Example problem: Wheel of Fortune

Amy is on the Wheel of Fortune and has to spin the wheel. She gives the wheel an initial angular speed of 3.40 rad/s. It then rotates through 1.25 revolutions and comes to rest on BANKRUPT.

(a) Find the wheel's angular acceleration, assuming it is constant.

(b) How long does it take for the wheel to stop?

$$\theta(t) = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega(t) = \omega_i + \alpha t$$

Clicker question

A student sees the following question on an exam:

A flywheel with mass 120 kg, and radius 0.6 m, starting at rest, has an angular acceleration of 0.1 rad/s^2. How many revolutions has the wheel undergone after 10 s? Which formula should the student use to answer the question?

A. $\omega = \omega_o + \alpha t$

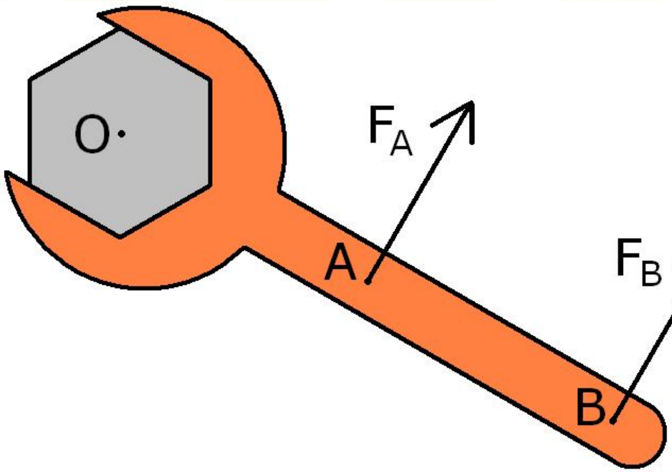
B. $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$

C. $\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$

Today's lecture

Rotational Equilibrium and Dynamics:

- Torque
- Center of Gravity
- Equilibrium



Torque

Until now we described an object's rotational motion. Now we will look at the reason, why objects start to rotate.

Again, this will be done in close analogy to linear motion, where we found forces to be the reason for linear acceleration.

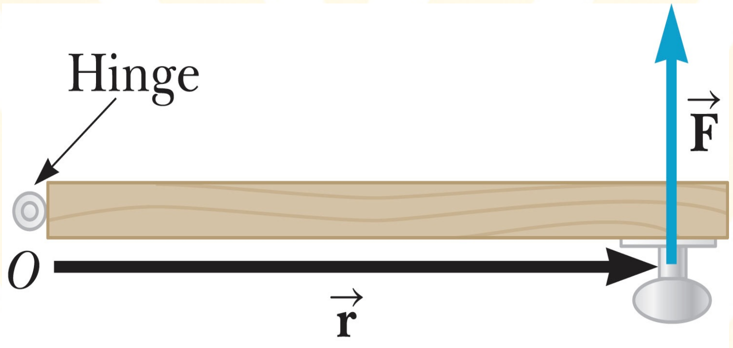
We have treated all objects as points until now, i.e. the point of application of a force was not considered. For rotational motion the position of application of a force, i.e. the distance from the center of rotation, is essential.

Definition of torque (magnitude):

$$\tau = rF$$

SI-unit: Nm (Newton · meter)

This equation will only be valid, if $\vec{r} \perp \vec{F}$



A net torque causes an object to rotate around an axis, O, at a radius r, i.e. torques cause angular acceleration.

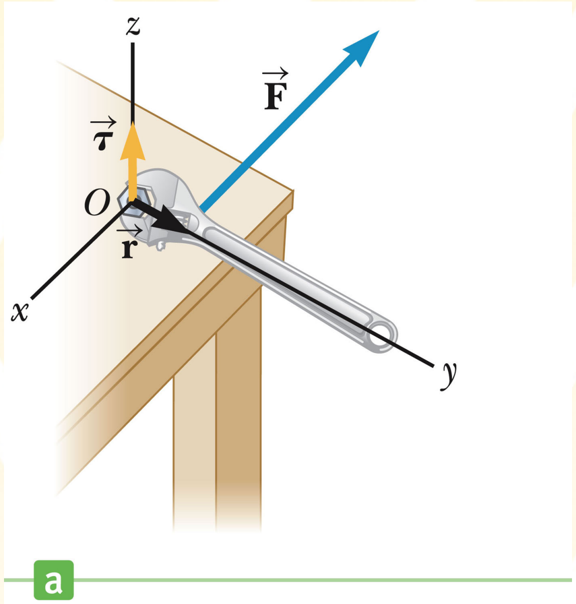
Torque is a vector

Torque is a vector. Its direction is perpendicular to the plane determined by the lever arm and the applied force vectors.

The direction of the torque vector can be determined based on the right-hand-rule:

1. Point your thumb into the direction of r.
2. Point your index finger into the direction of F.

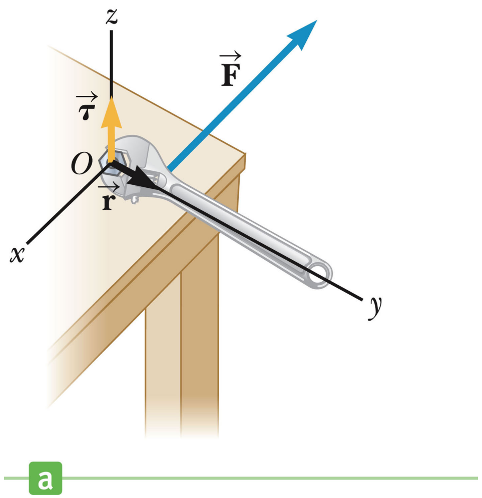
→ Your middle finger will point into the direction of the torque vector (all fingers perpendicular to each other).

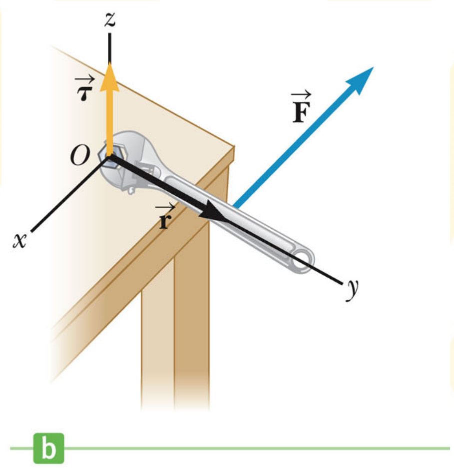


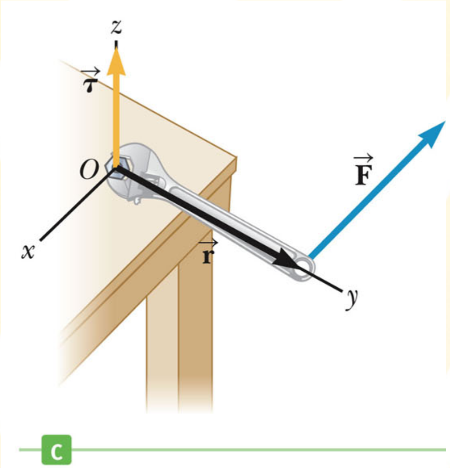
Positive torque: Object turns counterclockwise.

Negative torque: Object turns clockwise.

How can we change torque?







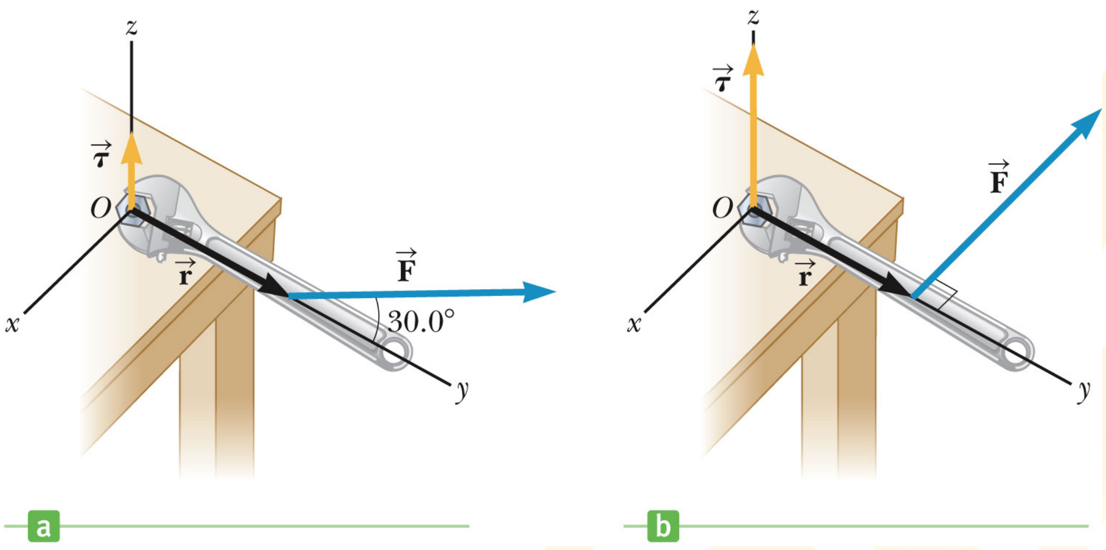
- The torque depends on the position of the applied force:

  The longer the lever arm, the stronger the torque:

- The torque is proportional to the magnitude of the applied force.

- The torque depends on the angle between the lever arm and the force:

Torque vs. Angle



The torque is proportional to the force component perpendicular to the lever arm.
General equation for torque:

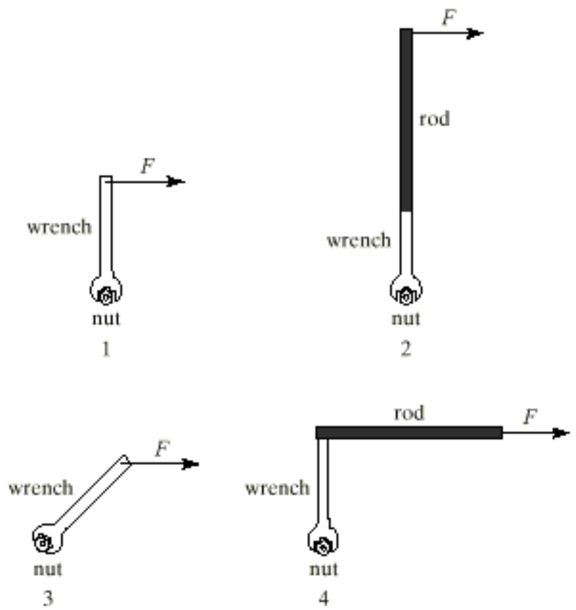
$$\tau = rF\sin(\theta)$$

The force component parallel to the lever arm does not yield any torque.

Movie: Torque

Clicker question

You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is most effective in loosening the nut? List in order of descending efficiency the following arrangements:



$$\tau = rF\sin(\theta)$$

Clicker question

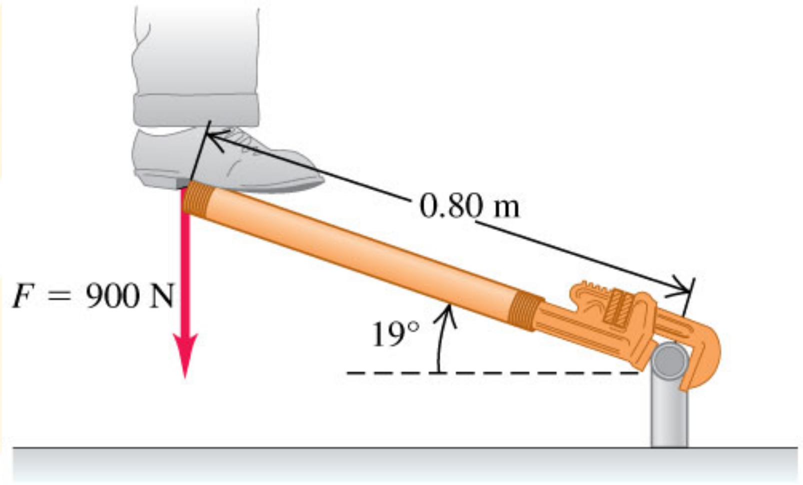
Joe the plumber pushes straight down on the end of a long wrench as shown. What is the magnitude of the torque Joe applies about the pipe?

A. (0.80 m)(900 N) sin 19°

B. (0.80 m)(900 N) cos 19°

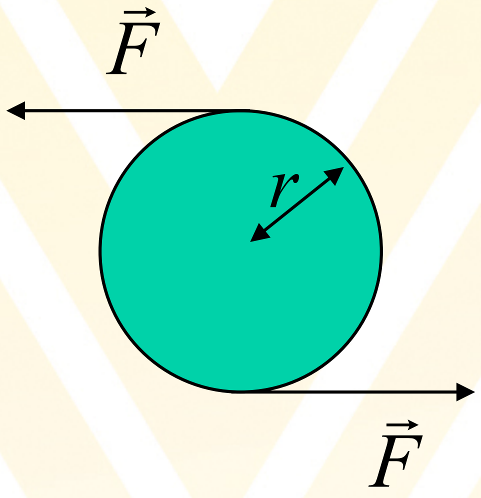
C. (0.80 m)(900 N) tan 19°

D. none of the above



Net torque

Similar to linear motion, where we calculated a net force acting on an object by adding all forces on the object up, we now calculate a net torque in case of rotational motion.



All torques exerted on an object are added:

$$\tau_{net} = \sum \tau$$

For the example shown here: $\tau_{net} = rF + rF = 2rF$

Linear vs. rotational motion:

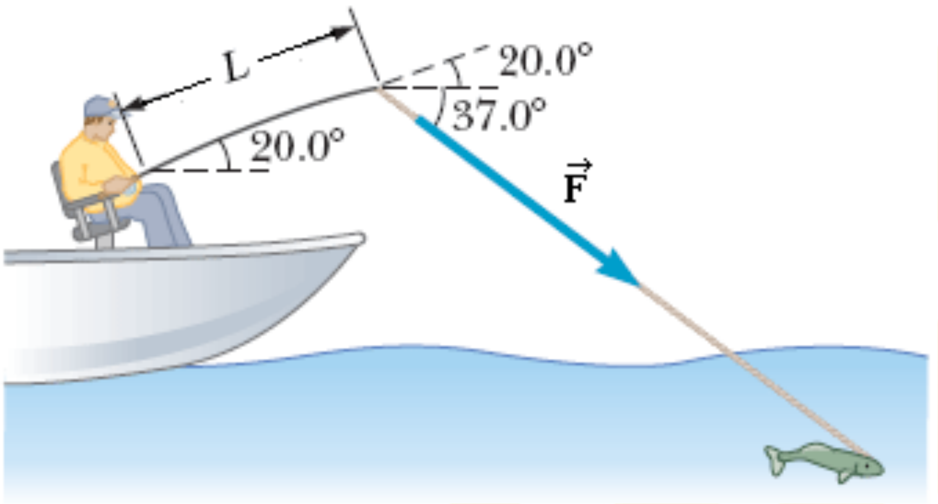
Linear motion - Newton's first law:

An object moves with a velocity that is constant in magnitude and direction unless a non-zero net force acts on it.

Rotational motion:

An object rotates at constant angular velocity unless a non-zero net torque acts on it.

Example problem: Torque



The fishing pole in the figure makes an angle of 20.0° with the horizontal. What is the magnitude of the torque exerted by the fish about an axis through the angler's hand if the fish pulls on the fishing line with a force = 100 N at an angle 37.0° below the horizontal? The force is applied at a point L = 2 m from the angler's hands.

Equilibrium

An object is in equilibrium, if

(i) there is no linear acceleration → no effective force.

$$\Sigma\vec{\mathbf{F}} = 0 \;\; or \;\; \Sigma\vec{\mathbf{F}}_x = 0 \;\; and \;\; \Sigma\vec{\mathbf{F}}_y = 0$$

(ii) there is no angular acceleration → no effective torque.

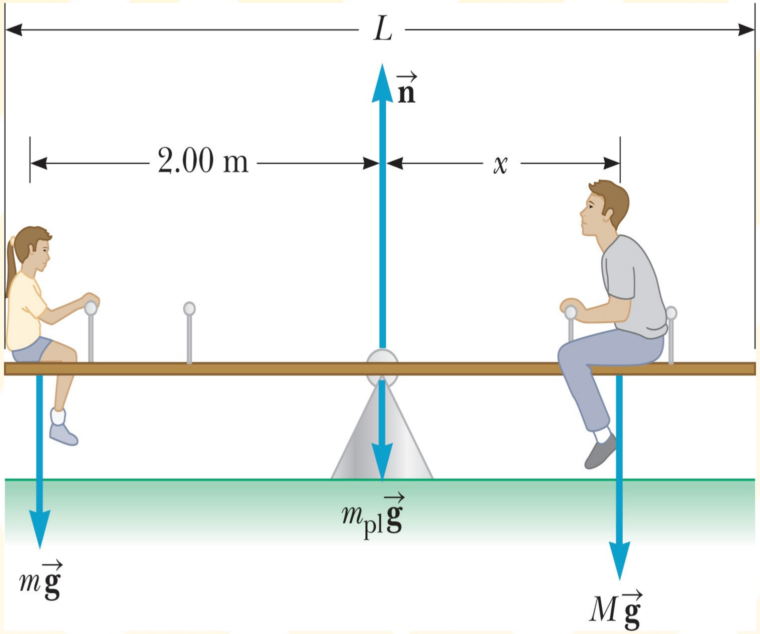
$$\Sigma\vec{\tau} = 0$$

Example problem: Equilibrium

A woman of mass m = 55 kg sits on the left end of a see-saw - a plank of length L = 4 m, pivoted in the middle.

(a) Compute the torques on the seesaw about an axis that passes through the pivot point. Where should a man of mass M = 75 kg sit if the system is to be balanced?

(b) Find the normal force exerted by the pivot if the plank has a mass of m_{pl} = 12 kg.



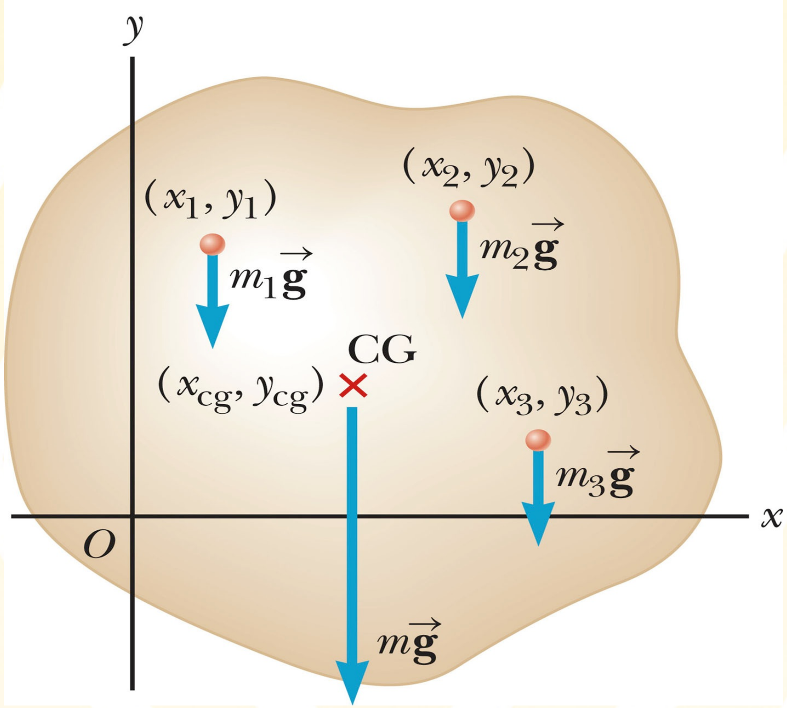
Center of gravity I

In the previous example, we neglected any torque produced by the force of gravity exerted on the plank itself.

This was justified, because generally we can compute the torque on a rigid body due to the force of gravity by assuming that its entire mass is concentrated in one single point - the center of gravity.

Then gravity is applied to this point and the resulting torque is calculated.

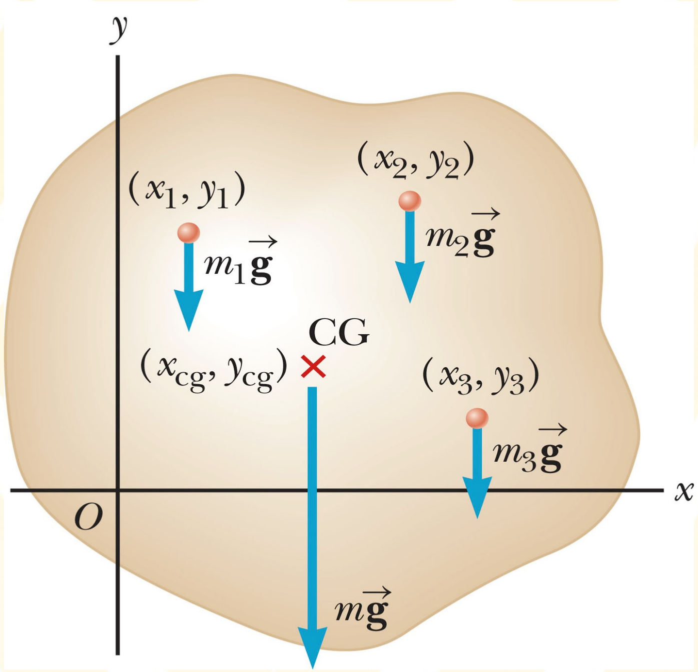
In the previous example the center of gravity was located at the axis of rotation (r = 0 m). Thus, there was no torque due to gravity.



But how do we find the center of gravity?

Center of gravity II

We divide the rigid body into mass segments and calculate the torque acting on every individual segment. Then, we look for a point, CG, where we can apply the force of gravity to get the same torque.



x - component:

$$m_1 g x_1 + m_2 g x_2 + m_3 g x_3 + \ldots = (m_1 + m_2 + m_3 + \ldots) g x_{cg}$$

$$\rightarrow \boxed{x_{cg} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum m_i x_i}{\sum m_i}}$$

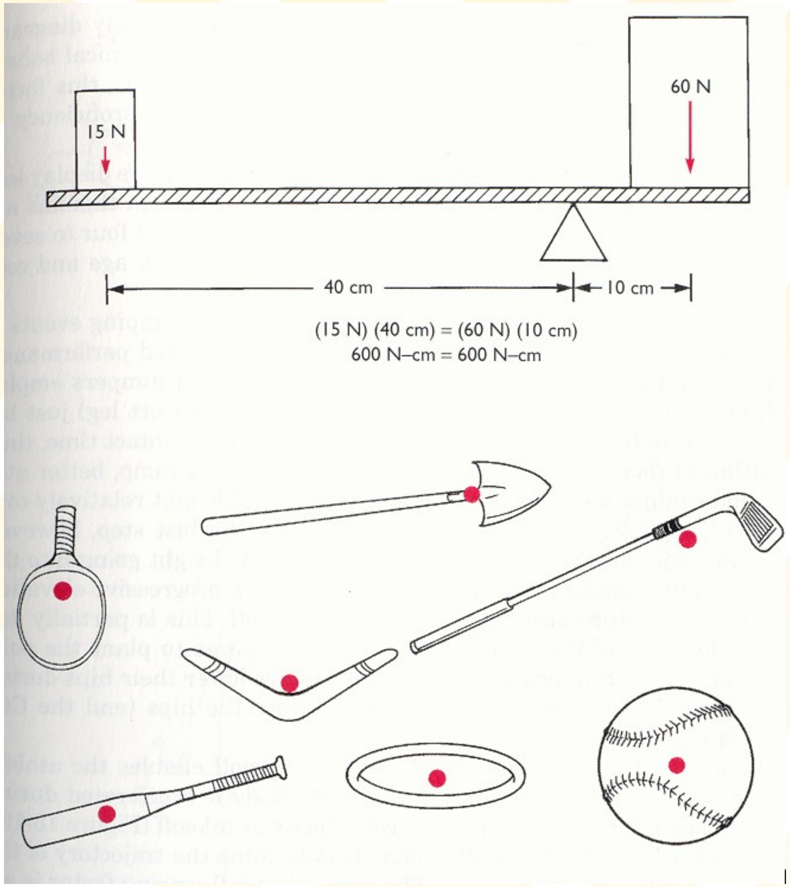
The same approach for the y- and z-component yields:

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} \qquad z_{cg} = \frac{\sum m_i z_i}{\sum m_i}$$

Examples: Center of gravity

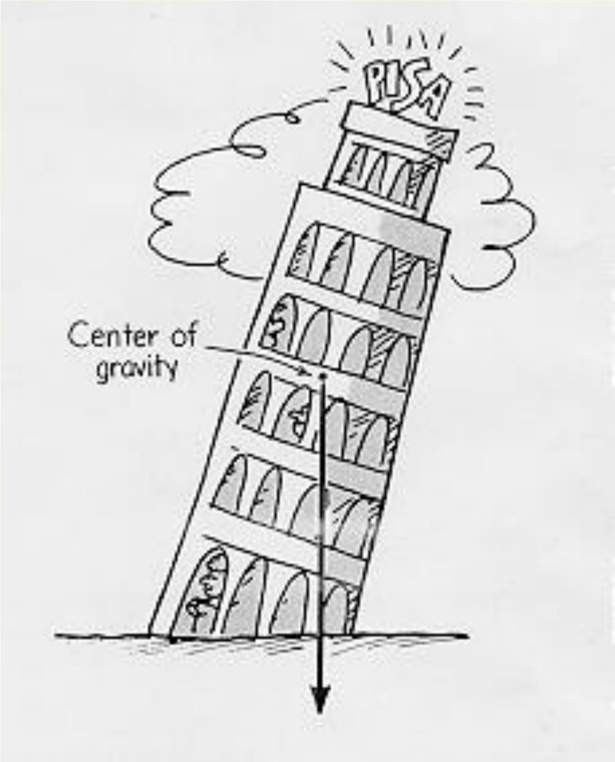
If the axis of rotation goes through the center of gravity, gravity will not cause any net torque on the object, i.e. it will not start to rotate from rest.

The center of gravity does not have to be part of the object itself.



Stability

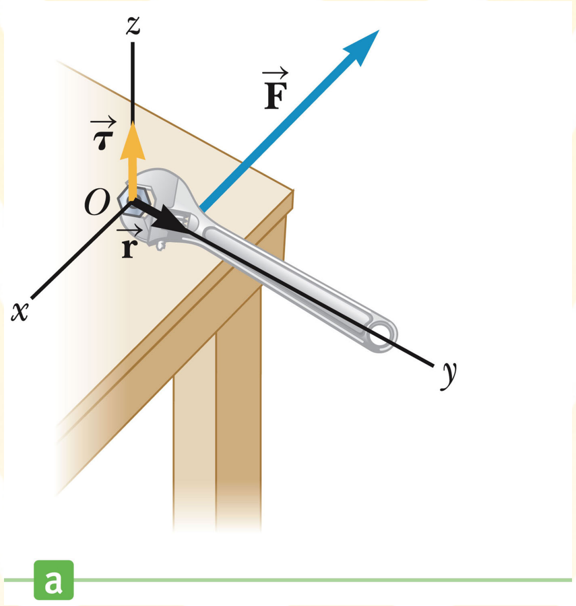
- If the CG of the object is above the area of support, the object will remain upright.
- If the CG is outside the area of support the object will topple.
- Leaning tower in Pisa, Italy - Center of gravity is still below its base at present



Summary

- A net torque, τ, causes an angular acceleration. This is equivalent to a force causing acceleration in linear motion:

$$\tau = rF\sin(\theta)$$



- Torque is a vector. Its direction is determined by the right-hand-rule.

- Its magnitude depends on (i) the lever arm, r, (ii) the magnitude of the applied force, F, and (iii) on the angle between force and lever arm.
- Translational equilibrium: $\sum \vec{F} = 0$ Rotational equilibrium: $\sum \vec{\tau} = 0$

- The net torque on a rigid body due to gravity is the same as if the force is applied only at the center of gravity:

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} \qquad y_{cg} = \frac{\sum m_i y_i}{\sum m_i} \qquad z_{cg} = \frac{\sum m_i z_i}{\sum m_i}$$