Example problem: Conservation of energy

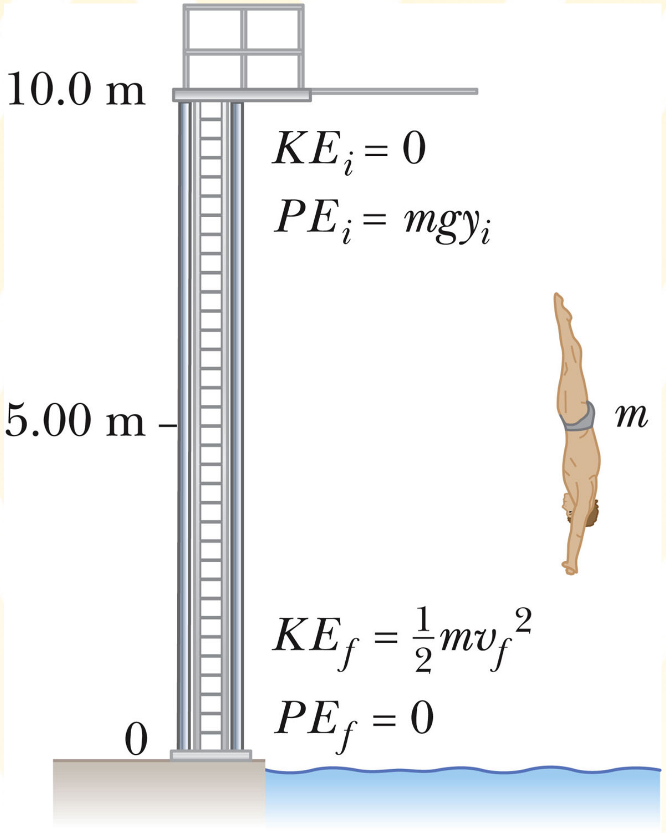
A diver of mass m drops from a board 10 m above the water's surface. Neglect air resistance.

A. What is his speed 5 m above the water surface?

B. Find his speed as he hits the water.

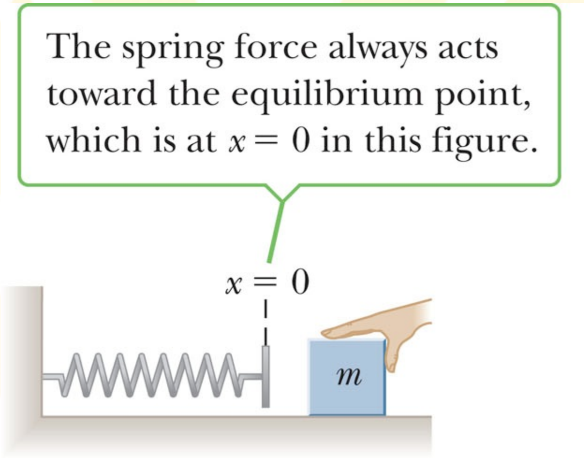
Conservation of energy:

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$



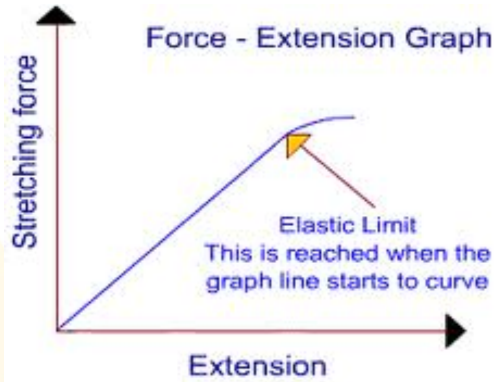
Springs: Hooke's law

The force a spring provides to an attached object is proportional to the amount that the spring is stretched or compressed from its equilibrium position. The force pulls/pushes the object back towards the equilibrium position (minus sign).



Hooke' law: $F_s = -kx$

k is the spring constant (unit: N/m). It is small for flexible spring and large for stiff springs.



Hooke's law is not always valid:

If you stretch a spring too much (elastic limit), the restoring force will no longer be linearly proportional to the extension, x.

Spring potential energy

Hooke's law: $F_s = -kx$

The spring force is conservative.

Thus, a potential energy - the spring potential energy, PE_s - can be associated with it.

In order to calculate PE_s we have to determine the work done by the spring:

$$W = F\Delta x$$

This equation is only valid for constant forces, but F_s depends on x, i.e. is not constant.

Therefore, we have to calculate the *average* force, when stretching the spring from its equilibrium position to x. This force can be treated as effectively constant.

$$\bar{F}_s = \frac{F_0 + F_1}{2} = \frac{0 - kx}{2} = -\frac{kx}{2}$$

With $\Delta x = x$ this yields: $W_s = -\frac{1}{2}kx^2 \quad \rightarrow \quad PE_s = \frac{1}{2}kx^2$

Work-Energy Theorem

Including the spring potential energy the Work-Energy Theorem is:

$$W_{nc} = (KE_f - KE_i) + (PE_{gf} - PE_{gi}) + (PE_{sf} - PE_{si})$$

Change of kinetic energy

Change of gravitational potential energy

Change of spring potential energy

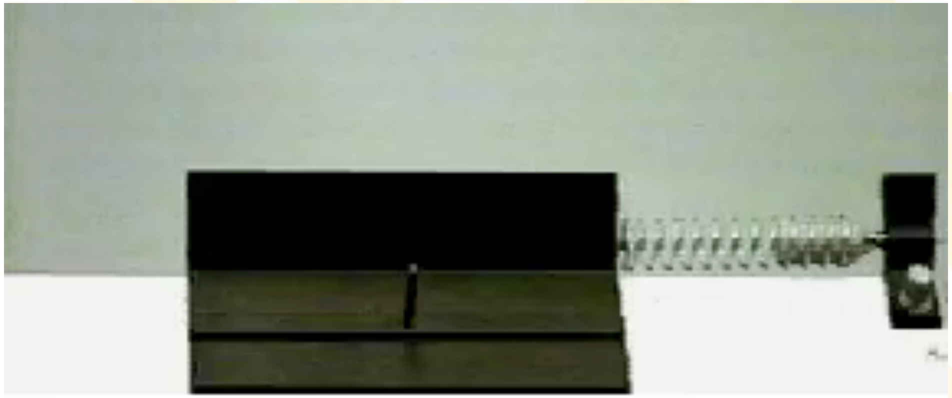
If non-conservative forces, e.g. friction, can be neglected, the mechanical energy is conserved:

$$0 = (KE_f - KE_i) + (PE_{gf} - PE_{gi}) + (PE_{sf} - PE_{si})$$

$$\rightarrow (KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$

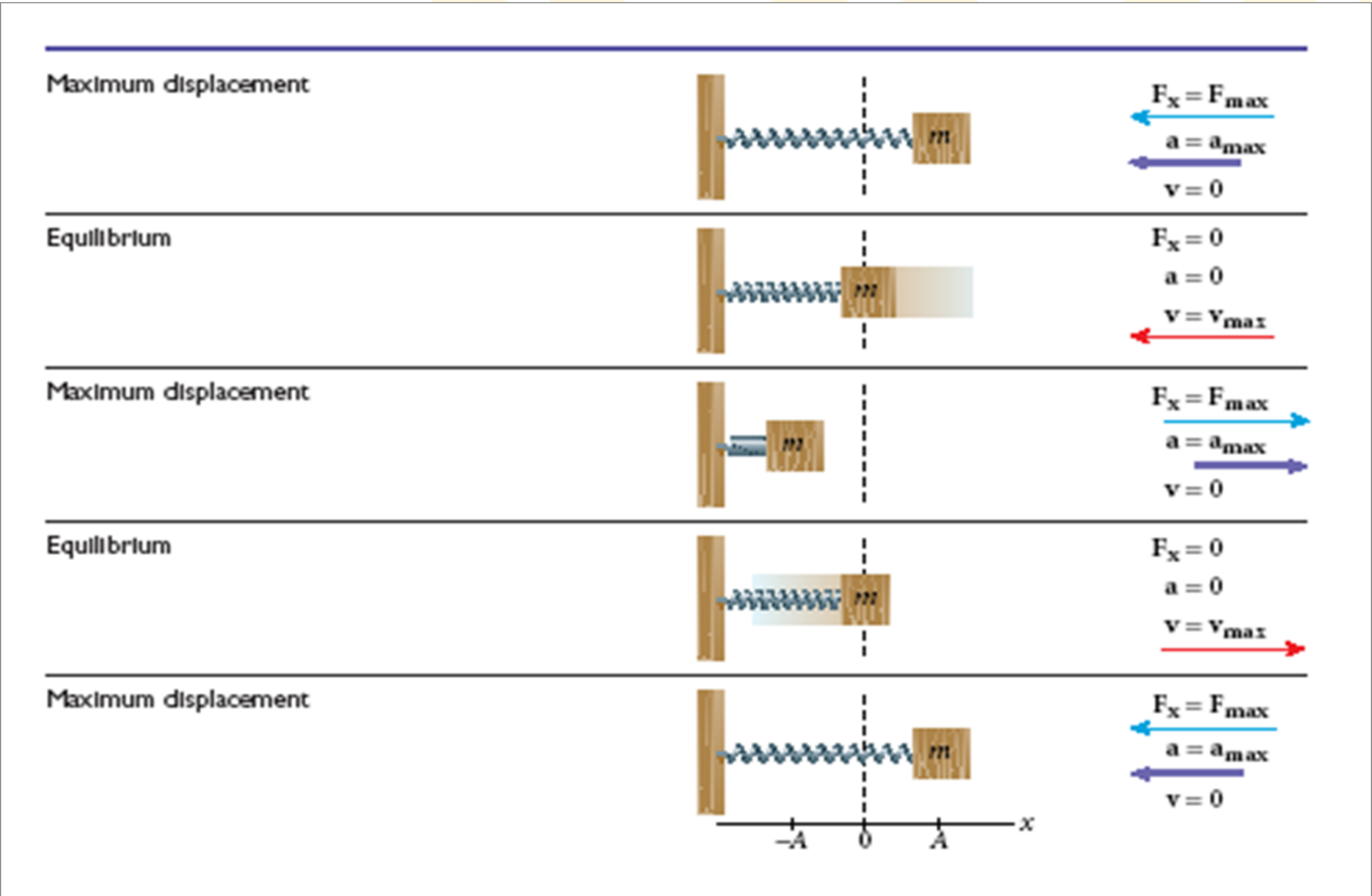
Generally, the total energy of a given system is always conserved. Energy is only transformed from one form to another. However, if non-conservative forces matter, energy will be transformed to e.g. heat, which cannot be easily transformed back to kinetic or potential energy.

Illustration - Hooke's law



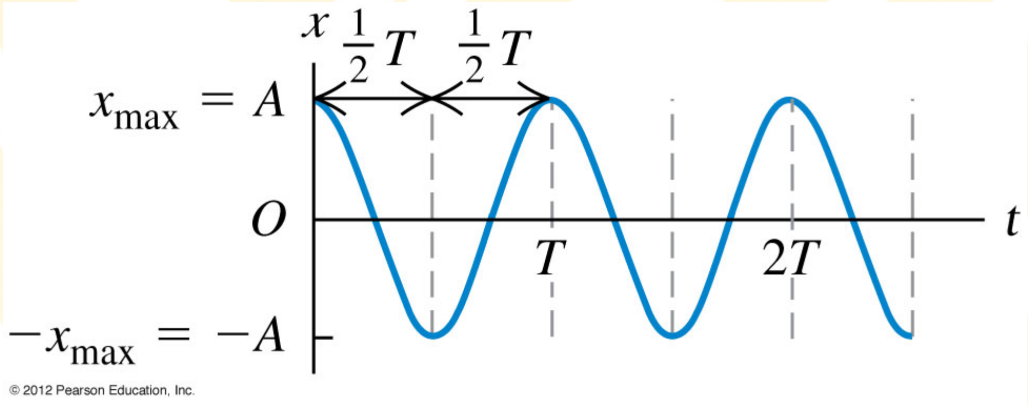
$$F_s = -kx$$

One oscillation cycle



Clicker question

This is an x-t diagram for an object attached to an oscillating spring. Friction is neglected, i.e. there are no non-conservative forces.



At which of the following times does the object have the most negative velocity, v_x?

A. t = T/4

B. t = T/2

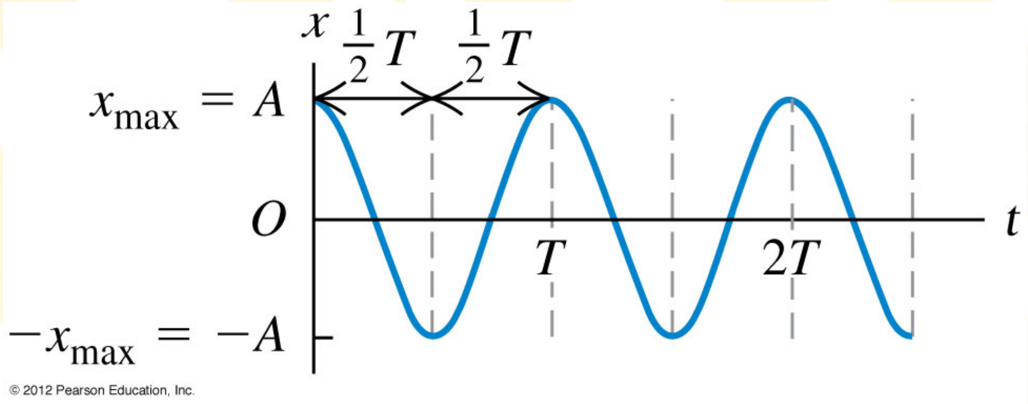
C. t = 3/4 T

D. t = T

Clicker question

This is an x-t diagram for an object attached to an oscillating spring. Friction is neglected, i.e. there are no non-conservative forces.

$$PE_s = \frac{1}{2}kx^2$$



At which of the following times is the potential energy of the spring the greatest?

A. t = T/8

B. t = T/4

C. t = 3/8 T

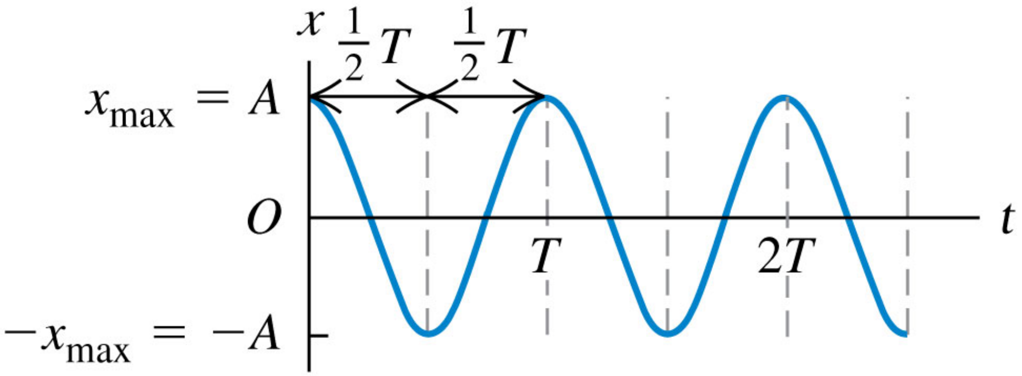
D. t = T/2

E. More than one of the above.

Clicker question

This is an x-t diagram for an object attached to an oscillating spring. Friction is neglected, i.e. there are no non-conservative forces.

$$KE = \frac{1}{2}mv^2$$



At which of the following times is the kinetic energy of the object the greatest?

A. t = T/8

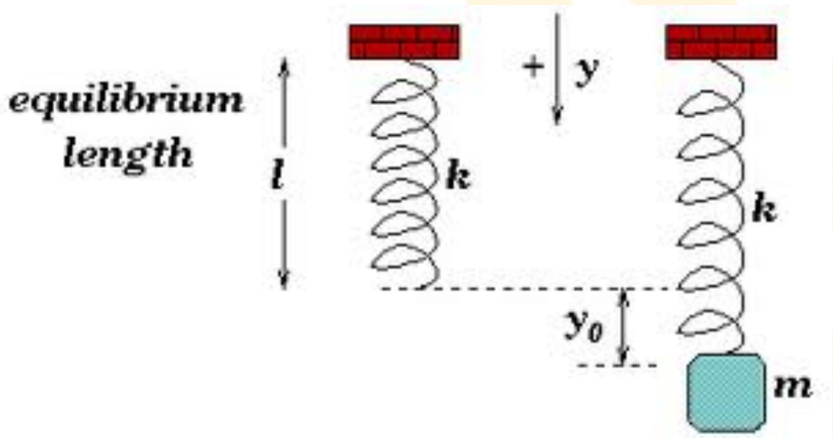
B. t = T/4

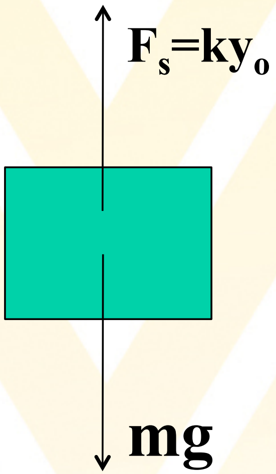
C. t = 3/8 T

D. t = T/2

E. More than one of the above.

Example problem: Vertical Springs





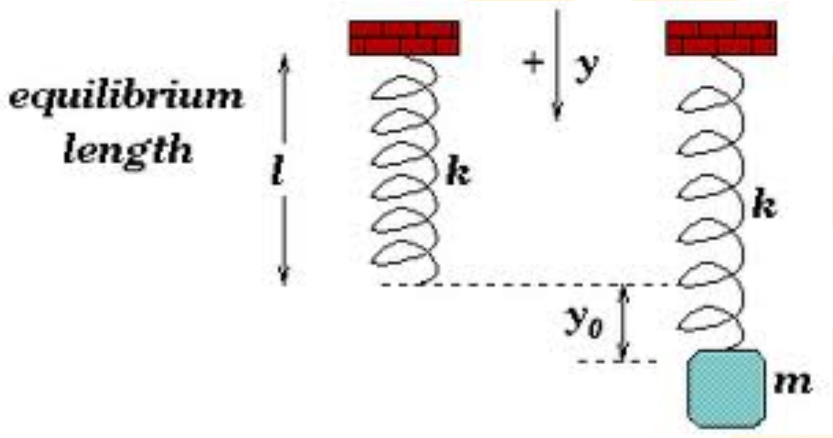
Free Body Diagram

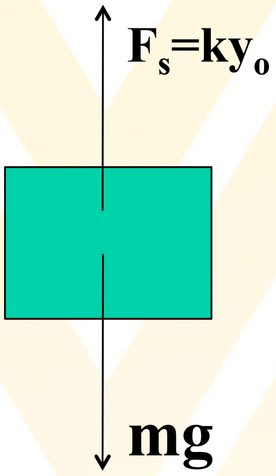
When a 2.5 kg object is hung vertically on a certain light spring, the spring stretches to a distance y_o. What **force** does the spring apply to the object?

If the string stretches 2.76 cm, what is the force constant of the spring?

What is the force if you stretch it 8 cm?

Example problem: Vertical Springs



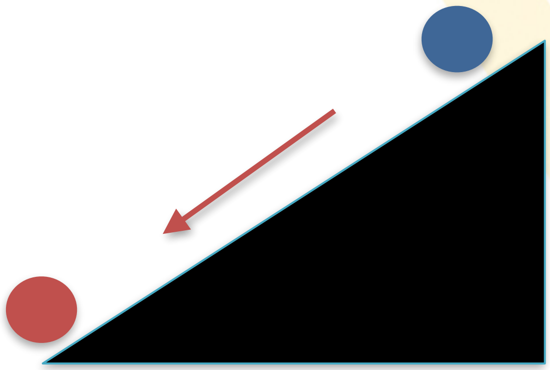


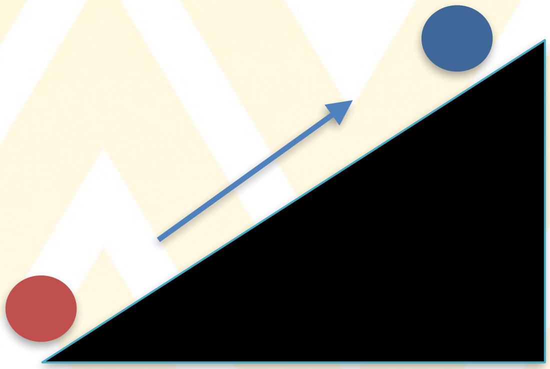
Free Body Diagram

A 2.5 kg object is hung vertically on a certain light spring with spring constant k=888N/m.

How much work must an external agent do to stretch the same spring 8.00 cm from its unstretched position?

Irreversible processes





A ball easily rolls down a hill and heats up via friction while going downhill.

Its energy is transformed from potential to kinetic energy and finally to heat.

A hot ball at rest at the bottom of a hill does not start moving uphill, while cooling down.

Its energy is not transformed from heat to kinetic energy and finally to potential energy.

No energy is lost. It just cannot be transformed back from heat to potential energy.

Roller coasters

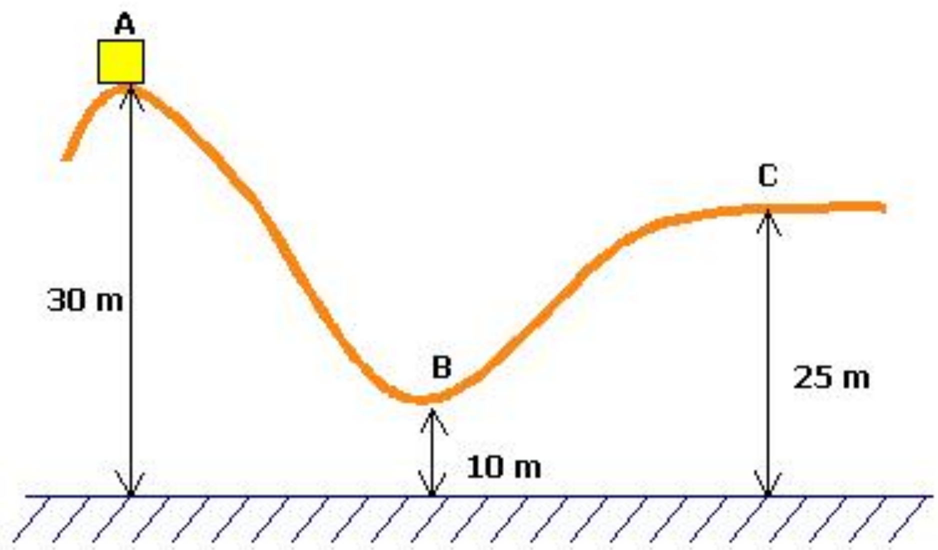
Riding a roller coaster is a typical application of energy conversation (neglecting friction).

Potential energy is transformed into kinetic energy and vice versa.

$$KE_i + PE_{gi} = KE_f + PE_{gf}$$

http://www.pbslearningmedia.org/resource/hew06.sci.phys.maf.rollercoaster/energy-in-a-roller-coaster-ride/

Example problem: Rollercoaster



A 1000 kg roller coaster car moves from point A to B and to C. Its initial velocity is 0 m/s. Neglect all non-conservative forces, e.g. friction.

A. What is its potential energy at points A, B, C?
B. What is its speed and kinetic energy at A, B, C?
C. What is its total energy at A, B, C?

Clicker question

An athlete jumping vertically on a trampoline leaves the surface with a velocity of 8.5 m/s upward.

What maximum height does he reach?

A. 13 m

B. 2.3 m

C. 3.7 m

D. 0.27 m

E. The answer cannot be determined because the mass of the athlete is not given.

Power

Power is the rate at which energy is transformed from one type to another:

Average power: $\bar{P} = \frac{W}{\Delta t}$ Power is a scalar quantity.

Unit: $1W = 1J/s = 1kg \cdot m/s^2 \cdot m \cdot s^{-1} = 1kg \cdot m^2/s^3$

Alternative expression for power:

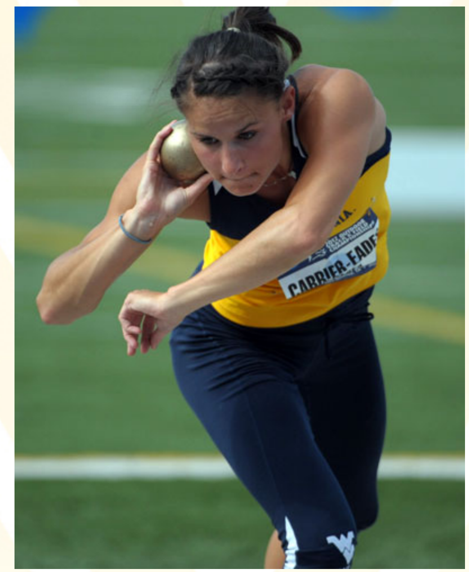
$W = F \cdot \Delta x$ if F is parallel to Δx.

$\rightarrow \quad \bar{P} = F \cdot \frac{\Delta x}{\Delta t} = F\bar{v}$

Example problem: Power

A shot-putter accelerates a 7.3-kg shot from rest to 14 m/s.

If this motion takes 2.0 s, what average power was produced?



Clicker question

Weightlifter A lifts a 100-kg weight to a height of 2.5 m above the ground in 1.0 s. Weightlifter B lifts a 75-kg weight to a height 2.5 m above the ground in 0.5 s. Which of the two weightlifters uses more power to lift the weights?

A. A

B. B

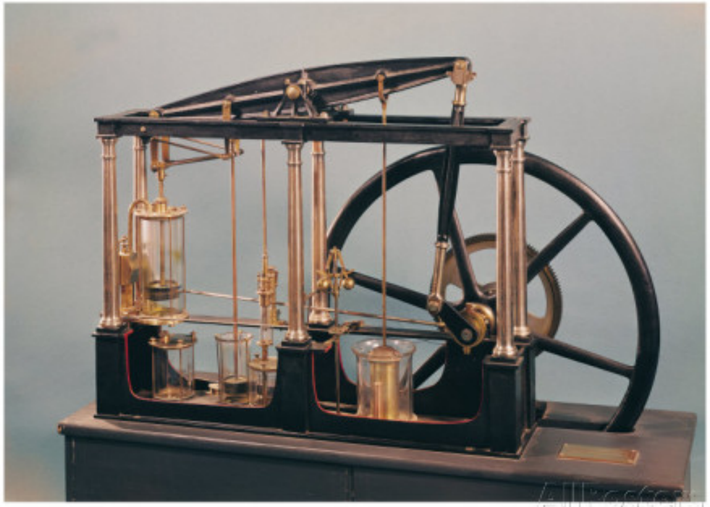
C. They both use the same amount of power.

D. Impossible to determine.

Horsepower - An alternative unit of Power

When James Watt invented the steam engine, he needed a large power unit to rate the output power of his new invention. He chose a standard horse.

$$1\ \text{hp} = 746\ \text{W}$$



Kilowatt-hour: A unit of energy

When you read your electricity bill, it will tell you how many kilowatt-hours (KWHRS, kWh) you consumed.

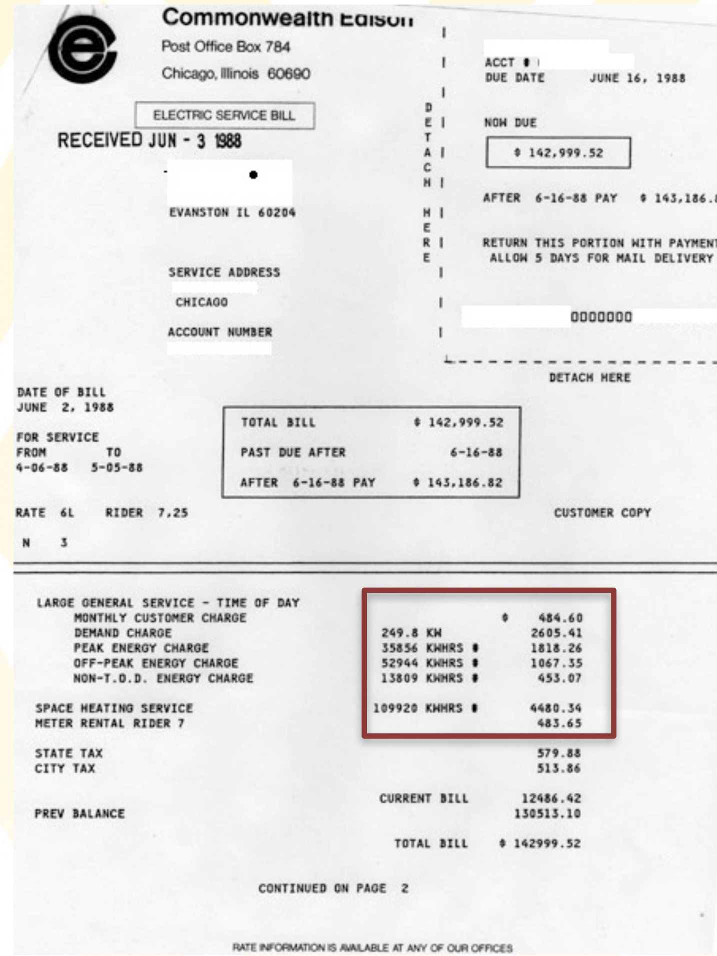
What does that mean?

1 kWh is the energy consumed in 1 hour at the constant rate of 1 W.

$$1\,kWh = (10^3\,W)\cdot(3600s)$$

$$= (10^3\,J/s)\cdot(3600s) = 3.6\cdot 10^6\,J$$

kWh is a unit of energy, not power!


Commonwealth Edison
Post Office Box 784
Chicago, Illinois 60690

ELECTRIC SERVICE BILL

RECEIVED JUN - 3 1988

EVANSTON IL 60204

SERVICE ADDRESS
CHICAGO
ACCOUNT NUMBER

DETACH HERE

ACCT
DUE DATE JUNE 16, 1988
NOW DUE
$ 142,999.52
AFTER 6-16-88 PAY $ 143,186.82
RETURN THIS PORTION WITH PAYMENT
ALLOW 5 DAYS FOR MAIL DELIVERY
0000000

DATE OF BILL
JUNE 2, 1988
FOR SERVICE
FROM TO
4-06-88 5-05-88

TOTAL BILL	$ 142,999.52
PAST DUE AFTER	6-16-88
AFTER 6-16-88 PAY	$ 143,186.82

RATE 6L RIDER 7,25
N 3

CUSTOMER COPY

LARGE GENERAL SERVICE - TIME OF DAY		
MONTHLY CUSTOMER CHARGE		$ 484.60
DEMAND CHARGE	249.8 KW	2605.41
PEAK ENERGY CHARGE	35856 KWHRS @	1818.26
OFF-PEAK ENERGY CHARGE	52944 KWHRS @	1067.35
NON-T.O.D. ENERGY CHARGE	13809 KWHRS @	453.07
SPACE HEATING SERVICE	109920 KWHRS @	4480.34
METER RENTAL RIDER 7		483.65
STATE TAX		579.88
CITY TAX		513.86
	CURRENT BILL	12486.42
PREV BALANCE		130513.10
	TOTAL BILL	$ 142999.52

CONTINUED ON PAGE 2

RATE INFORMATION IS AVAILABLE AT ANY OF OUR OFFICES

Get a feeling for the value of energy

How many hamsters running on wheels would it take to provide enough power for a house?

Let's assume a hamster weighing 50 grams can run up a 30-degree slope at 2 m/s.

120 hamsters to keep a 60-watt bulb lit

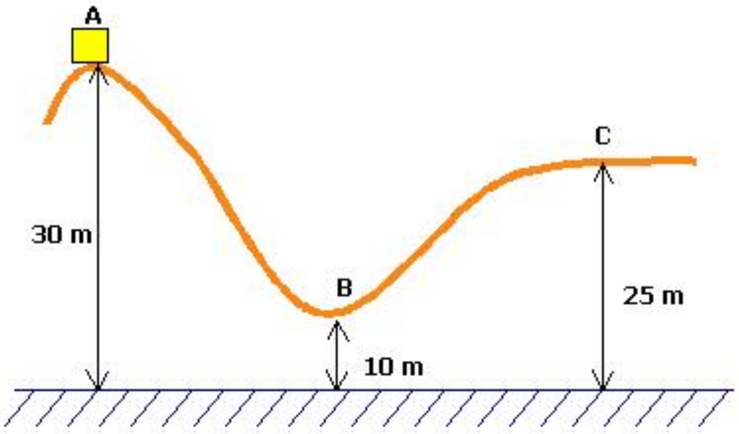
Average hamster probably spends ~5 % of its life running, so we would need 2,400 hamsters just for lightbulb

The average household needs a constant power consumption of about ~2.5 kW. Each house would need ~100,000 hamsters.

Summary

- If non-conservative forces can be neglected, the sum of potential and kinetic energy will be conserved.

- $$KE_i + PE_{gi} = KE_f + PE_{gf}$$



- Typical problems that can be solved by energy conservation: Rollercoasters, Pendulums, Jumping.

- Power is defined as the rate of energy transfer with time (scalar quantity):

$$\bar{P} = \frac{W}{\Delta t} = F\bar{v}$$ Unit: 1 W = 1 J/s

- 1 horsepower is an alternative unit for power (non SI-unit): 1 hp = 746 W

- 1 kilowatt-hour is a unit for energy. It is the energy consumed within 1 hour at the rate of 1 W:

$$1\,kWh = 3.6 \cdot 10^6\,J$$

Example problem: Horsepower

An advertisement claims that a certain 1200 kg car can accelerate from rest to a speed of 25 m/s in a time of 8.0 s. What power (in units of horsepower) must the motor produce in order to cause this acceleration?

Ignore losses due to friction. (1 hp=746 W)

Example problem: Cost of Energy

What is the cost of forgetting to turn off your bathroom light for the day?

Let's say you have three 75W bulbs in this light and you are gone for 12 hours.

Electricity costs about $0.10 per kWh.