Recap

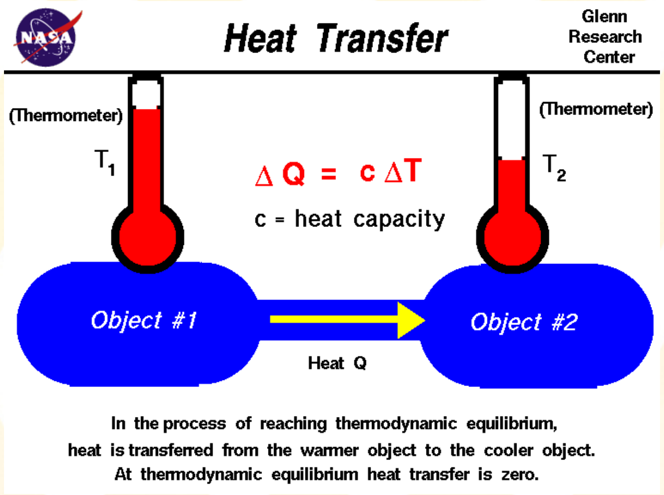
- Internal Energy, U, is the energy associated with the atoms and molecules of the system.
- Heat, Q: Energy *transferred* between a system and its environment due to a *temperature difference:*

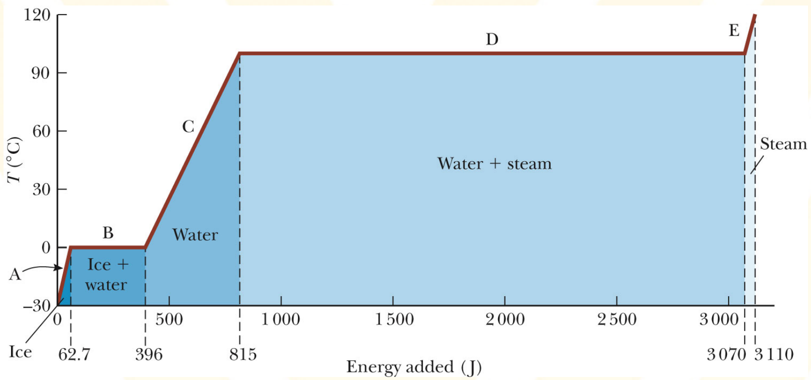
$$\boxed{Q = mc\Delta T}$$ Unit: J/(kg °C)

- c is the material dependent specific heat, i.e. the energy required to raise the temperature of 1 kg of the material by 1 °C.
- Latent heat, L, is the energy required to chance the phase a substance and does not cause its temperature to increase.

$$\boxed{Q = mL}$$ Unit of L: J/kg

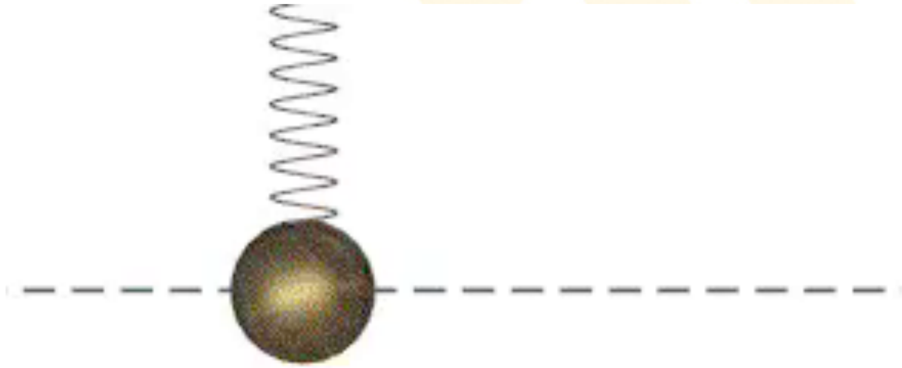
- 3 ways to transfer thermal energy: Conduction, convection, and radiation



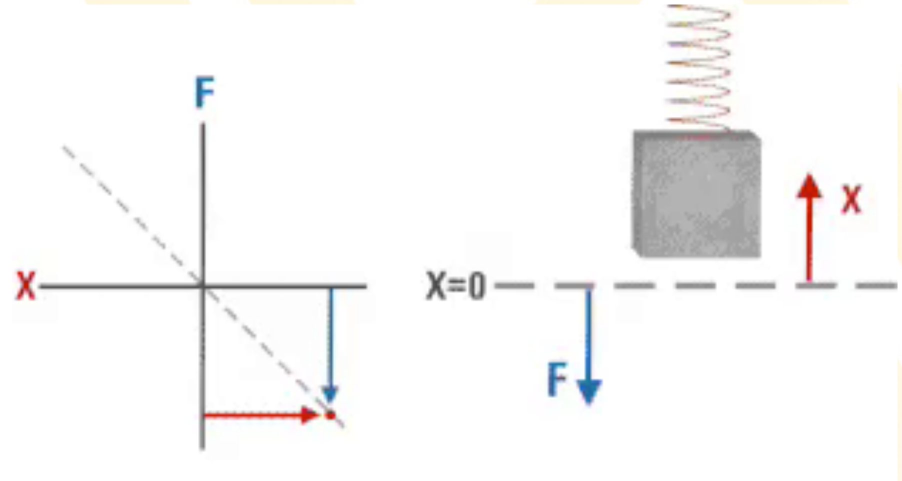


Chapter 13: Vibrations and Waves

- Hooke’s Law, Spring potential Energy
- Simple harmonic vs. circular motion
- Pendulums



Hooke's Law and Spring Potential Energy



Objects attached to springs undergo a periodic motion - they oscillate.

For short extensions from their equilibrium position Hooke's Law describes the force on the attached object:

$$\boxed{F_s = -kx}$$

This is a restoring force accelerating the object back to its equilibrium position. It linearly depends on the extension, x.

The spring potential energy is (see chapter 5):

$$\boxed{PE_s = \frac{1}{2}kx^2}$$

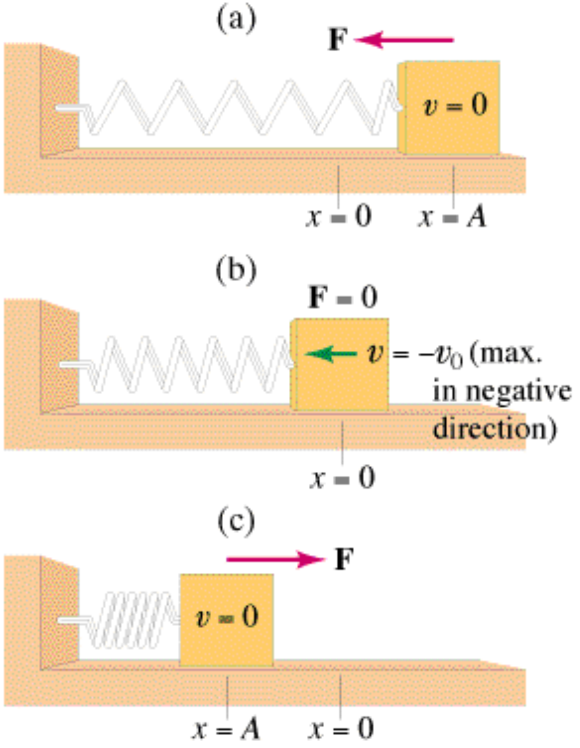
Some Definitions

The Amplitude, A, is the greatest distance from the equilibrium position.

The period, T, is the time it takes for the object to complete one complete cycle of motion from x = A to x = - A and back to x = A.

The frequency, f, is the number of complete cycles per unit time (f = 1/T).

When the net force along the direction of motion obeys Hooke's Law, Simple Harmonic Motion (SHM) occurs.

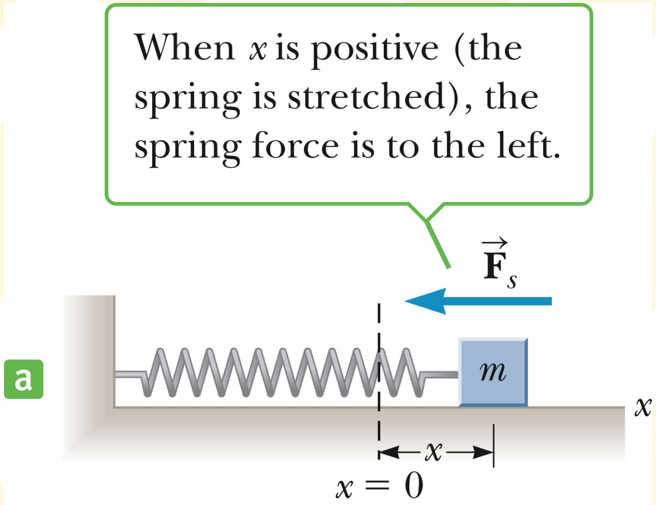


Acceleration in Simple Harmonic Motion

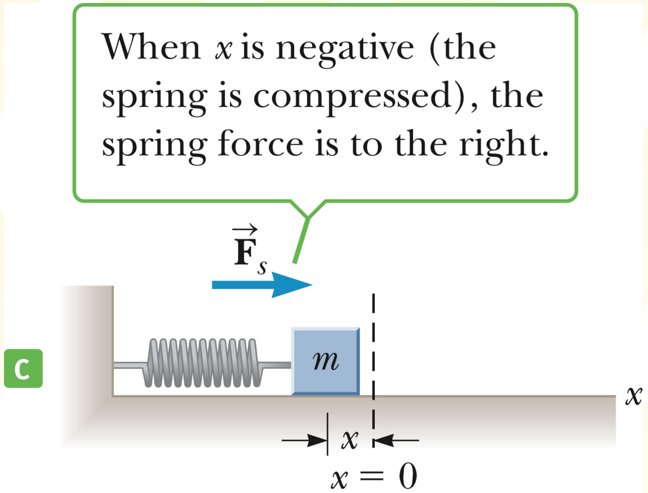
The acceleration can be found from Newton's second law and Hooke's Law:

$$F = ma = -kx$$

$$\rightarrow \quad \boxed{a = -\frac{k}{m}x}$$



The acceleration is not constant in time, since the position, x, is time dependent.



Simple harmonic motion is not a uniform motion with constant acceleration.

Clicker question

A block on the end of a horizontal spring is pulled from equilibrium at x = 0 m to x = A.

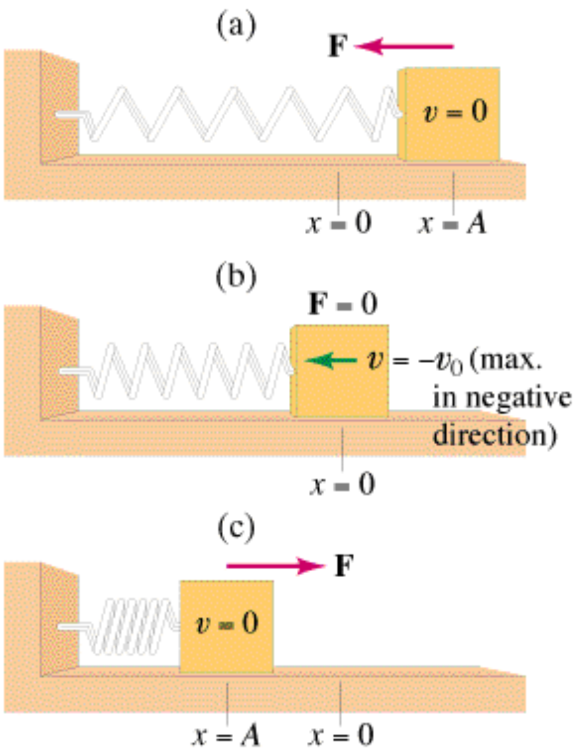
Through what total distance does it travel in one full cycle?

A. A/2

B. A

C. 2 A

D. 4 A



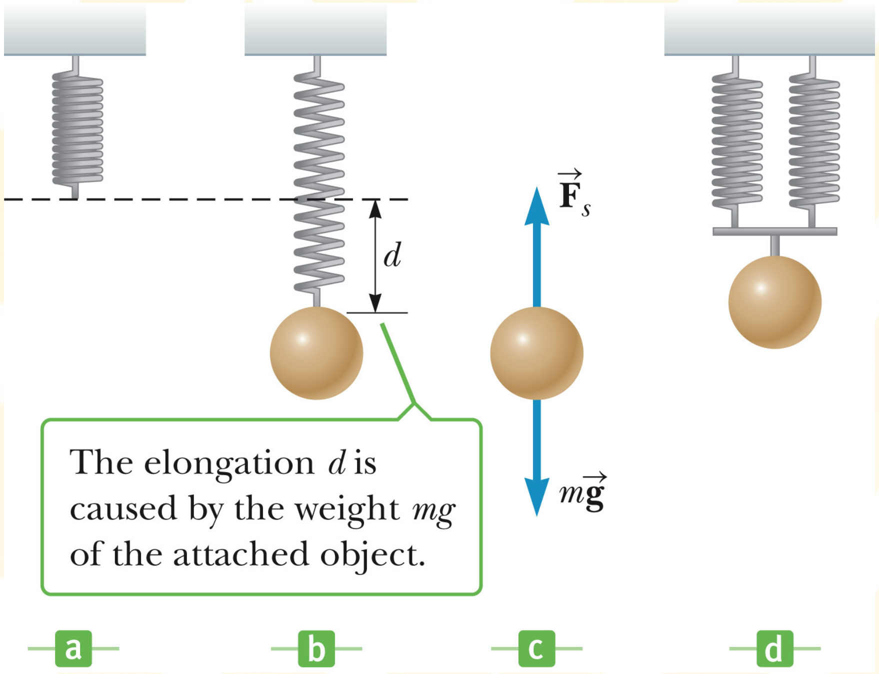
Example problem: Mass on a vertical spring

A spring is hung vertically and an object of mass m attached to the lower end is slowly lowered a distance d to the equilibrium point.

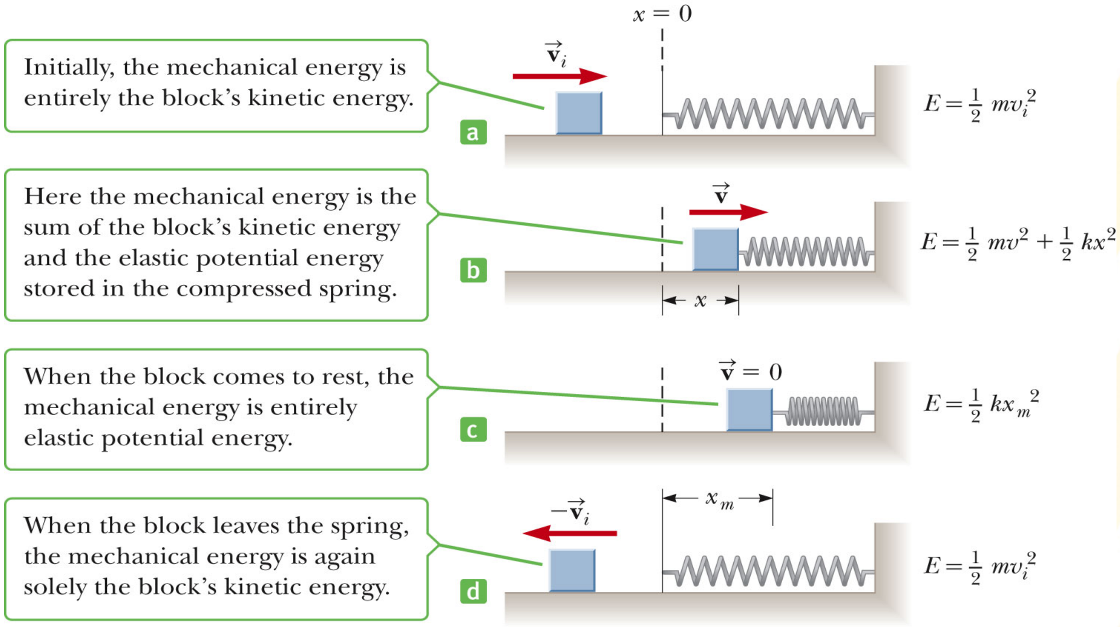
A. Find the value of the spring constant, if the spring is displaced by d = 2 cm and the mass is 0.55 kg.

B. If a second identical spring is attached to the object n parallel with the first spring, where is the new equilibrium point of the system?

C. What is the effective spring constant of the two springs acting as one?



Energy conservation (SHM)



Initial time

Final time

Energy conservation: $$(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$

At initial and final time (see plot): $$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$\rightarrow \boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}}$$

The velocity is maximum at the equilibrium point (x = 0 m)

Example problem: Energy conservation

A 0.5 kg object connected to a light spring with k = 20 N/m oscillates on a frictionless horizontal surface.

A. Calculate the total energy of the system and the maximum speed of the object if the amplitude is 3 cm.

B. What is the velocity of the object when the displacement is 2 cm?

C. Compute the kinetic and potential energies of the system when the displacement is 2 cm.

$$\rightarrow \quad v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

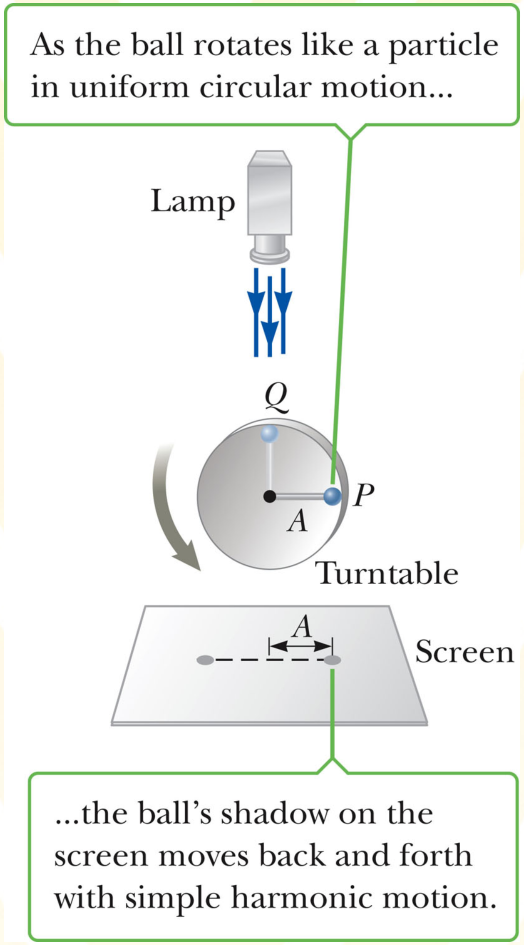
Simple Harmonic Motion vs. Circular Motion

As a turntable rotates with constant angular speed, the shadow of the ball (projection) moves back and forth with simple harmonic motion.

We can prove that this is true. For SHM the following equation for the velocity holds:

$$v = C\sqrt{A^2 - x^2}$$

If we can show that the same equation holds for the shadow of the rotating ball, we prove that it undergoes a Simple Harmonic Motion, too.



Simple Harmonic Motion vs. Circular Motion

For the rotating ball (small triangle):

$$\sin\theta = \frac{v}{v_0}$$

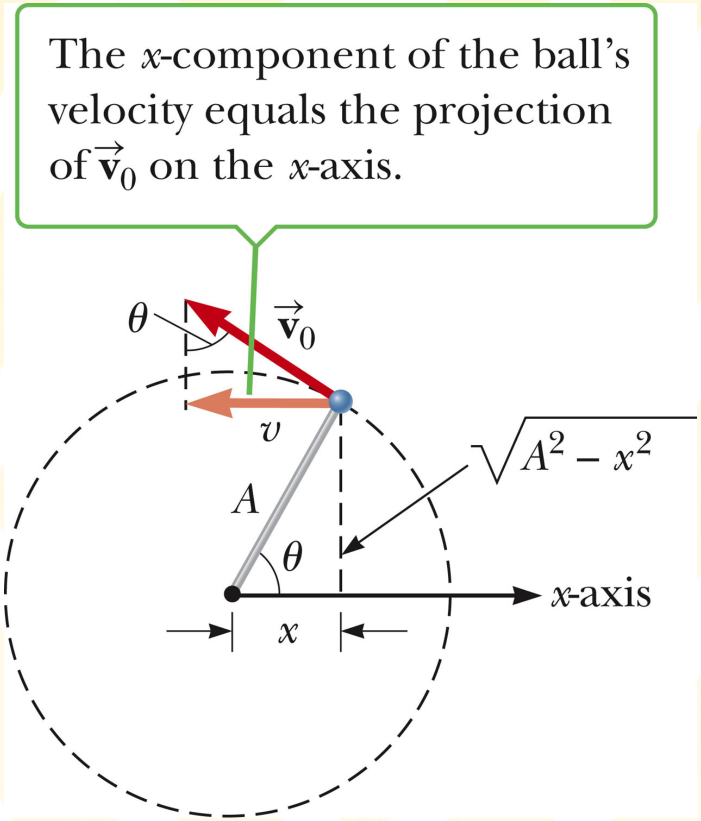
v is the x-component (shadow) of its total velocity v_0.

From the large triangle we get:

$$\sin\theta = \frac{\sqrt{A^2 - x^2}}{A}$$

$$\rightarrow \quad \frac{v}{v_0} = \frac{\sqrt{A^2 - x^2}}{A}$$

$$\rightarrow \quad v = \frac{v_0}{A}\sqrt{A^2 - x^2} = C\sqrt{A^2 - x^2}$$



The x-component of the velocity undergoes simple harmonic motion.

Period and Frequency

The period of the shadow of the rotating ball is the time it takes the ball to make one complete revolution on the turntable:

$$v_0 = \frac{2\pi A}{T} \quad \rightarrow \quad T = \frac{2\pi A}{v_0}$$

If the ball makes one quarter of a revolution, its shadow moves from x = A (only potential spring energy) to x = 0 m (only kinetic energy):

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 \quad \rightarrow \quad \frac{A}{v_0} = \sqrt{\frac{m}{k}}$$

Substituting this into the first equation for T yields:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Or for the angular frequency:

$$\boxed{\omega = 2\pi f = 2\pi\frac{1}{T} = \sqrt{\frac{k}{m}}}$$

Clicker question

An object of mass m is attached to a horizontal spring, stretched to a displacement A from equilibrium and released undergoing harmonic oscillations on a frictionless surface with period T_0. The experiment is then repeated with a mass of 4m.

What is the new period of oscillation?

A. $2T_0$ B. T_0 C. $T_0/2$ D. $T_0/4$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Position, velocity, Acceleration

The x-component of the position vector of the rotating ball is:

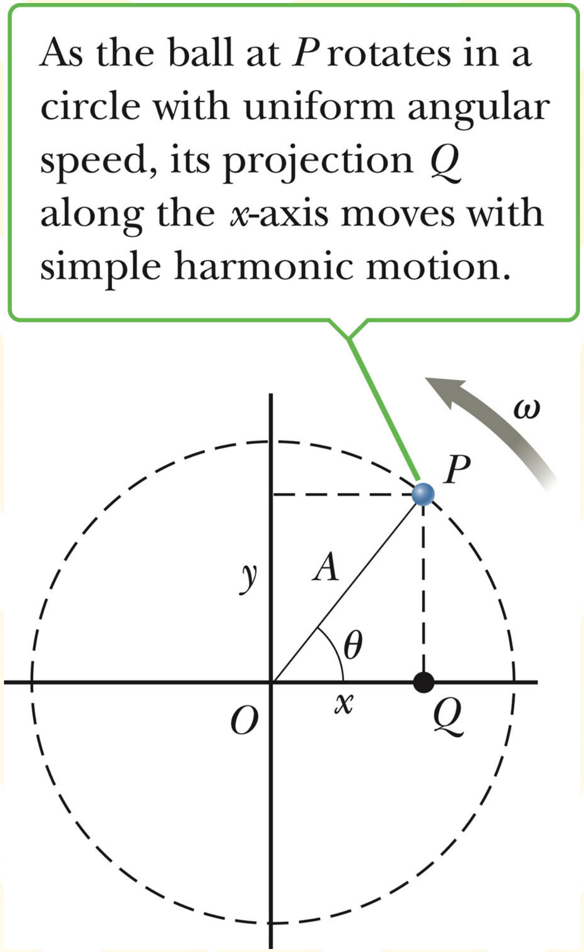
$$x = A\cos\theta = A\cos(\omega t)$$

Using ω = 2πf yields:

$$\boxed{x = A\cos(2\pi f t)}$$

After some algebra we find similar equations for the velocity and acceleration as a function of time:

$$\boxed{\begin{aligned} v &= -A\omega\sin(2\pi f t) \\ a &= -A\omega^2\cos(2\pi f t) \end{aligned}}$$

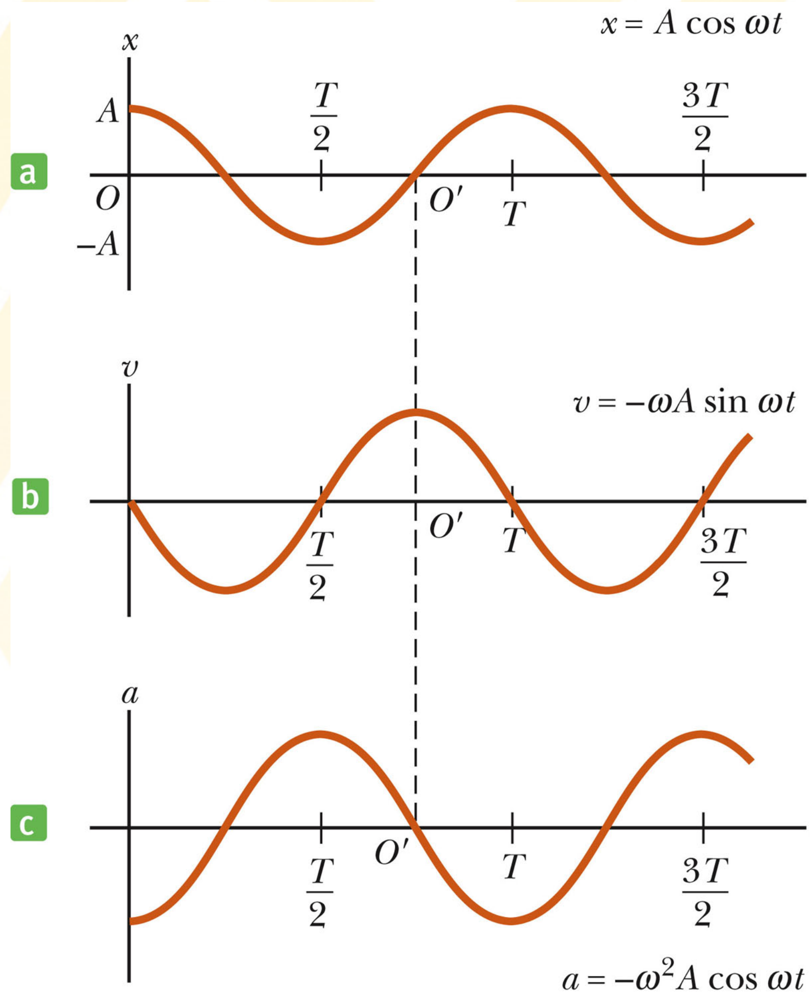


Position, velocity, Acceleration

There is a phase shift of 90 degree between position, velocity, and acceleration in simple harmonic motion.

When x = 0 m (equilibrium point), the velocity is maximum and the acceleration is zero.

When x = A, the velocity is zero and the acceleration is at its negative extremum.



The pendulum

A pendulum of length L oscillates because of the force of gravity acting on it.

The restoring force is the tangential component of F_g:

$$F_t = -mg\sin\theta = -mg\sin\left(\frac{s}{L}\right)$$

For small angles $\sin\theta \approx \theta = s/L$:

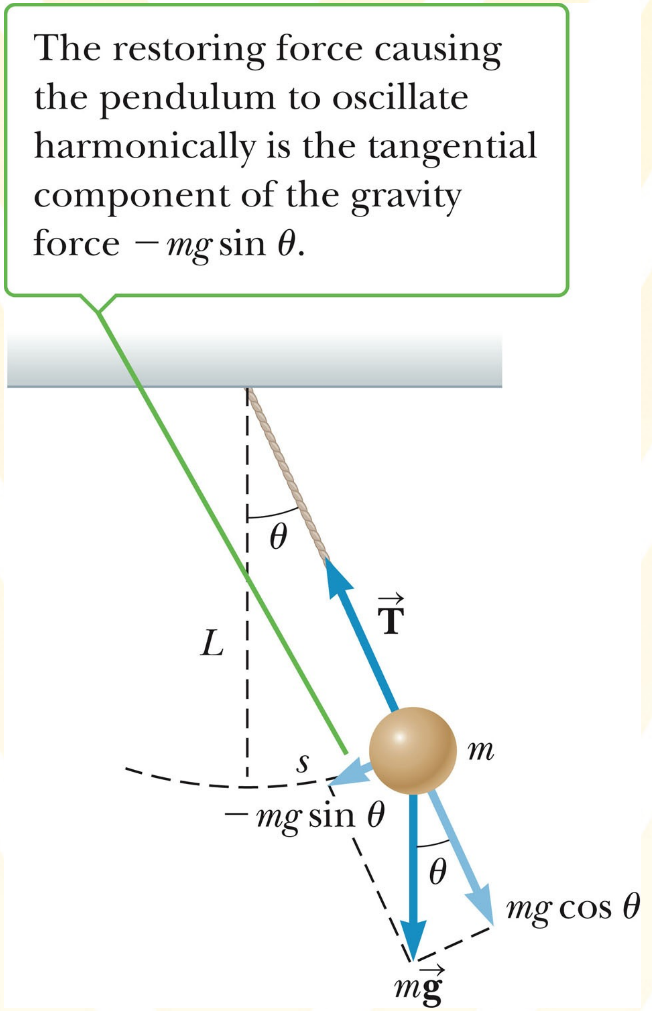
$$F_t = -\left(\frac{mg}{L}\right)s$$

This equation has the form of Hooke's Law ($F = -kx$) with $k = mg/L$.

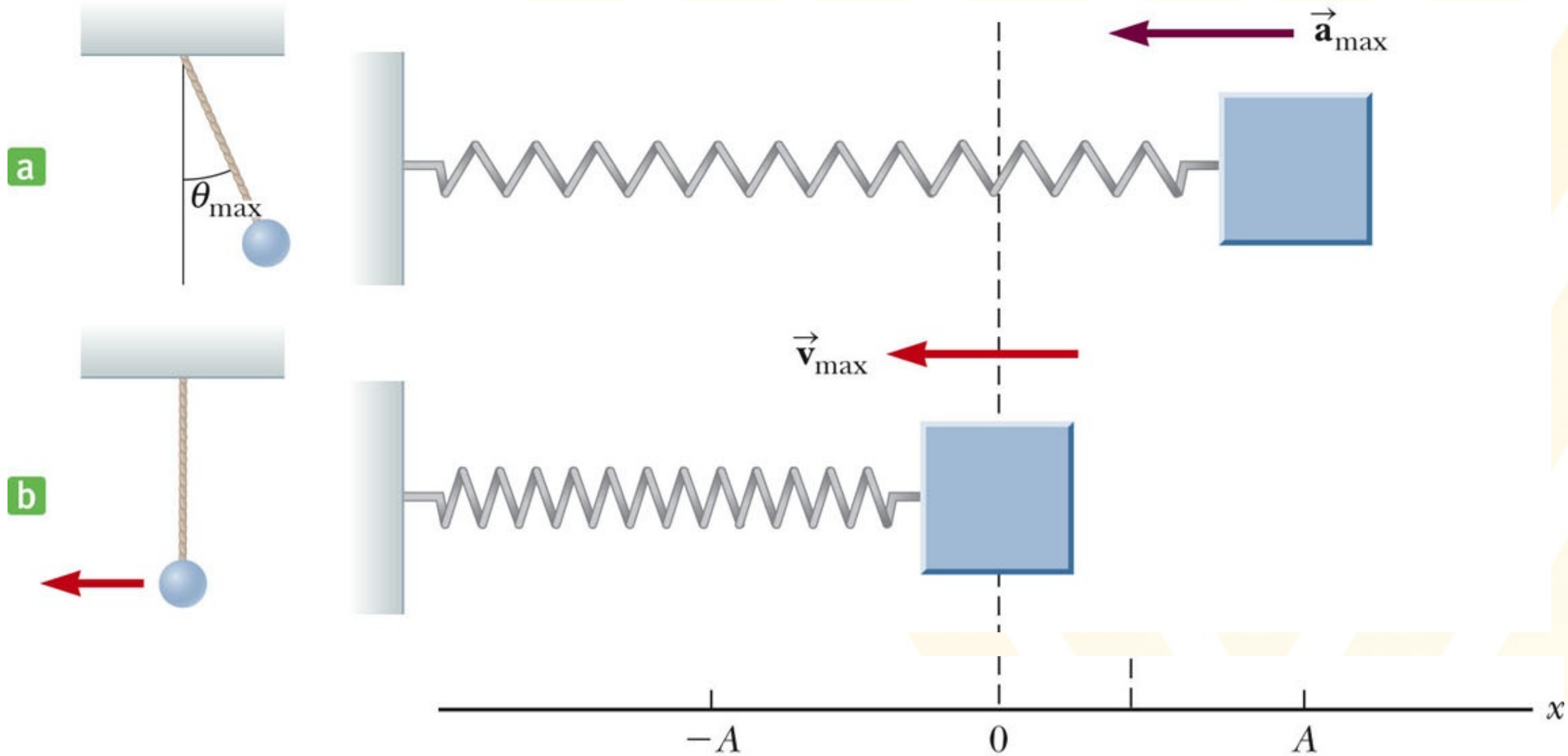
Using $\omega = \sqrt{\frac{k}{m}}$ yields: $\omega = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$

$$\rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

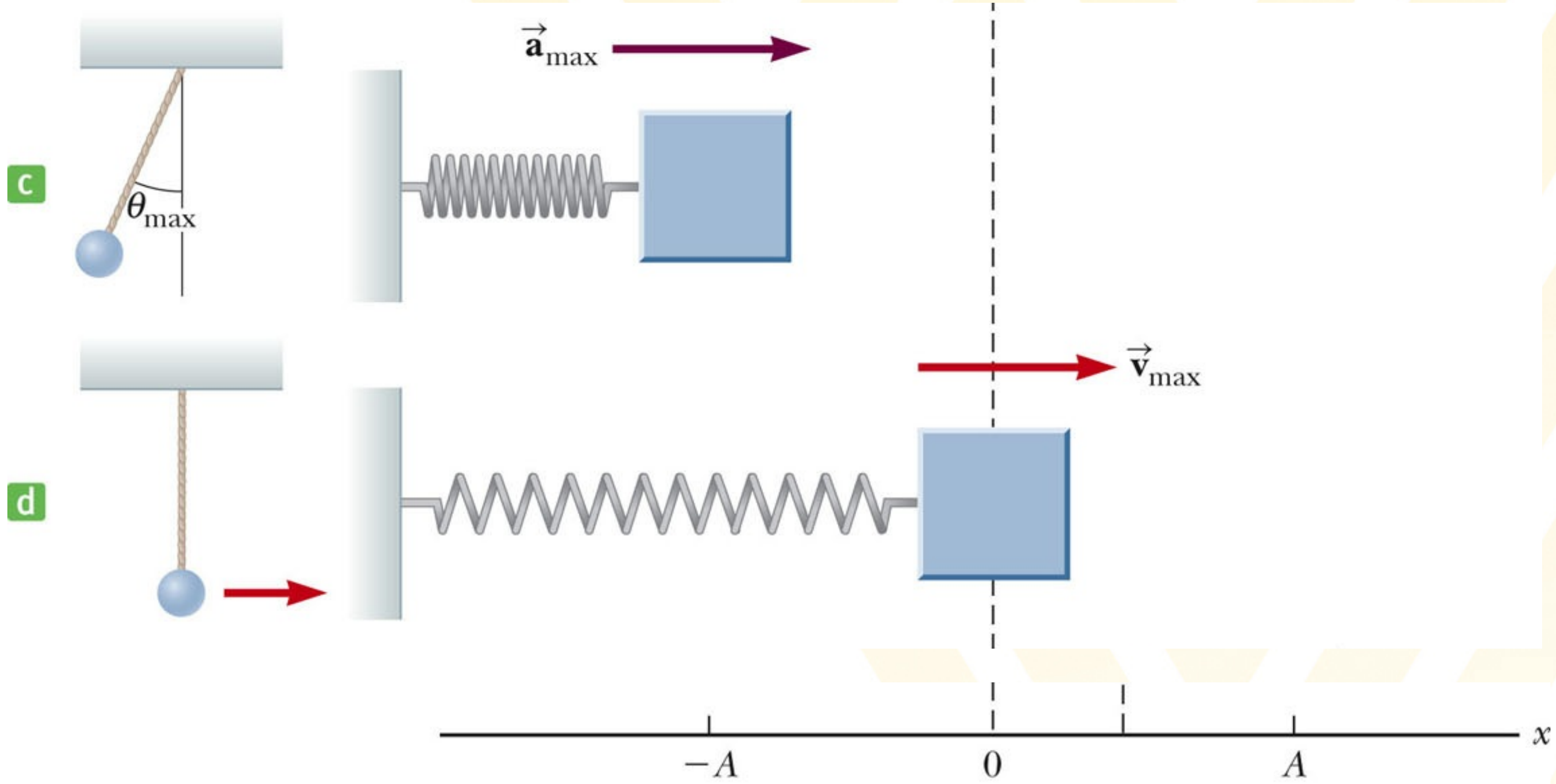
The period does not depend on the mass and amplitude.



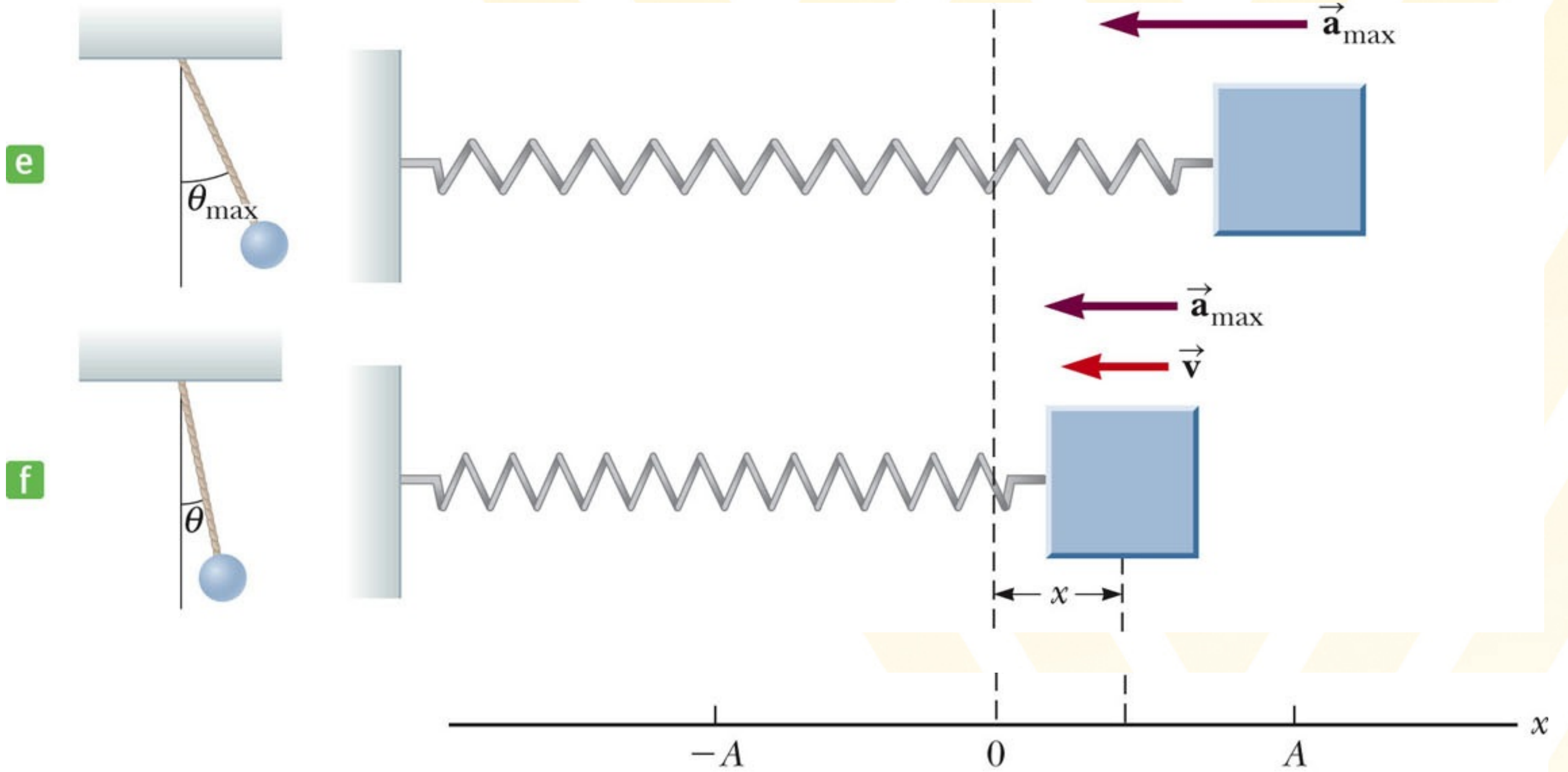
Analogy: Spring - pendulum



Analogy: Spring - pendulum

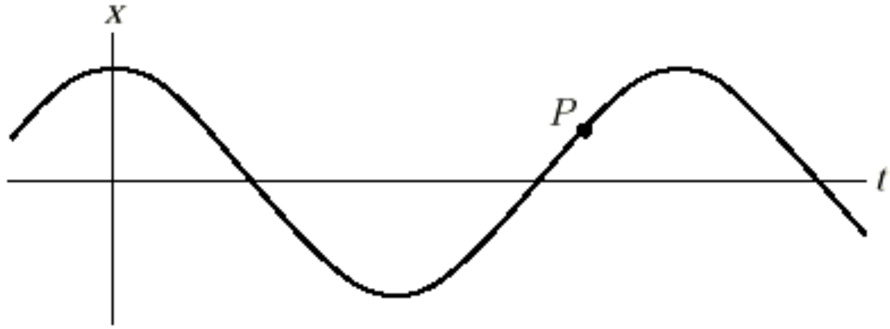


Analogy: Spring - pendulum



Clicker question

A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot below. At point P, the mass has



A. positive velocity and positive acceleration.
B. positive velocity and negative acceleration.
C. positive velocity and zero acceleration.
D. negative velocity and positive acceleration.
E. negative velocity and negative acceleration.