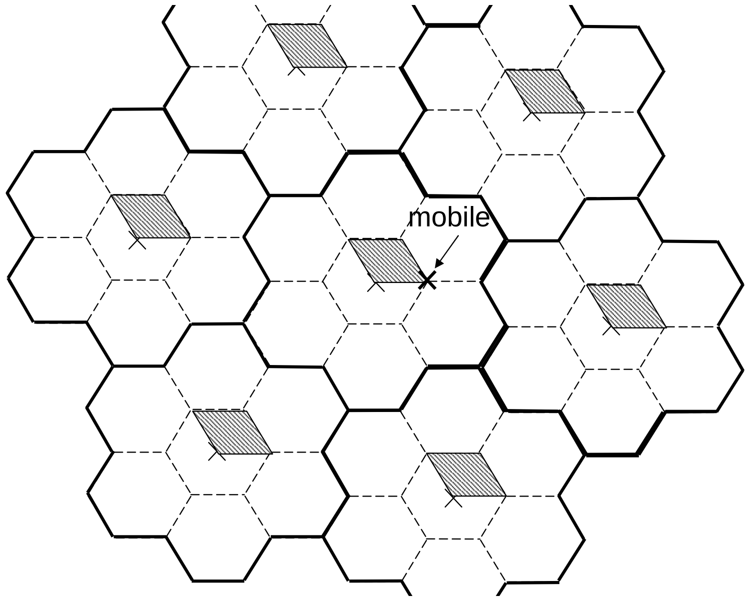
α	Exact	Approx.
2		
3		
4		

2.6 Cell Sectorization

Usually, base station antennas are *omnidirectional*. That is, they radiate power equally well in all directions. However, performance can be improved by using *sectorized* antennas. While an omnidirectional antenna radiates 360 degrees around the cell, a sectorized antenna will radiate only 120 degrees or 60 degrees around a cell. As an example, consider a wireless network with base stations that each use three 120 degree sectorized antennas. For a mobile located in the corner of a cell, how many interferers are there?

Compute the SIR when 120 degree sectorized antennas are used, first using the rough approximation for $\alpha = \{3, 4\}$ and $N = \{3, 4, 7, 12\}$.

Next, compute the exact SIR for the $N = 7$ case and $n = \{3, 4\}$.



What is a disadvantage of using sectorized antennas?

2.7 Cell Splitting

One way to increase coverage in a cellular network is to design it with a higher density of base stations. However, when a network is initially deployed, base stations are usually few and far between. As the network evolves, more and more base stations are added. For instance, a new base station could be placed between two existing base stations. When new base stations are added, the existing cells are split into smaller cells. For this reason, the process of overlaying additional base stations is sometimes called *cell splitting*. Note that when new base stations are added, they typically operate at a lower transmit power than previous base stations, to assure that the footprint of the new cells are smaller.

Chapter 3

Probability and Its Application to Wireless Networks

3.1 Introduction

Probability is essential to the understanding of wireless systems. This is because there is a lot going on in the system that can be described by random variables and random processes. At a high level, the *location* of each mobile users is random, as is each user's *traffic activity* (how often they make a call, and for how long; how often they transmit a packet, and the size of the packet). The characteristics of the channel are also characterized by random quantities; in particular *noise* and *fading* are modeled as random variables. This chapter provides a review of probability theory, with an emphasis on how probability is applied to the analysis of wireless networks.

3.2 Continuous Random Variables

3.2.1 Definitions

- **Random Experiment:** An experiment with an unpredictable *outcome*, even if repeated under the same conditions.

- **Outcome:** The result of a random experiment. Outcomes are mutually exclusive; i.e., you can't get heads *and* tails on the same coin toss.

- **Random variable:** A *number* that describes the outcome of a random experiment. Random variables can be *discrete* or *continuous*. They are typically represented by capital letters, such as X or Y.

3.2.2 Probability Density Function

The random variable X may be described by its *probability density function* (pdf) $f_X(x)$. The pdf satisfies the property that integrating it over the range $(a, b]$ gives the probability that X lies in that range. We may write this property out as:

$$P[a < X \leq b] \quad = \quad \int_{a^+}^{b^+} f_X(x)dx. \tag{3.1}$$

If there is a delta function[1] located at a, then it is not included in the integral given by (3.1), but if there is a delta function located at b it is included. The pdf is a non-negative function, i.e. $f_X(x) \geq 0$, $\forall x$, and it integrates to one:

$$\int_{-\infty}^{\infty} f_X(x)dx \quad = \quad 1$$

Note that the Rappaport book uses the notation $p_X(x)$ for pdf's instead of the $f_X(x)$ used in these notes.

Uniform Random Variables

One of the most basic **continuous random variable** is the *uniform* random variable. A uniform random variable takes on values over the continuous range (a, b) with equal probability. The pdf is as follows (see table G.6 in the book):

$$f_X(x) \quad = \quad \begin{cases} c & \text{for } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

where $b > a$ and c is a some constant.
Exercise: Determine the value of c such that this is a valid pdf:

Exercise: Suppose that $a = 0$ and $b = 2$. Determine the following probabilities:

1. $P[1/2 < X \leq 3/2]$

2. $P[X \leq 1]$

3. $P\left[|X| \leq 1/2\right]$

[1] This detail is not important now, but it will be later. In particular, as will be seen shortly, delta functions arise when random variables are *discrete*.

Exponential Random Variables

The pdf of an *exponential* random variable X is:

$$f_X(x) = ce^{-\lambda x}u(x), \tag{3.2}$$

where c is a constant and $u(x)$ is the unit step function,

$$u(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{3.3}$$

Exercise: Determine the value of c such that this is a valid pdf:

Exercise: Suppose that $\lambda = 2$, determine:

1. $P[1/2 < X \leq 3/2]$

2. Determine $P[X \leq 1]$

3. Determine $P[|X| \leq 1/2]$

Applications:

1. Suppose that a signal is received over a so-called *Rayleigh* fading channel[2] with an *average* SNR of Γ. Let X be a random variable representing the *instantaneous* SNR of the signal (as an absolute ratio; i.e. *not* in dB). It follows that X is exponential with parameter $\lambda = 1/\Gamma$.

2. The *time between phone calls* by a given subscriber is often modeled as being exponential, as is the *duration* of each call.

Exponential random variables are said to be *memoryless.* Loosely stated, this means that the time from right now until your next call is independent from how long it has been since your last call.

[2] In a fading channel, the amplitude of the transmitted signal is multiplied by a random gain. In a Rayleigh fading channel, the amplitude gain is a Rayleigh random variable. The pdf for a Rayleigh variable is given by (5.49) on page 210 of the text. The received signal power is the square of the amplitude, and the square of a Rayleigh variable is exponential.

Exercise: Suppose that a signal is received over a Rayleigh fading channel with an *average* SNR of -3 dB. Suppose that the threshold for reliable communications is $\beta = 0$ dB; i.e., reliable communications is possible as long as the received signal power is above β. Determine the probability that the received signal power is above the threshold and the fraction of time it is below the threshold

3.2.3 Moments

The *mean*, or first-moment, of a random variable is given defined by:

$$\begin{aligned} m_X &= E[X] && (3.4) \\ &= \int_{-\infty}^{\infty} x f_X(x) dx && (3.5) \end{aligned}$$

Exercise: Find the mean of an exponential random variable.

More generally, the expectation of *any* function $g(X)$ of X can be found as:

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx \qquad (3.6)$$

A key example is the *n-th moment*, which is $E[x^n]$, i.e. the expected value when the function $g(X) = X^n$. Another important expectation function is $g(x) = (x - m_X)^n$, which is the *n-th central moment*. The second central moment is also called the *variance*:

$$\begin{aligned} \sigma_X^2 &= E\left[(x - m_X)^2\right] \\ &= E\left[X^2\right] - m_X^2, \end{aligned}$$

where the last expression follows from linearity of the expectation operation.

Exercise: Find the variance of the exponential random variable.

3.3 Diversity (Section 7.10)

Fading can have a negative impact on performance. The performance can be improved by observing the signal over multiple, diverse fading channels. The signal could be observed at different locations, different frequencies, and different times. Suppose we observe the signal at two different locations and each signal undergoes independent fading. Suppose that the signal amplitude must be over some threshold. With diversity, the signal can be reliably received as long as it is above the threshold at either location.

3.3.1 Types of Diversity

There are several types of diversity

1. **Space:** Use two (or more) antennas at different locations, and then combine the received signals.

2. **Frequency:** Transmit the same signal at two different frequencies.

3. **Time:** Repeat the signal, i.e. transmit at two different times.

4. **Polarization:** Like light, electromagnetic waves may be *vertically* or *horizontally* polarized, or they may be CW and CCW circularly polarized. By using polarization, two independent channels can be realized.

3.3.2 Performance with Diversity

Suppose we have L branches of diversity. Let γ_i be the instantaneous SNR of the i^{th} diversity branch. With diversity, the overall SNR is the sum of the individual SNRs[3],

$$\gamma = \sum_{i=1}^{L} \gamma_i \tag{3.7}$$

If each γ_i is exponential with mean $E[\gamma_i] = \Gamma$ and if all the γ_i are independent, then γ is an *m-Erlang* random variable with pdf [equation (7.68) in Rappaport [4]]:

$$f(\gamma) = \frac{\gamma^{L-1} e^{-\gamma/\Gamma}}{\Gamma^L (L-1)!} u(\gamma). \tag{3.8}$$

where $u(\gamma)$ is the unit-step function.

Exercise: Suppose a signal with a SNR of -3 dB is received using $L = 3$ diverse antennas and combined (using MRC combining). Determine $P[\gamma > \beta]$, where γ is given by (3.7) and $\beta = 1$ (or 0 dB) is the *outage threshold*.

3.4 Discrete Random Variables

Discrete random variables take on values from a finite set or a countably infinite set of values. Examples include the number of heads in n tosses, and the number of tosses until the first occurrence of heads. While discrete random variables can be described in terms of their pdf, the pdf requires the use of a delta function. Instead, discrete variables are usually characterized by their *probability mass function (pmf)*, which is

$$p_X[x_k] = P[X = x_k]$$

for all possible x_k that the variable can assume. Like the pdf, the pmf is a non-negative function. It must sum to unity, i.e $\sum_{k=0}^{N} p_X[x_k] = 1$.

[3]This requires what is known as *maximal ratio combining* (MRC), which is a way to optimally combine *all* the received signals. The performance with *selection diversity* and *equal gain combining* will be worse than MRC.

[4]Rappaport uses M to indicate the number of diversity branches, but I prefer L to distinguish this from the number of signals in the constellation. Also, Rappaport says that γ is "Chi-square with 2M degrees of freedom" which is the same thing as "m-Erlang".

Exercise: Determine the pmf of a fair coin toss and the pmf of a fair die roll.

Poisson Random Variables

Consider the following random experiment. Assume that you have a timer that expires after a random amount of time, and this random amount of time is an exponential random variable with parameter $\lambda > 0$. The mean length of the timer is therefore $1/\lambda$ units (where a unit could be a second, minute, hour, etc). At time zero, set the timer and let it run. Once it expires, reset it and let it run again. Repeat this process until a specified amount of time has passed (one unit). Let X be the number of times the timer goes off during a time unit. The discrete random variable X is *Poisson* and has pmf:

$$p_X[k] = \frac{\lambda^k e^{-\lambda}}{k!} \quad k = 0, 1, 2, ... \tag{3.9}$$

There is no upper bound on how high k can be, but the average number of events (timer expirations) is equal to λ.

Poisson random variables are useful for modelling network traffic. If the time between a customer's cell phone calls is exponential, then the number of calls he/she makes per day is Poisson.

Exercise: On average, Mary makes two calls on her cell phone per day. Her calling plan allows her to make up to 20 calls per week until she gets penalized with an overage charge. What is the probability that on a given week she must pay an overage charge? Assume that the time between her calls is exponential.

3.5 Trunking Theory (Section 3.6)

At this point, we know how to determine the number of channels assigned to a cell. If there are an equal number of channels per cell, then the number of channels per cell is found by simply dividing the total number of channels by the cluster size N. So the next question is, how many subscribers can be enrolled in a wireless network? Continuing our example, the most conservative choice would guarantee that there is never more than 90 users in a particular cell. Because everyone in the cell has a channel, they are 100% certain that they will be able to connect to the network. But the only way to provide such a guarantee is to enroll only 90 paying subscribers. These users will hardly ever be in the same cell, and even if they are in the same cell, they are not likely to be using the network at the exact same time.

Instead of having such a rigid service guarantee, wireless providers try to guarantee that a user will be able to obtain an open channel a certain percentage of the time. For instance, a typical system might be engineered such that a user has no more than a 5% chance of having a *blocked call* during peak calling hours. Understanding how many subscribers can be allocated to a cell with a certain number of channels is an exercise in probability. One must know something about the calling statistics of the users (frequency of calls and their duration), and can use this knowledge along with sound probability theory to be able to provide the necessary service guarantees. The concept called *trunking* is what allows a large number of users to share a relatively small pool of channels by allocating them on an as-needed basis.

In a trunked system, there will always be more subscribers than available channels. Because of this, there will always be a possibility that a call is either *blocked* (a new call attempt can't get a channel) or *dropped* (an existing call can't handoff to a new cell). The best that can be done under these circumstances is to minimize the likelihood that these events occur by guaranteeing a certain *grade of service* (GoS), also called *quality of service* (QoS). The GoS may be quantified in terms of a probability, namely the probability that a call is blocked.

3.5.1 Definitions Related to Traffic Intensity

The following definitions are required to understand Trunking theory (see table 3.3 in text):

- **Request rate:** Assume that each subscriber makes λ calls per hour, on average. The variable λ is called the *per-user call request rate* or simply *request rate*.

- **Holding time:** Assume that the duration of each call is H hours, on average. This is called the *holding time*.

- **Traffic intensity:** The per user *traffic intensity* is given by $A_u = \lambda H$. This is the number of minutes per hour that a user is actually generating traffic, on average.

- **Erlang:** Although strictly speaking, intensity is a unitless quantity, it is customary to ascribe a unit to them. The basic unit of traffic intensity is called an *Erlang*.

- **Load:** If there are U users in the cell, then the offered load in the cell is $A = UA_u$ Erlangs. This is the total intensity offered to the cell.

Note that the above definitions assumed time units of hours. However, the actual time units do not matter as long as they are consistent. So, for instance, you could substitute minutes instead of hours in all of the above definitions.

3.5.2 Erlang-B Formula

It is assumed that the time between calls is exponential, and therefore the number of calls that arrive per hour is Poisson. Furthermore, it is assumed that the call duration is exponentially distributed. Under these assumptions, the probability that a call is blocked is given by (equation 3.16, which is derived in Appendix A):

$$P[\text{Blocking}] \quad = \quad \frac{A^C}{C! \sum_{k=0}^{C} \frac{A^k}{k!}} \tag{3.10}$$

where C is the number of channels and A is the offered load. This function is plotted in Fig. 3.6 of the text and tabulated in Table 3.4.

Example: Suppose that each user makes an average of $\lambda = 3$ calls per hour, and that each call lasts for an average of $H = 5$ minutes. There are $C = 10$ channels per cell, and it is assumed that the number of subscribers is much larger than C. How many users can be supported such that the call blocking probability does not exceed 1 percent?

Repeat the example for the case of $C = 20$ channels. Comment on how the number of subscribers is related to the number of channels.