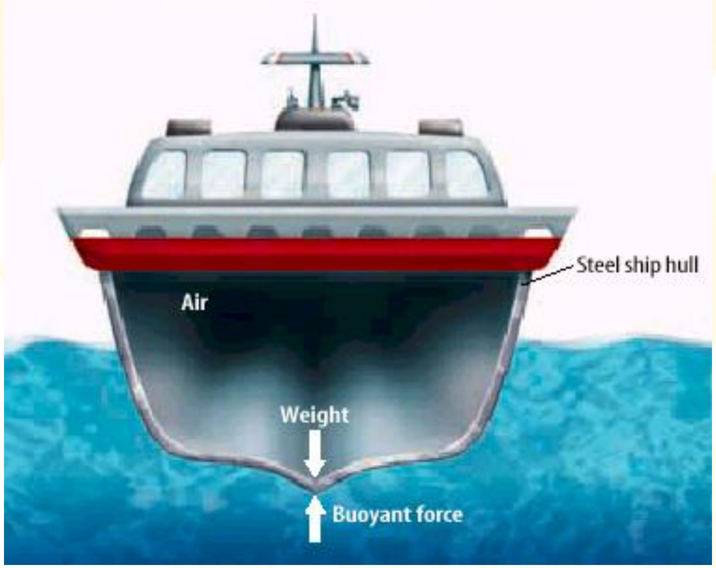
Announcements

- The third midterm exam takes place in White B51 on April 10th, 5 PM - 7 PM.
- The makeup exam takes place in Clark 317 on April 9th, 5 PM - 7 PM.
- The exams covers sections 6.5 (rockets), 7.0 - 7.5 (rotation, gravity), 8.0 - 8.7 (rotational dynamics), 9.0 - 9.7 (states and properties of matter), and 10.0-10.4 (thermodynamics) if we get to it by April 8.
- The formula sheet and an old exam are available on the course webpage.
- Best way to prepare: first, go through all worked examples and clickers from lecture slides, then, if you have time, go through homework and WebAssign "practice tests" (i.e. extra problems). Next, try the old exam without reference materials, and re-review the worked examples from step 1 for any problem areas. Finally (if you still have more time after all that) try additional problems from the text.

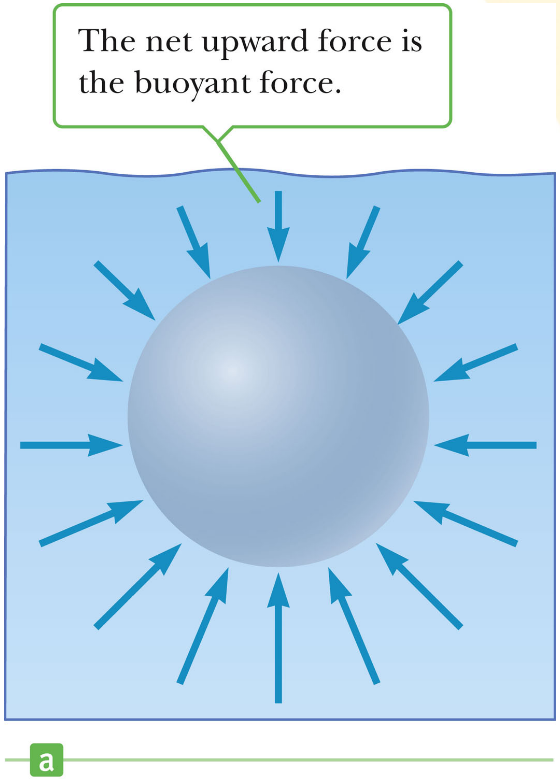
Today's lecture

Solids and Fluids

- Archimedes Principle + Buoyant Force
- Fluids in Motion



Why do objects float or sink?



Pressure changes as a function of depth, h:

$$P = P_0 + \rho g h$$

Multiplication by A yields:

$$PA - P_0 A = \Delta F = Mg$$

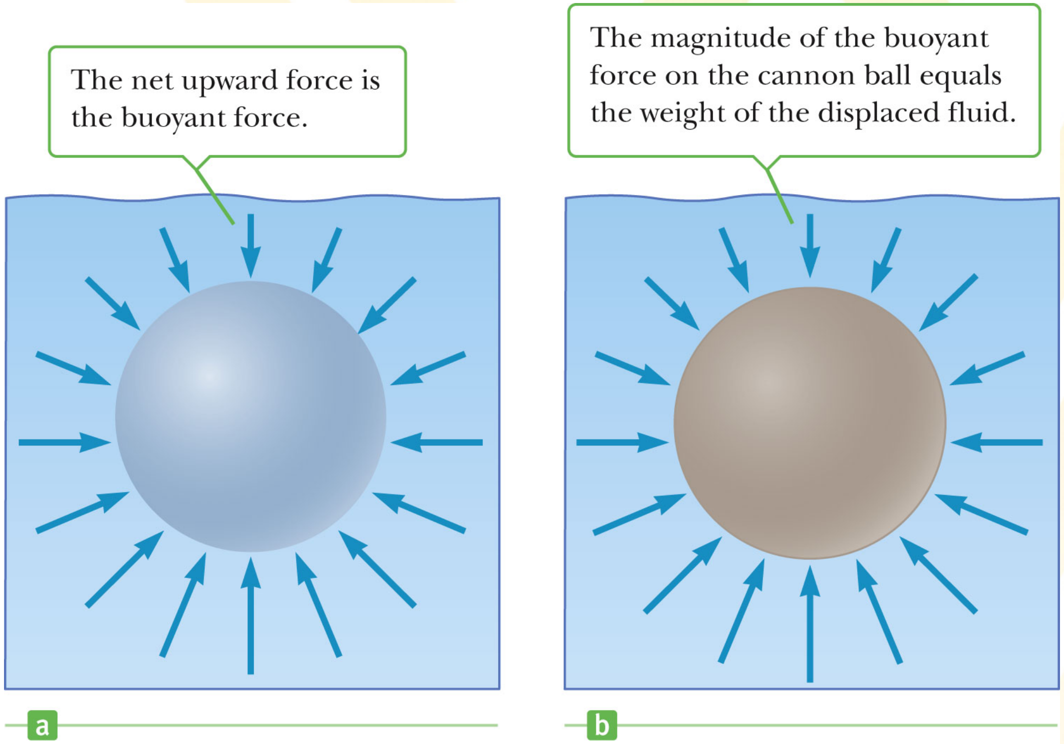
Here, P is the pressure at the bottom of a fluid volume and P_0 is the pressure at the top of this volume (V = Ah).

There is no pressure difference in horizontal direction at any given depth.

ΔF is an upward force onto any submerged object and corresponds to the weight of the fluid displaced by the object (Mg). This is *not the weight of the object itself*!

ΔF = B = Mg is called the Buoyant force.

Buoyant force and Archimedes principle



<u>Archimedes principle:</u> "Any object completely or partially submerged in a fluid is buoyed up by a force whose magnitude is equal to the weight of the fluid displaced by the object"

The myth says that this principle was invented by Archimedes, when he had a bath and realized that his body displaced water and felt lighter, when sitting in the bath tub.

https://www.youtube.com/watch?v=ijj58xD5fDI

Buoyant force and Archimedes principle

287 – 212 BC

$$B = Mg = w_{fluid} = \rho_{fluid} \cdot V_{fluid} \cdot g$$

- The magnitude of the buoyant force <u>always</u> equals the weight of the displaced fluid.
- The buoyant force is the same for a totally submerged object *of any size, shape, or density.*

We now calculate the net force exerted on the object:

Upward force: $B = \rho_{fluid} V_{obj} g$ (Buoyant force)

Downward force: $w = mg = \rho_{obj} V_{obj} g$ (Gravity)

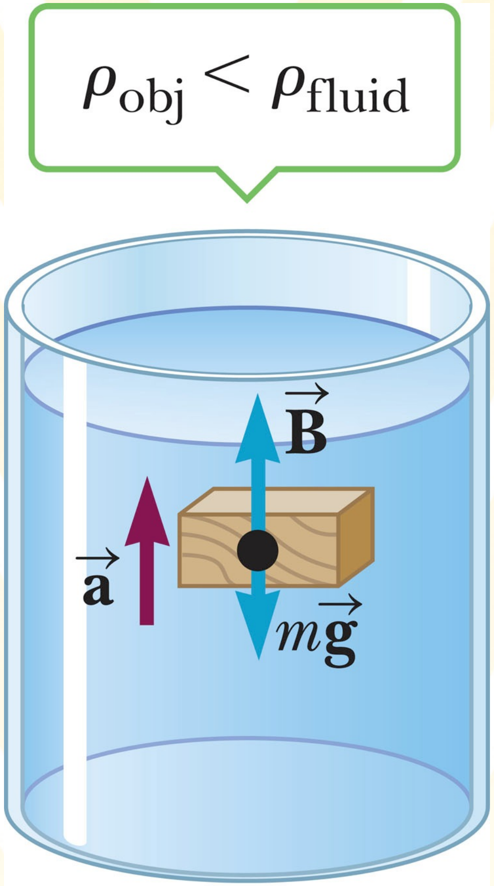
Net force on the object: $$B - w = (\rho_{fluid} - \rho_{obj}) \cdot V_{obj} \cdot g$$

Rising objects

If the object is less dense than the fluid, then the object experiences a net upward force.

$$B - w = (\rho_{fluid} - \rho_{obj}) \cdot V_{obj} \cdot g > 0N$$

The object will rise and finally float at the surface, where it is only partially submerged.

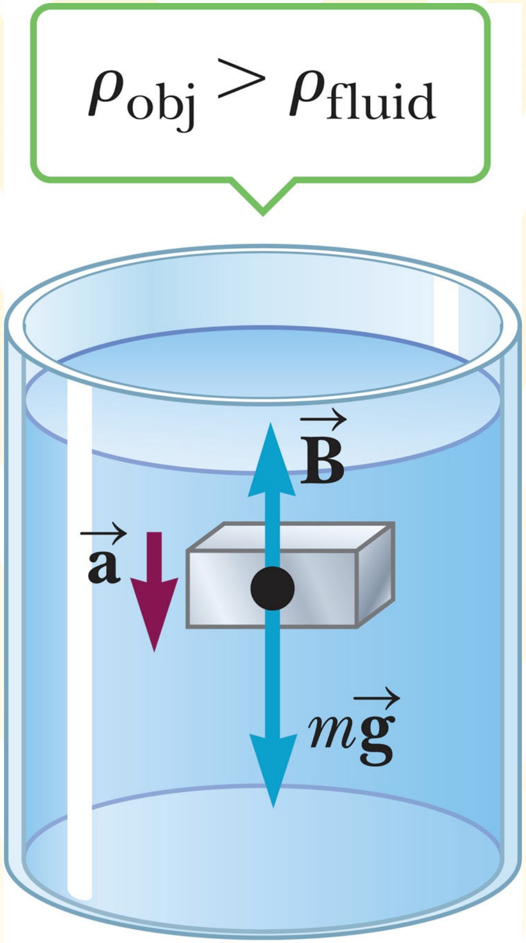


Sinking objects

If the object is more dense than the fluid, then the net force is *downward*, and the object accelerates *downward*.

$$B - w = (\rho_{fluid} - \rho_{obj}) \cdot V_{obj} \cdot g < 0N$$

Consequently, the object sinks.



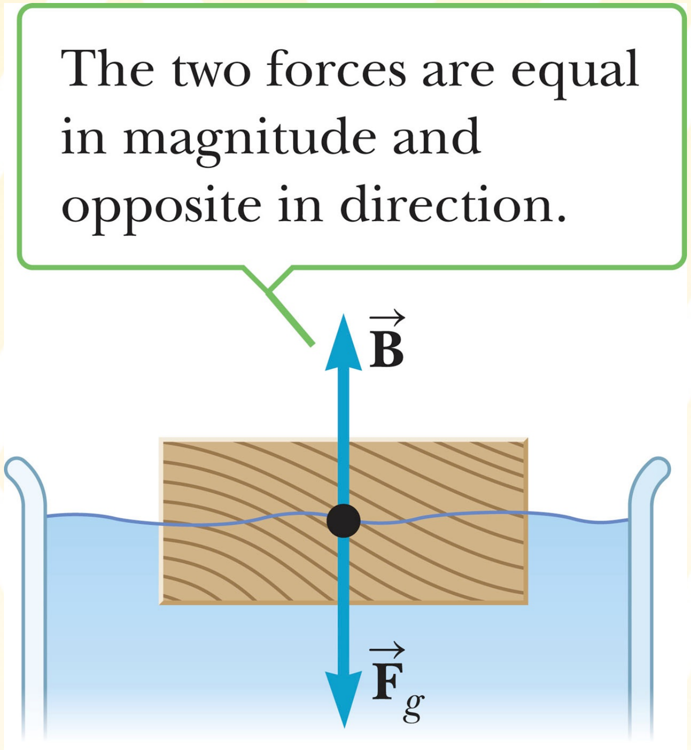
Floating objects

Floating object:

- The object is only partially submerged.
- The displaced volume of fluid yields a Buoyant force sufficient to compensate the complete weight of the object:

$$\rho_{fluid} \cdot V_{fluid} \cdot g = \rho_{obj} \cdot V_{obj} \cdot g$$

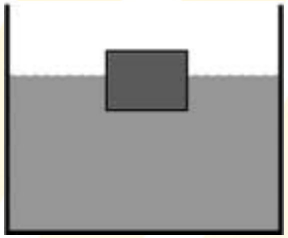
$$\rightarrow \quad \frac{\rho_{obj}}{\rho_{fluid}} = \frac{V_{fluid}}{V_{obj}}$$



Objects of lower density than the density of the fluid they are submerged in float, others sink.

Clicker question

Two blocks (A and B) have the same size and shape. Block A floats in the water, but Block B sinks in the water. Which block has the larger buoyant force on it?



A. Block A has the larger buoyant force on it.

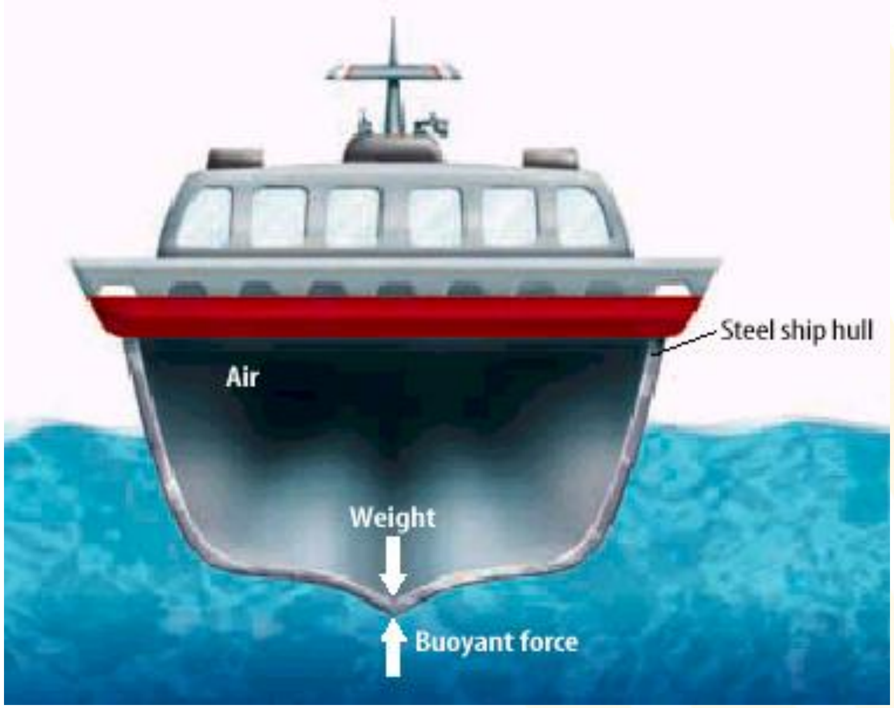
B. Block B has the larger buoyant force on it.

C. Neither; they have the same.

D. Not enough information

$$B = Mg = w_{fluid} = \rho_{fluid} \cdot V_{fluid} \cdot g$$

How can a steel ship float?

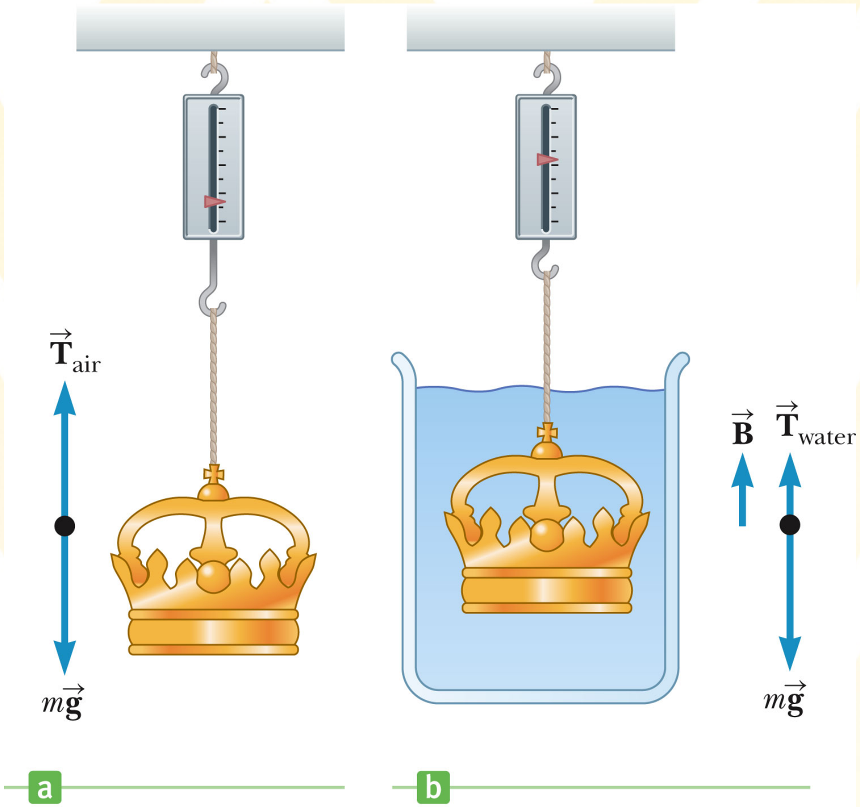


The hull contains mostly air and displaces a lot of water...enough so that $F_b = F_g$ and it floats.

Example problem: Buoyant force

A bargain hunter purchases a „gold“ crown at a flea market. After she gets home, she hangs it from a scale and finds its weight to be 7.84 N. She then weighs the crown while it is immersed in water. Now, the scale reads 6.86 N.

Is the crown made of pure gold?



Fluids in Motion: Equation of Continuity

Now we consider a fluid moving through a pipe of changing diameter.

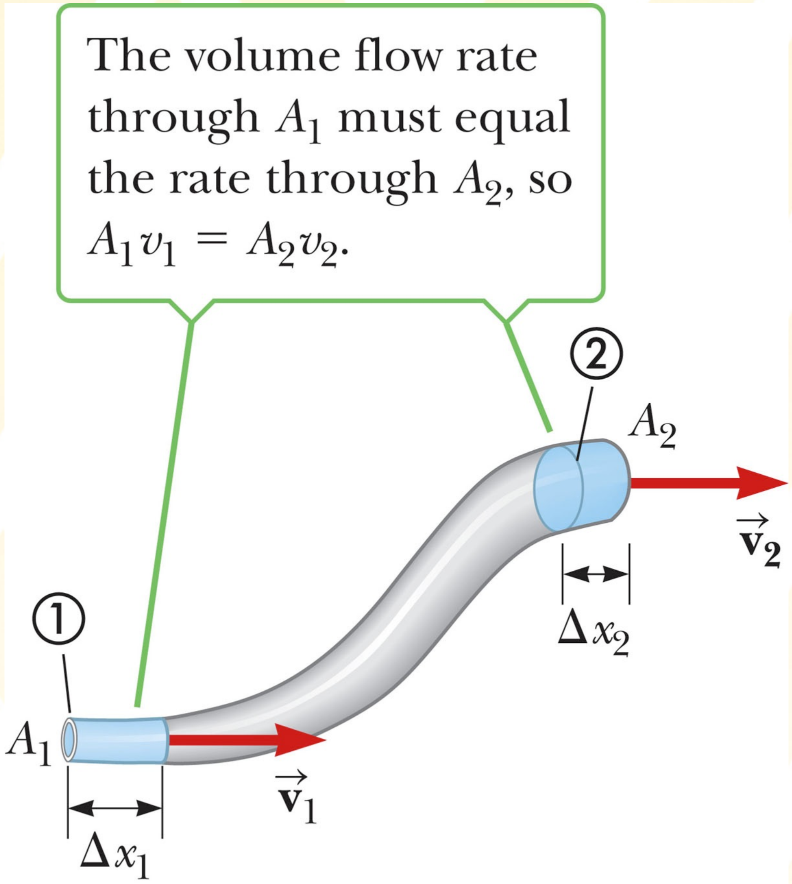
The fluid entering at the bottom crosses a distance $\Delta x_1 = v_1 \Delta t$ in a time interval Δt. v_1 is the fluid velocity at the bottom.

Then, the mass, ΔM_1, contained in the bottom blue region is $\Delta M_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$.

The same mass must leave the pipe at the top during Δt: $\Delta M_1 = \Delta M_2 = \rho A_2 v_2 \Delta t$.

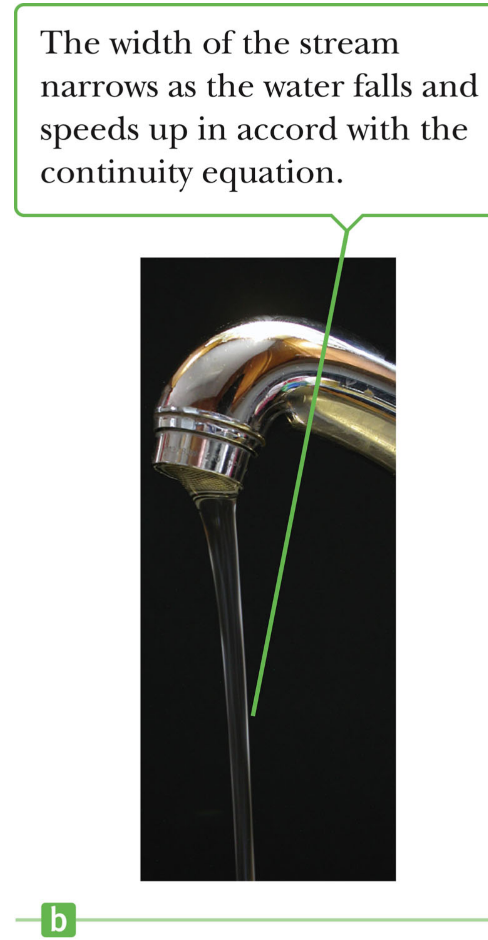
$$\rightarrow \quad \rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$$

$$\rightarrow \quad \boxed{A_1 V_1 = A_2 V_2}$$



The speed is high, where the tube is constricted, and the speed is low, where the tube is wide.

Applications: Equation of Continuity

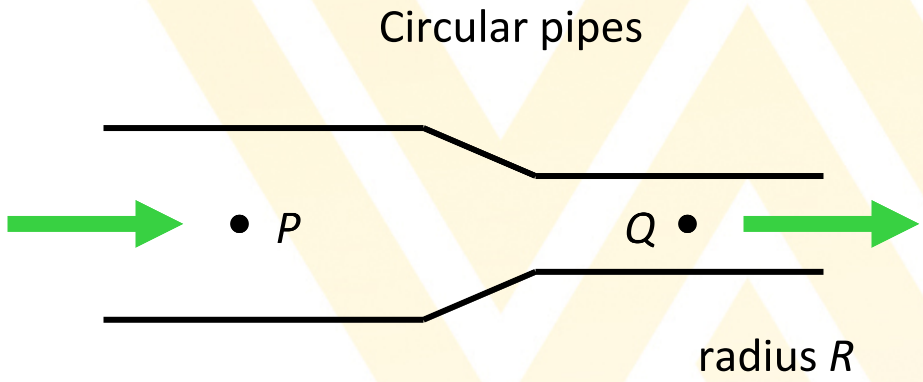


Definition of flow rate: $flow = A \cdot v$ unit: m^3/s

You can reduce the cross-sectional area of a garden hose by pressing your thumb on it.

This will cause the water to spray out faster and the stream to go further, since A v = const.

Clicker question



$$A_1 V_1 = A_2 V_2$$

A fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point *P*, the fluid at point *Q* has

A. 4 times the fluid speed.

B. 2 times the fluid speed.

C. the same fluid speed.

D. 1/2 the fluid speed.

E. 1/4 the fluid speed.

Bernoulli's Equation

We will now look at the net work done on a fluid, when it flows through a pipe of varying cross-section and elevation.

Within a time, Δt, the fluid moves a distance Δx_1 at the bottom. The force on the fluid on the lower end is $P_1 A_1$. The work done on the lower end is then:

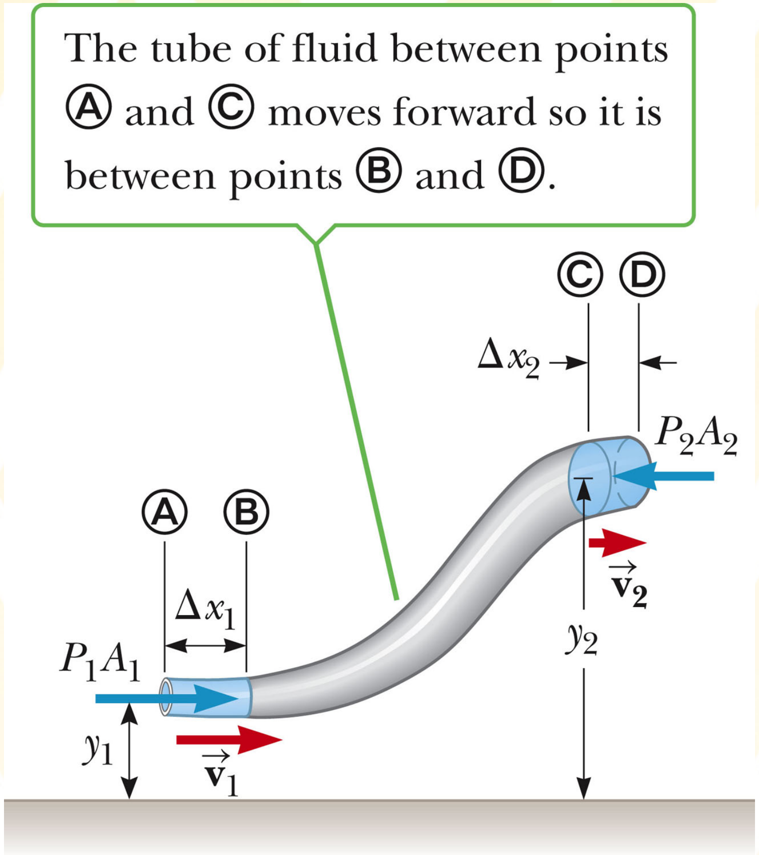
$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 V$$

The work done on the fluid on the upper end is:

$$W_2 = -P_2 V$$

Force and displacement are in opposite directions (minus sign).

The net work is then: $W_{fluid} = P_1 V - P_2 V$



Bernoulli's Equation

$$W_{fluid} = P_1V - P_2V$$

Part of this work goes into changing the fluid's kinetic energy:

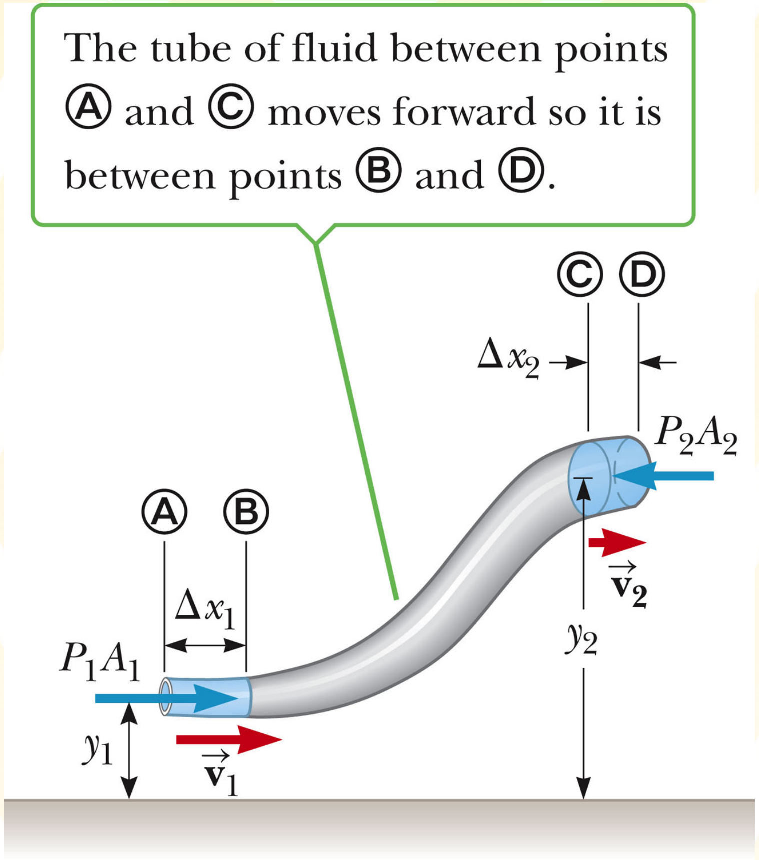
$$\Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The rest goes into changing the fluid's potential energy:

$$\Delta PE = mgy_2 - mgy_1$$

Thus: $$W_{fluid} = \Delta KE + \Delta PE$$

$$\rightarrow \quad P_1V - P_2V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$



Dividing by V and rearranging yields: $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's Equation

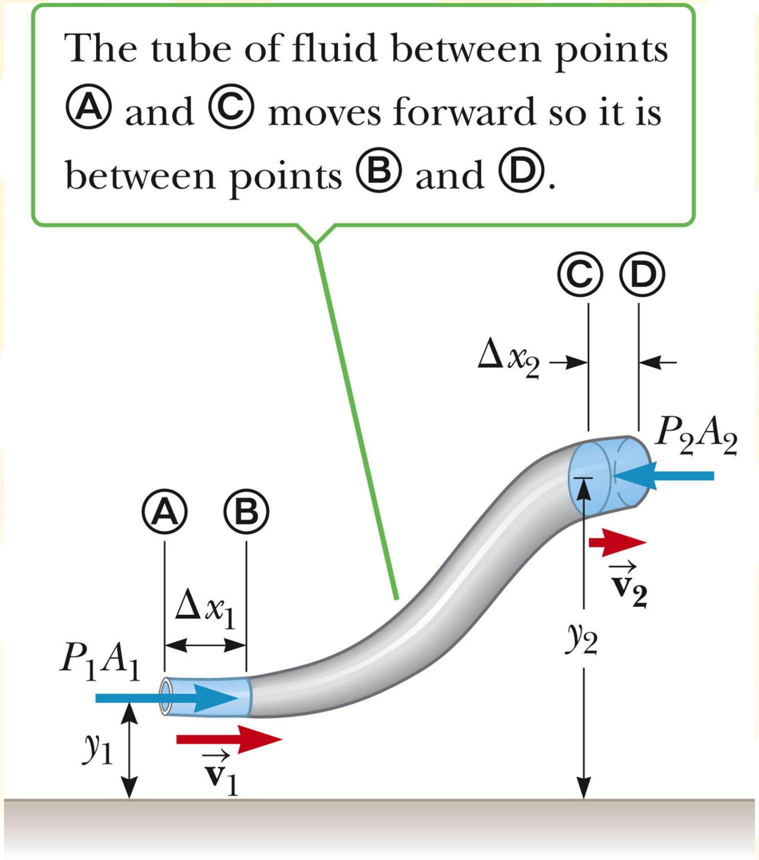
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This equation is the analogon to energy conservation and can also be written as:

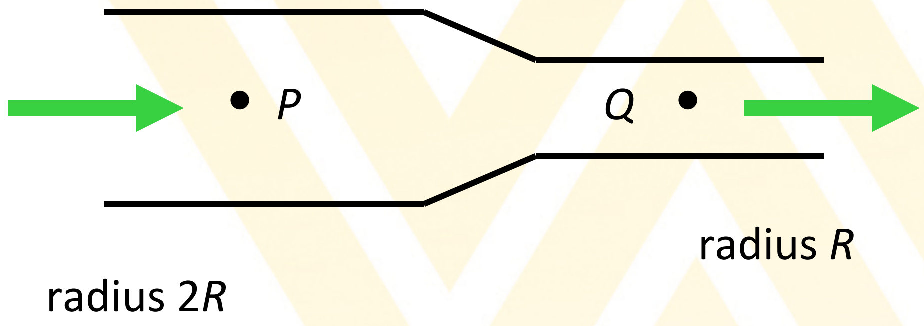
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = const.$$

This equation is called Bernoulli's equation.

It says that the sum of pressure, kinetic energy per volume, and potential energy is constant everywhere along the streamline.



Clicker question



An incompressible fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point *P*, the fluid at point *Q* has

A. greater pressure and greater volume flow rate.

B. greater pressure and the same volume flow rate.

C. the same pressure and greater volume flow rate.

D. lower pressure and the same volume flow rate.

E. none of the above

$$A_1 V_1 = A_2 V_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + mgy_1 = const.$$