Announcements

- The third midterm exam takes place in White B51 on April 10th, 5 PM - 7 PM.

- The makeup exam takes place in Clark 317 on April 9th, 5 PM - 7 PM.

- The exams covers sections 6.5 (rockets), 7.0 - 7.5 (rotation, gravity), 8.0 - 8.7 (rotational dynamics), 9.0 - 9.7 (states and properties of matter), and 10.0-10.4 (thermodynamics) if we get to it by April 8.

- The formula sheet and an old exam are available on the course webpage.

- Best way to prepare: first, go through all worked examples and clickers from lecture slides, then, if you have time, go through homework and WebAssign "practice tests" (i.e. extra problems). Next, try the old exam without reference materials, and re-review the worked examples from step 1 for any problem areas. Finally (if you still have more time after all that) try additional problems from the text.

Fluids in Motion: Equation of Continuity

Now we consider a fluid moving through a pipe of changing diameter.

The fluid entering at the bottom crosses a distance $\Delta x_1 = v_1 \Delta t$ in a time interval Δt. v_1 is the fluid velocity at the bottom.

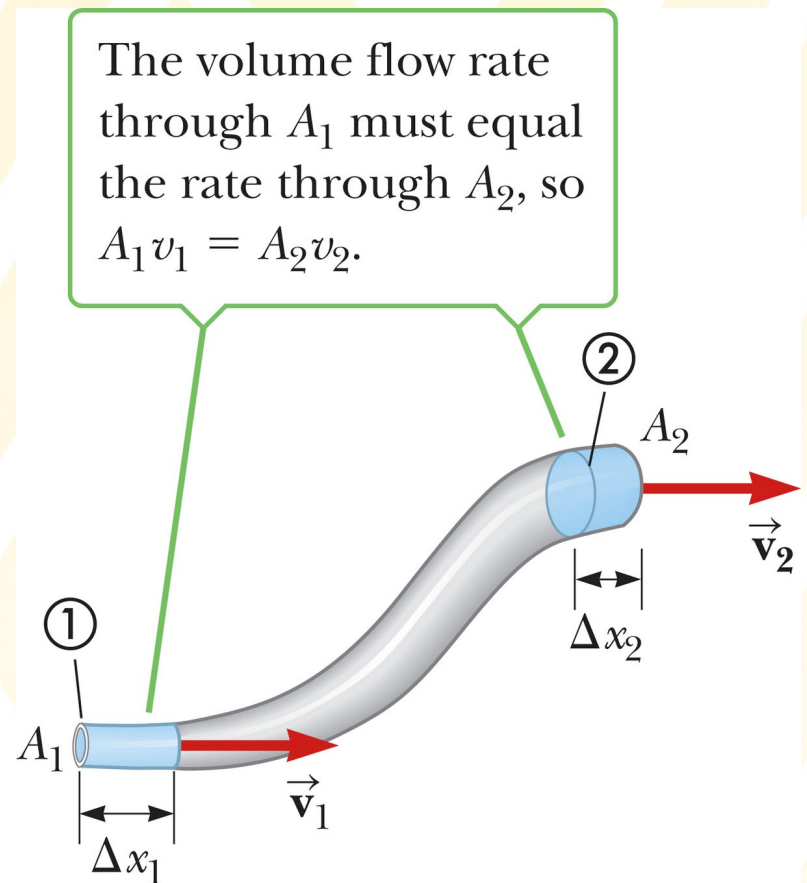Then, the mass, ΔM_1, contained in the bottom blue region is $\Delta M_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$.

The same mass must leave the pipe at the top during Δt: $\Delta M_1 = \Delta M_2 = \rho A_2 v_2 \Delta t$.

$$\rightarrow \quad \rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$$

$$\rightarrow \quad \boxed{A_1 V_1 = A_2 V_2}$$

The speed is high, where the tube is constricted, and the speed is low, where the tube is wide.

The volume flow rate through A_1 must equal the rate through A_2, so $A_1 v_1 = A_2 v_2$.

Applications: Equation of Continuity

The width of the stream narrows as the water falls and speeds up in accord with the continuity equation.
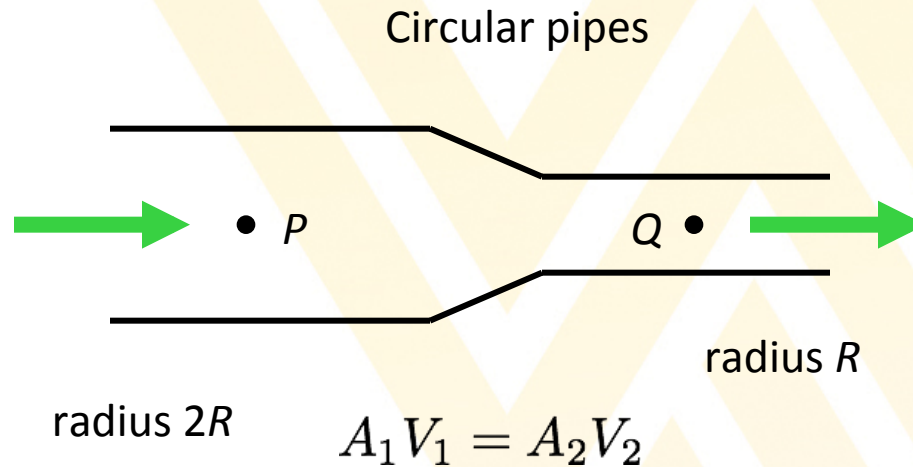

b

Definition of flow rate: $flow = A \cdot v$ unit: m³/s

You can reduce the cross-sectional area of a garden hose by pressing your thumb on it.

This will cause the water to spray out faster and the stream to go further, since A v = const.

Clicker question

Circular pipes

radius 2R

$$A_1 V_1 = A_2 V_2$$

radius R

A fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point P, the fluid at point Q has

A. 4 times the fluid speed.

B. 2 times the fluid speed.

C. the same fluid speed.

D. 1/2 the fluid speed.

E. 1/4 the fluid speed.

Bernoulli's Equation

We will now look at the net work done on a fluid, when it flows through a pipe of varying cross-section and elevation.

Within a time, Δt, the fluid moves a distance Δx_1 at the bottom. The force on the fluid on the lower end is $P_1 A_1$. The work done on the lower end is then:
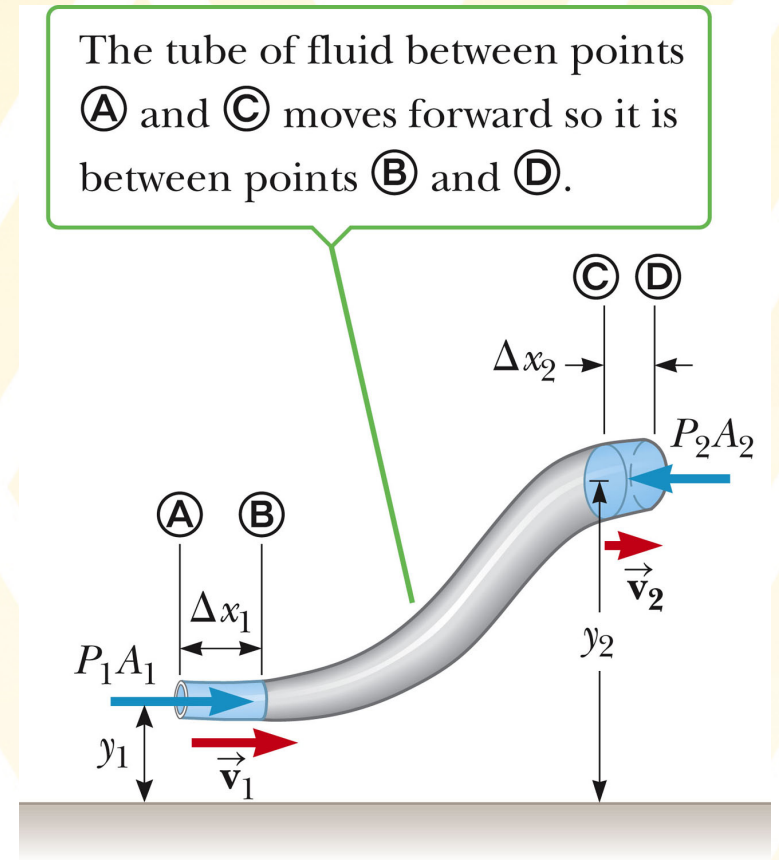
$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 V$$

The work done on the fluid on the upper end is:

$$W_2 = -P_2 V$$

Force and displacement are in opposite directions (minus sign).

The net work is then: $\quad W_{fluid} = P_1 V - P_2 V$

The tube of fluid between points Ⓐ and Ⓒ moves forward so it is between points Ⓑ and Ⓓ.

Bernoulli's Equation

$$W_{fluid} = P_1 V - P_2 V$$

Part of this work goes into changing the fluid's kinetic energy:
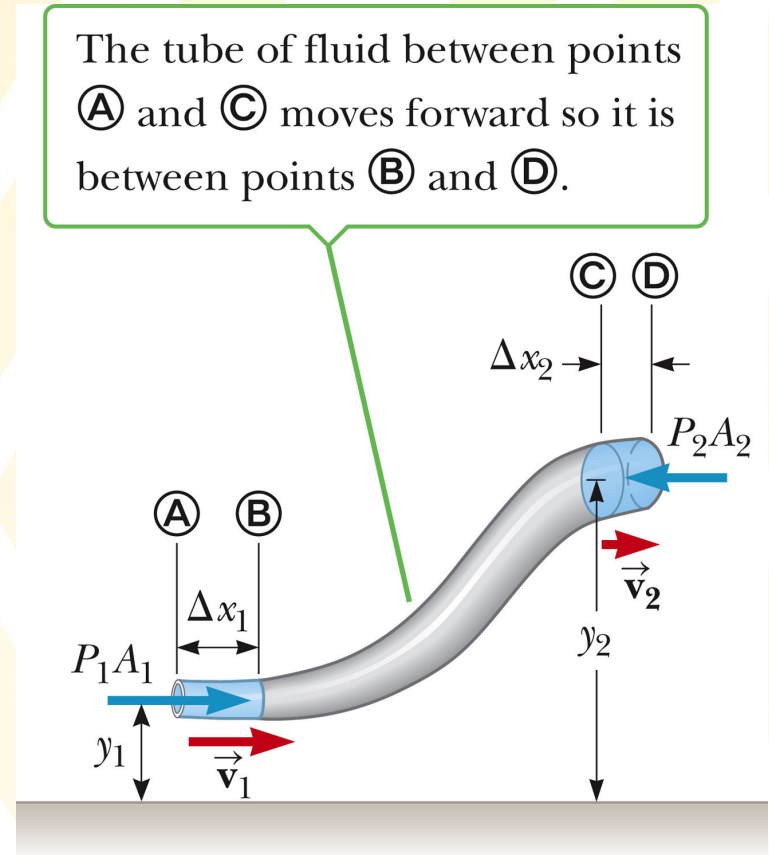
$$\Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The rest goes into changing the fluid's potential energy:

$$\Delta PE = mgy_2 - mgy_1$$

Thus: $W_{fluid} = \Delta KE + \Delta PE$

$$\rightarrow \quad P_1 V - P_2 V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

Dividing by V and rearranging yields: $\quad P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

The tube of fluid between points Ⓐ and Ⓒ moves forward so it is between points Ⓑ and Ⓓ.

Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$
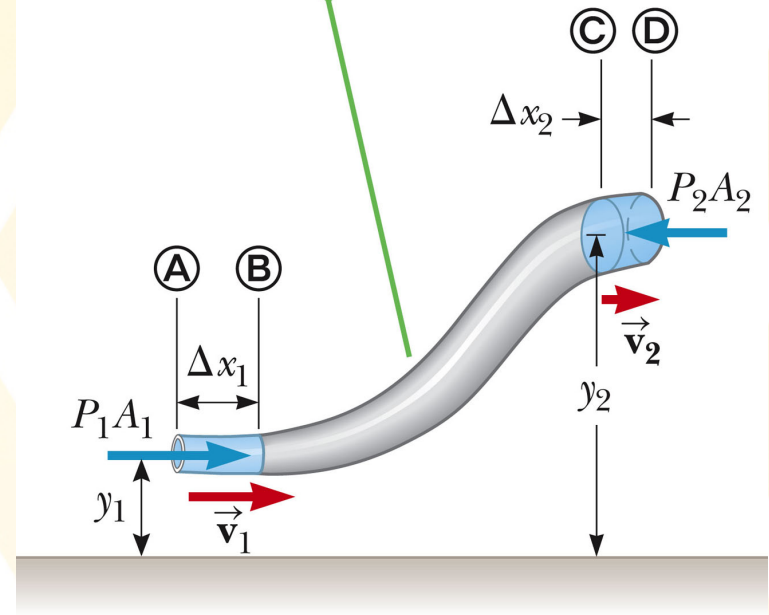
This equation is the analog to energy conservation and can also be written as:

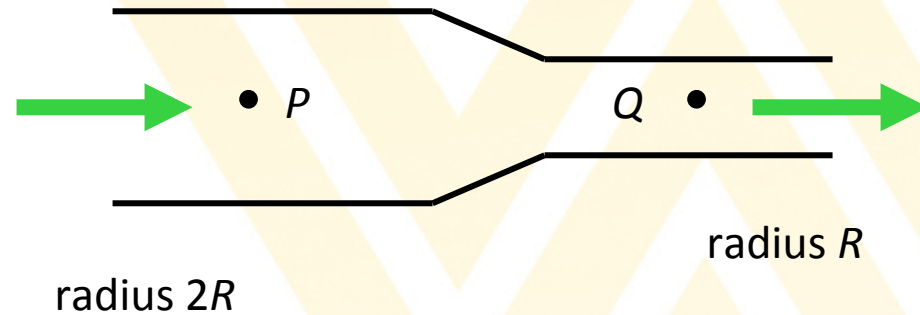$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = const.$$

This equation is called Bernoulli's equation.

It says that the sum of pressure, kinetic energy per volume, and potential energy is constant everywhere along the streamline.

The tube of fluid between points Ⓐ and Ⓒ moves forward so it is between points Ⓑ and Ⓓ.

Clicker question

radius $2R$

radius R

An incompressible fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point P, the fluid at point Q has

A. greater pressure and greater volume flow rate.

B. greater pressure and the same volume flow rate.

C. the same pressure and greater volume flow rate.

D. lower pressure and the same volume flow rate.

E. none of the above

$$A_1 V_1 = A_2 V_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + mgy_1 = const.$$

Midterm 3 Review

Rockets

- Classical rocket propulsion works because of momentum conservation. Exhaust gas ejected from a rocket pushes the rocket forwards, i.e. accelerates it.

$$v_f - v_i = v_e ln \left(\frac{M_i}{M_f} \right)$$

- The bigger the exhaust speed, v_e, the higher the gain in velocity of the rocket.

- The higher the ratio of initial to final mass of the rocket, the bigger the gain in velocity.

- Multi-stage rockets are used to minimize the final mass, M_f.

- Plasma thrusters are used for precise thrust control of satellites. They are based on ejecting ions at high velocities.

Example Problem: Rocket propulsion

A rocket has a total mass of 1×10^5 kg and a burnout mass of 1×10^4 kg, including engines, shell, and payload. The rocket blasts off from earth and exhausts all its fuel in 4 min, burning the fuel at a steady rate with an exhaust velocity of 4.5×10^3 m/s.

(a) If air, friction, and gravity are neglected, what is the speed of the rocket at burnout?

(b) What thrust does the engine develop at liftoff?

Rotational motion

- Angular position: $\theta = \dfrac{s}{r}$

- Angular displacement: $\Delta\theta = \theta_f - \theta_i$

- Angular velocity: $\omega_{av} = \dfrac{\theta_f - \theta_i}{t_f - t_i} = \dfrac{\Delta\theta}{\Delta t}$

- Angular Acceleration: $\alpha_{av} = \dfrac{\omega_f - \omega_i}{t_f - t_i} = \dfrac{\Delta\omega}{\Delta t}$

- Every point on a rotating rigid object has the same angular, but not the same linear motion!
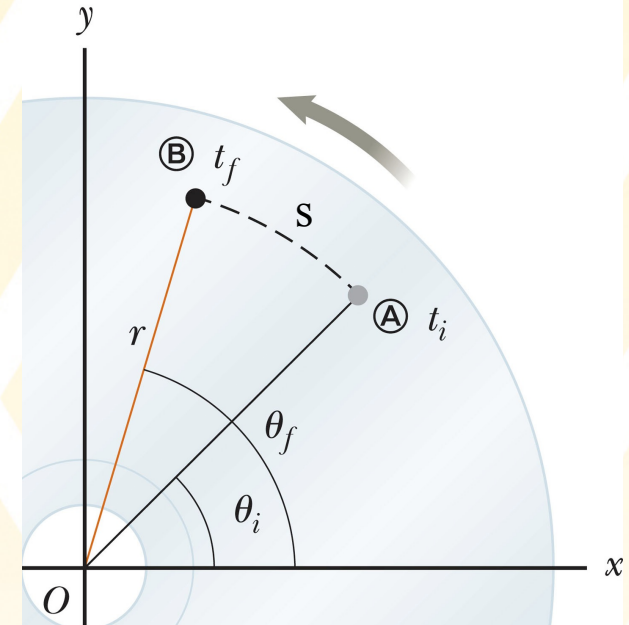
Linear Motion with a Constant (Variables: x and v)	Rotational Motion About a Fixed Axis with α Constant (Variables: θ and ω)
$v = v_i + at$	$\omega = \omega_i + \alpha t$
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$

Example problem: Rotational motion

A wheel rotates at a constant angular acceleration of 6 rad/s².
It starts from rest at an angular position of 90 degrees.

What is its angular position after 4 seconds?

Linear Motion with a Constant (Variables: x and v)	Rotational Motion About a Fixed Axis with α Constant (Variables: θ and ω)
$v = v_i + at$	$\omega = \omega_i + \alpha t$
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$

Central forces

- Circular motion at constant speed is an accelerated motion (the direction of the velocity vector changes).
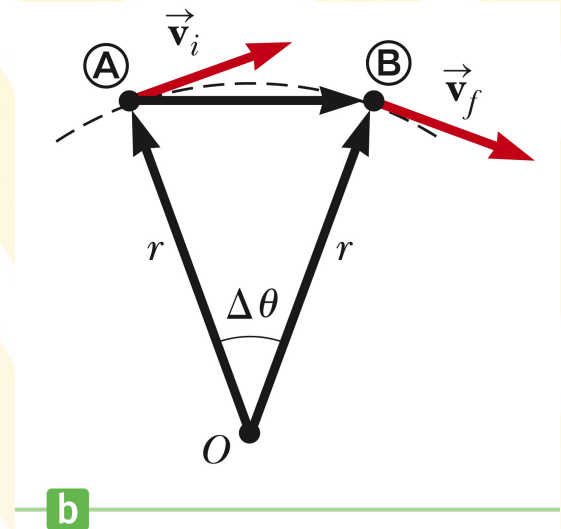
- The corresponding centripetal acceleration is:

$$a_c = \frac{v^2}{r} = r\omega^2$$

- The centripetal force is:

$$F_c = ma_c = m\frac{v^2}{r} = mr\omega^2$$

- If the centripetal force stops, the object will continue to move straight based on Newton's 1st law.

- The centripetal force is always directed towards the center.

- Two masses separated by a distance, r, attract each other based on Newton's law of gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

Different types of accelerations

The centripetal acceleration, a_c, causes an object to move on a circle.

$$a_c = \frac{v^2}{r} = r\omega^2$$

This could happen at constant or changing speed.

Additionally, there can be a tangential acceleration of magnitude a_t, that is linked to the angular acceleration, α.
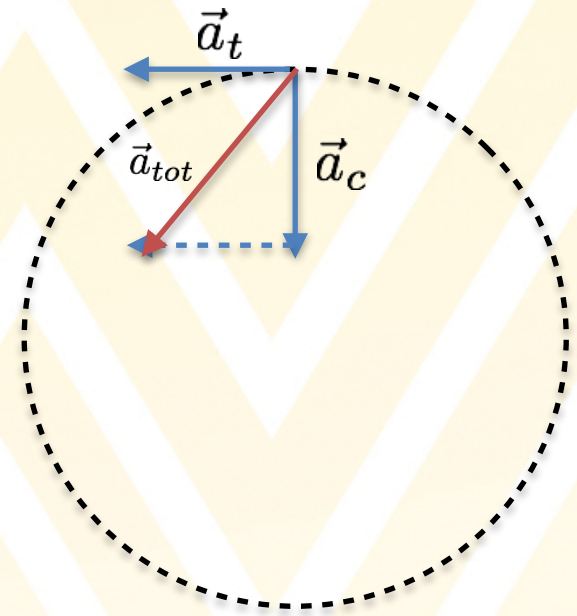
$$a_t = \alpha r$$

Both accelerations are vectors. The tangential acceleration causes the speed of the object to change.

In order to find the total acceleration of the object both vectors must be added:

$$\vec{a}_{tot} = \vec{a}_c + \vec{a}_t \qquad \rightarrow \qquad \boxed{|\vec{a}_{tot}| = \sqrt{a_c^2 + a_t^2}}$$

Example problem: circular acceleration

A race car accelerates uniformly from a speed of 40 m/s to a speed of 60 m/s in 5 s while traveling counterclockwise around a circular track of radius 400 m. When the car reaches a speed of 50 m/s, find

(a) the magnitude of the car's centripetal acceleration.

(b) the angular speed.

(c) the magnitude of the tangential acceleration.

(d) the magnitude of the total acceleration.

Torque, Equilibrium, center of gravity

- A net torque, τ, causes an angular acceleration. This is equivalent to a force causing acceleration in linear motion:

$$\tau = rF\sin(\theta)$$

- Torque is a vector. Its direction is determined by the right-hand-rule.

- Its magnitude depends on (i) the lever arm, r, (ii) the magnitude of the applied force, F, and (iii) on the angle between force and lever arm.

- Translational equilibrium: $\sum \vec{F} = 0$ Rotational equilibrium: $\sum \vec{\tau} = 0$

- The net torque on a rigid body due to gravity is the same as if the force is applied only at the center of gravity:

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} \qquad y_{cg} = \frac{\sum m_i y_i}{\sum m_i} \qquad z_{cg} = \frac{\sum m_i z_i}{\sum m_i}$$
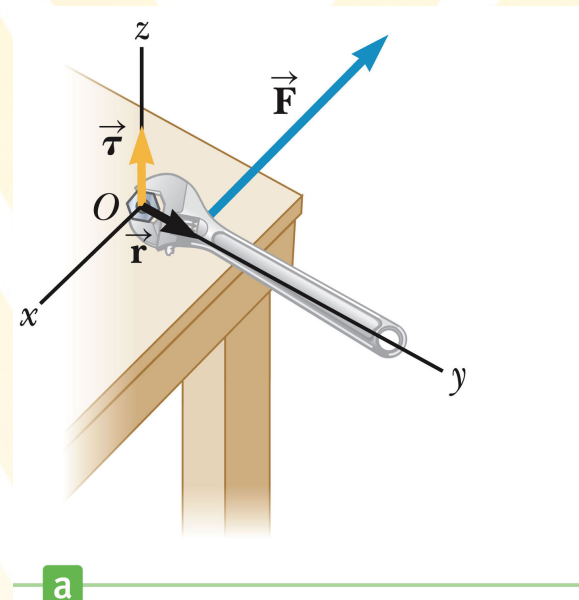
Example problem: Center of gravity

m_1 .... m_2

Find the center of gravity of a system consisting of two point masses, $m_1 = 10$ kg and $m_2 = 20$ kg, separated by a distance of 1 m.

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

Moment of Inertia, Rotational kinetic energy, Angular momentum

- Definition of the Moment of Inertia of this system: $I = \sum mr^2$ Unit: kg m^2

- The moment of inertia links torque with angular acceleration: $\tau = I\alpha$

- Rotational kinetic energy: $KE_r = \dfrac{1}{2}I\omega^2$

- Angular momentum: $L = I\omega$ $\sum \tau = \dfrac{L_f - L_i}{\Delta t} = \dfrac{\Delta L}{\Delta t}$

- Angular momentum will be conserved, if there is no net torque on the system.

- The moment of inertia can be changed by rearranging the distribution of mass of an object. In the absence of a net torque, this will change the angular velocity.

$$I = \sum mr^2 \qquad L_f = I_f\omega_f = L_i = I_i\omega_i$$

Example problem: Moment of Inertia

$$m_1 \cdots\cdots\cdots\cdots\cdots\cdots m_2$$

There are again two point masses, $m_1 = 10$ kg and $m_2 = 20$ kg, separated by a distance of 1 m. Find the moment of inertia about an axis perpendicular to the page and located exactly between both masses.

$$I = \sum mr^2$$

# Example problem: Rotational kinetic energy

Two balls, both of mass M and radius R start from rest at a height of 2 m. One of them rolls down an incline of 30 degree slope, and the other slides down the same incline.

What is the linear speed of the rolling ball when it leaves the incline?

What is the linear speed of the sliding ball when it leaves the incline?

Moment of Inertia for a solid sphere:

$$I = \frac{2}{5}MR^2$$

M

R

2.00 m

30.0°

$\vec{v}$
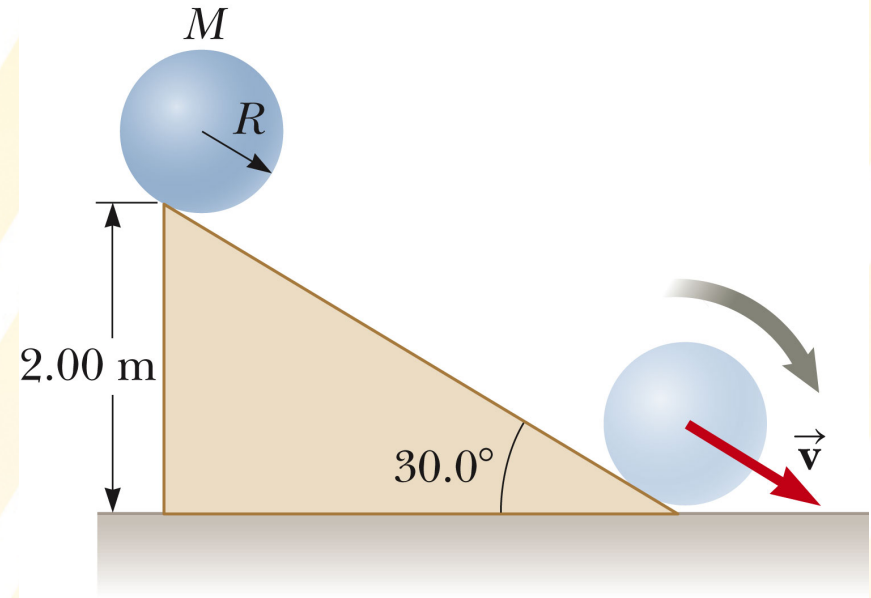
Density and Pressure

- There are 4 aggregates states of matter:

  - Solid: Strong interatomic bonds, particles cannot move freely.

  - Liquid: Weaker bonds, particles move more freely

  - Gas: No interatomic bonds, particles move freely.

  - Plasma: Ionized gas, electrons and ions are separated.

- Transitions from one state into another are initiated by heating/cooling the material.

- Density is mass per volume: $\rho \equiv \dfrac{m}{V}$

- Pressure is force per area: $P = \dfrac{F}{A}$
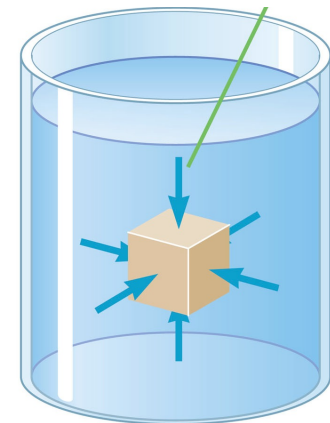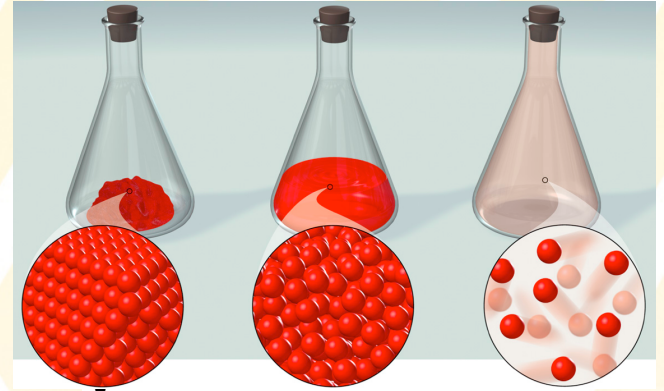
Elasticity, Pressure vs. depth, Pascal's principle

- All objects are deformable. Until a maximum stress (F/A) the object will tend to its original shape, when the force is removed (elasticity).
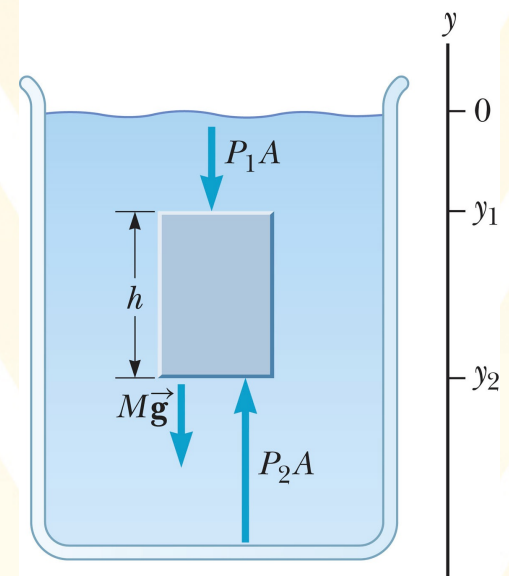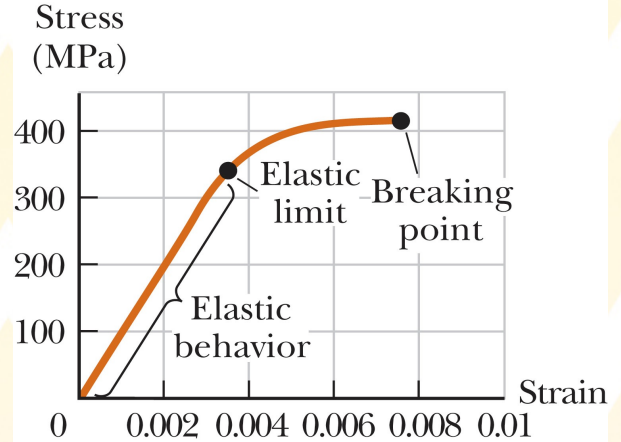
- There are 3 types of elasticities:

  (i) Elasticity of length: $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_0}$

  (ii) Elasticity of shape: $\dfrac{F}{A} = S\dfrac{\Delta x}{h}$

  (iii) Elasticity of volume: $\Delta P = -B\dfrac{\Delta V}{V}$

- Pressure depends on depth: $P = P_0 + \rho g h$

- Pascal's principle: $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ (hydraulic press)

Example problem: Pascal's principle

In a hydraulic press, a force, F_1, is exerted on a small piston of quadratic cross section (side length a = 10 cm). This pressure is transmitted through the fluid to a second larger piston of quadratic cross sectional area (side length b = 30 cm).

What force must be exerted on the smaller piston to lift a 20000 N object via the larger piston?

A small force $\vec{F}_1$ on the left produces a much larger force $\vec{F}_2$ on the right.
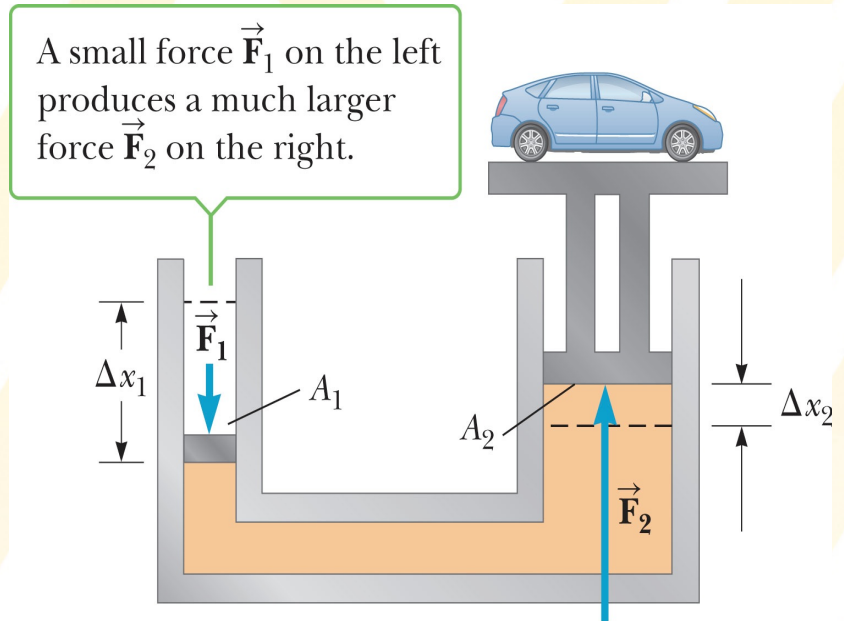
$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

# Archimedes principle, continuity, Bernoulli

- The Buoyant Force acts on every object immersed in a liquid (or gas):

$$B = \rho_{fluid} \cdot V_{fluid} \cdot g$$

It corresponds to the weight of the fluid displaced by the object (Archimedes principle).

Such an object also feels the force of gravity due to its own mass. The balance of the Buoyant force and gravity determines if the object floats or sinks (ρ_{obj} vs. ρ_{fluid}).
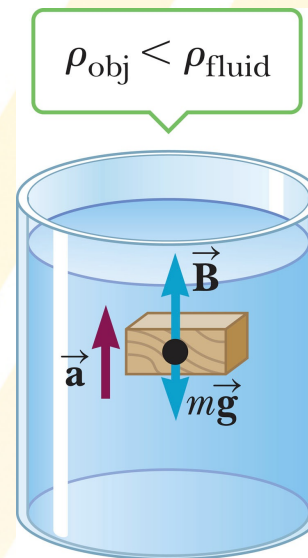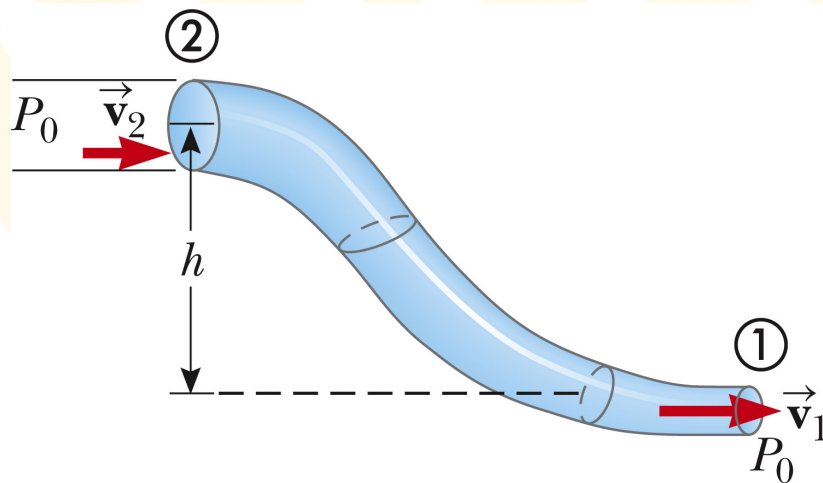
- Equation of continuity:

$$A_1 V_1 = A_2 V_2$$

- Bernoulli equation (energy consevation):

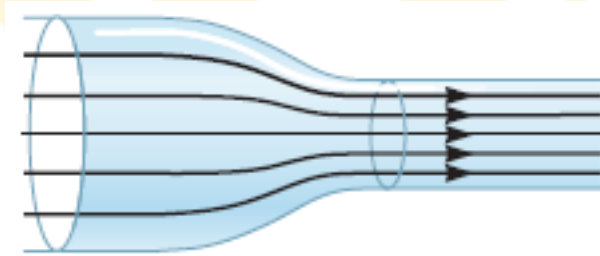$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = const.$$

$\rho_{obj} < \rho_{fluid}$

Example problem: Continuity

radius 2R

radius R

A liquid flows through a cylindrical pipe of diameter d_1 = 1 m at a velocity of 1 m/s. This pipe is connected to another pipe of diameter d_2 = 0.5 m.

How fast does the liquid flow in the second pipe?

Example problem: Bernoulli equation

A liquid (ρ = 1.65 g/cm³) flows through a horizontal pipe of varying cross section as in the figure below. In the first section, the cross-sectional area is 10.0 cm²,the flow speed is 289 cm/s, and the pressure is 1.20 × 10⁵ Pa. In the second section, the cross-sectional area is 3.50 cm².

(a) Calculate the smaller section's flow speed.

(b) Calculate the smaller section's pressure.