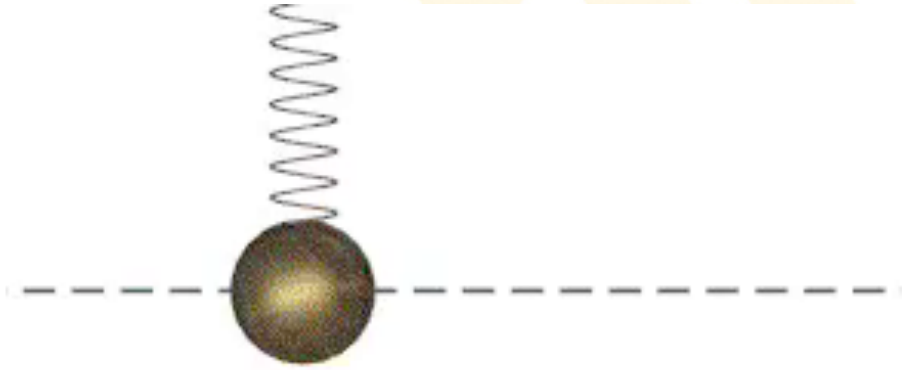
Announcements

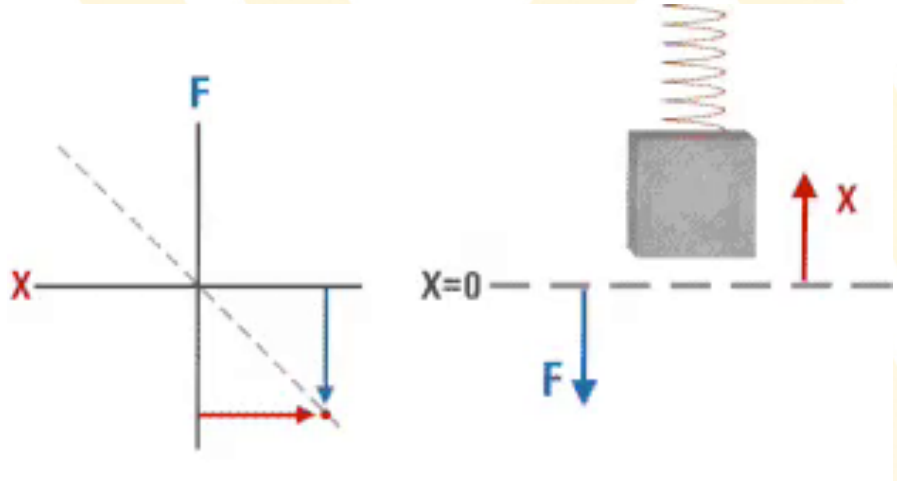
- The final exam takes place in White B51 on **Thursday, May 2, 8 PM - 10 PM**.
- The exams covers **all** sections that we have covered in class through the end of class today (i.e. Chapter 13 will be included, Chapter 14 will **not**).
- The formula sheet, some old exam and extra problems, and my own worked solutions to them, will be available on the course webpage on/after Wednesday.
- Best way to prepare: first, go through all worked examples and clickers from lecture slides, then, if you have time, go through homework and WebAssign "practice tests" (i.e. extra problems). Next, try the old exam without reference materials, and re-review the worked examples from step 1 for any problem areas. Finally (if you still have more time after all that) try additional problems from the text.

Chapter 13: Vibrations and Waves

- Hooke’s Law, Spring potential Energy
- Simple harmonic vs. circular motion
- Pendulums



Hooke's Law and Spring Potential Energy



Objects attached to springs undergo a periodic motion - they oscillate.

For short extensions from their equilibrium position Hooke's Law describes the force on the attached object:

$$F_s = -kx$$

This is a restoring force accelerating the object back to its equilibrium position. It linearly depends on the extension, x.

The spring potential energy is (see chapter 5):

$$PE_s = \frac{1}{2}kx^2$$

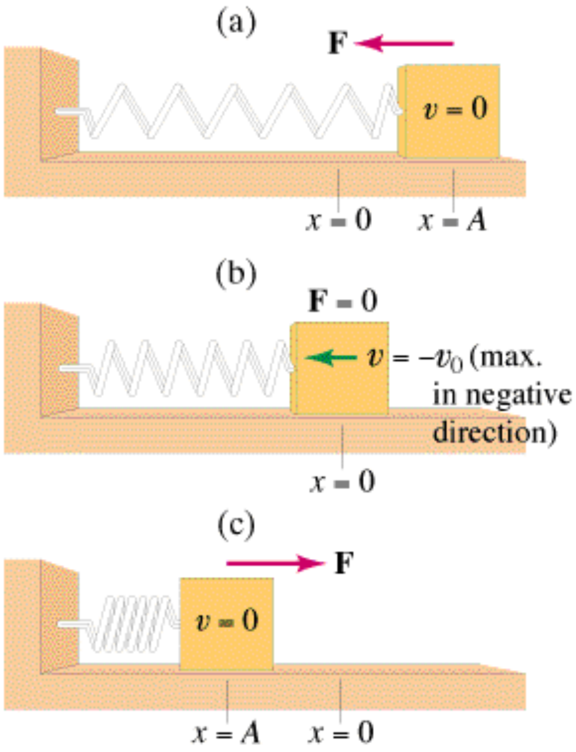
Some Definitions

The Amplitude, A, is the greatest distance from the equilibrium position.

The period, T, is the time it takes for the object to complete one complete cycle of motion from x = A to x = - A and back to x = A.

The frequency, f, is the number of complete cycles per unit time (f = 1/T).

When the net force along the direction of motion obeys Hooke's Law, Simple Harmonic Motion (SHM) occurs.



Acceleration in Simple Harmonic Motion

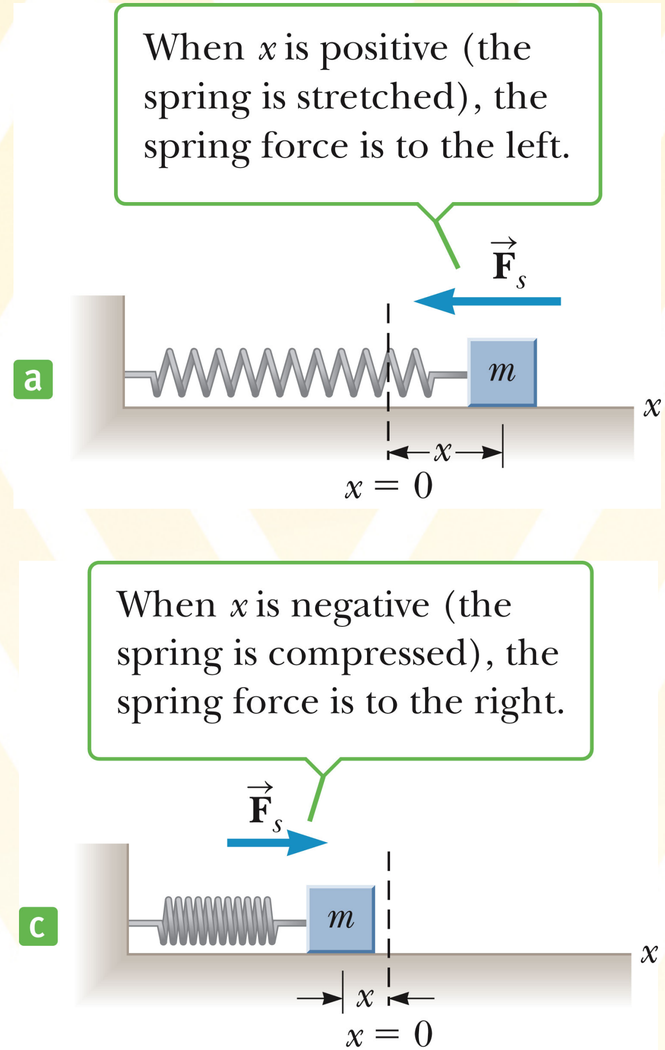
The acceleration can be found from Newton's second law and Hooke's Law:

$$F = ma = -kx$$

$$\rightarrow \quad \boxed{a = -\frac{k}{m}x}$$

The acceleration is not constant in time, since the position, x, is time dependent.

Simple harmonic motion is not a uniform motion with constant acceleration.



Clicker question

A block on the end of a horizontal spring is pulled from equilibrium at x = 0 to x = A.

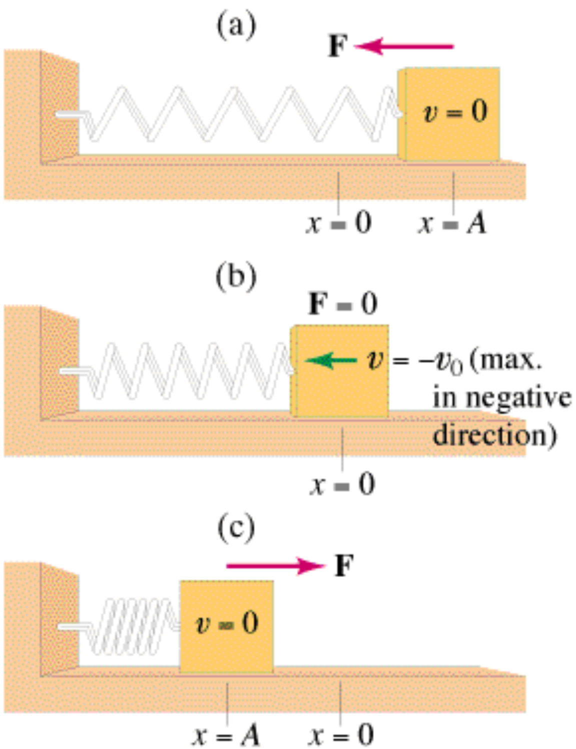
Through what total distance does it travel in one full cycle?

A. A/2

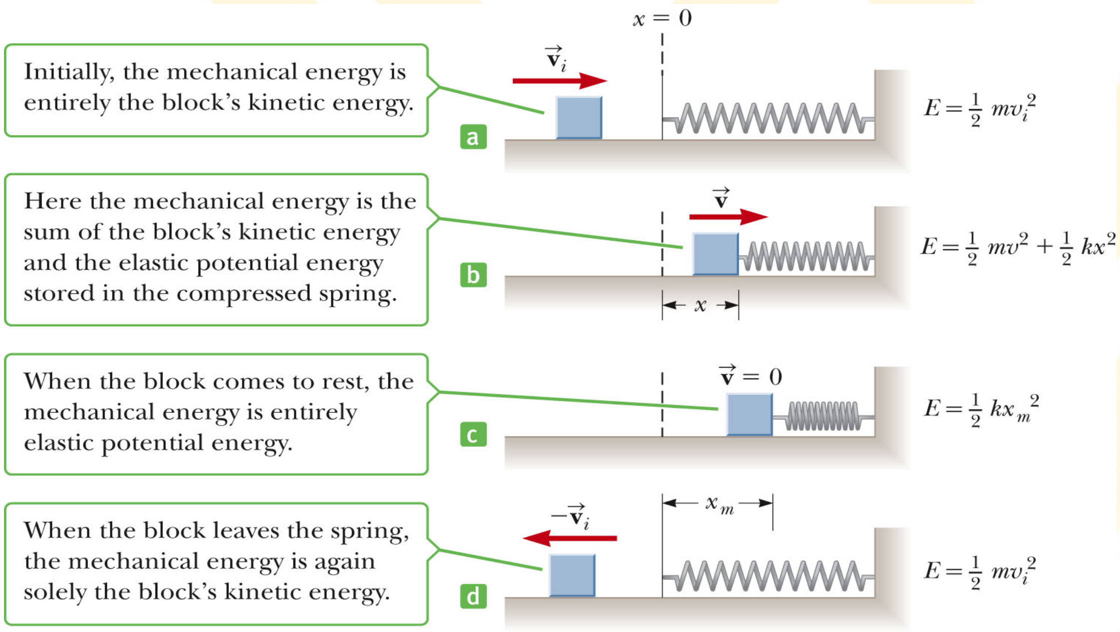
B. A

C. 2 A

D. 4 A



Energy conservation (SHM)



Initial time

Final time

Energy conservation: $(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$

At initial and final time (see plot): $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$$\rightarrow \quad v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

The velocity is maximum at the equilibrium point (x = 0 m)

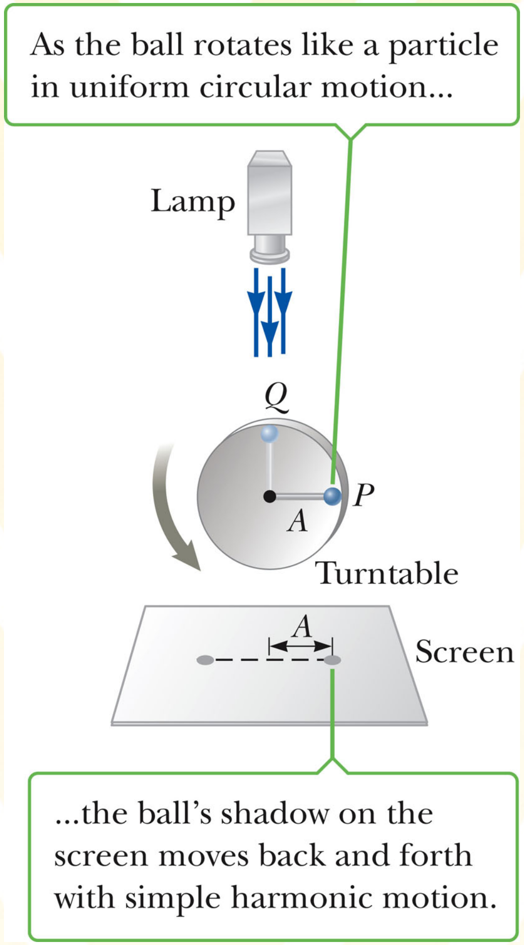
Simple Harmonic Motion vs. Circular Motion

As a turntable rotates with constant angular speed, the shadow of the ball (projection) moves back and forth with simple harmonic motion.

We can prove that this is true. For SHM the following equation for the velocity holds:

$$v = C\sqrt{A^2 - x^2}$$

If we can show that the same equation holds for the shadow of the rotating ball, we prove that it undergoes a Simple Harmonic Motion, too.



Simple Harmonic Motion vs. Circular Motion

For the rotating ball (small triangle):

$$\sin\theta = \frac{v}{v_0}$$

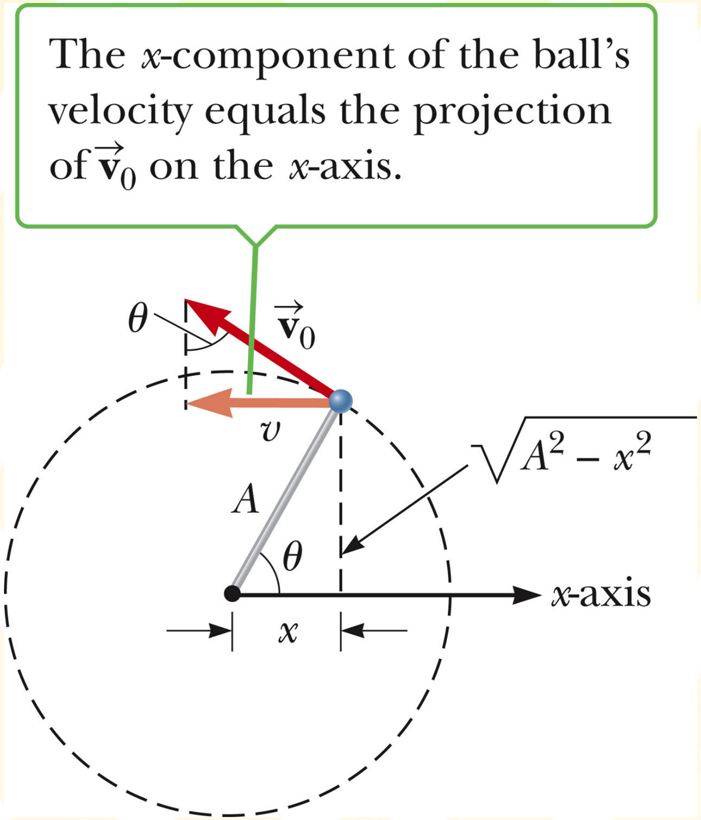
v is the x-component (shadow) of its total velocity v_0.

From the large triangle we get:

$$\sin\theta = \frac{\sqrt{A^2 - x^2}}{A}$$

$$\rightarrow \quad \frac{v}{v_0} = \frac{\sqrt{A^2 - x^2}}{A}$$

$$\rightarrow \quad v = \frac{v_0}{A}\sqrt{A^2 - x^2} = C\sqrt{A^2 - x^2}$$



The x-component of the velocity undergoes simple harmonic motion.

Period and Frequency

The period of the shadow of the rotating ball is the time it takes the ball to make one complete revolution on the turntable:

$$v_0 = \frac{2\pi A}{T} \quad \rightarrow \quad T = \frac{2\pi A}{v_0}$$

If the ball makes one quarter of a revolution, its shadow moves from x = A (only potential spring energy) to x = 0 m (only kinetic energy):

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 \quad \rightarrow \quad \frac{A}{v_0} = \sqrt{\frac{m}{k}}$$

Substituting this into the first equation for T yields:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Or for the angular frequency:

$$\omega = 2\pi f = 2\pi\frac{1}{T} = \sqrt{\frac{k}{m}}$$

Clicker question

An object of mass m is attached to a horizontal spring, stretched to a displacement A from equilibrium and released undergoing harmonic oscillations on a frictionless surface with period T_0. The experiment is then repeated with a mass of 4m.

What is the new period of oscillation?

A. $2T_0$ B. T_0 C. $T_0/2$ D. $T_0/4$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Position, velocity, Acceleration

The x-component of the position vector of the rotating ball is:

$$x = A\cos\theta = A\cos(\omega t)$$

Using ω = 2πf yields:

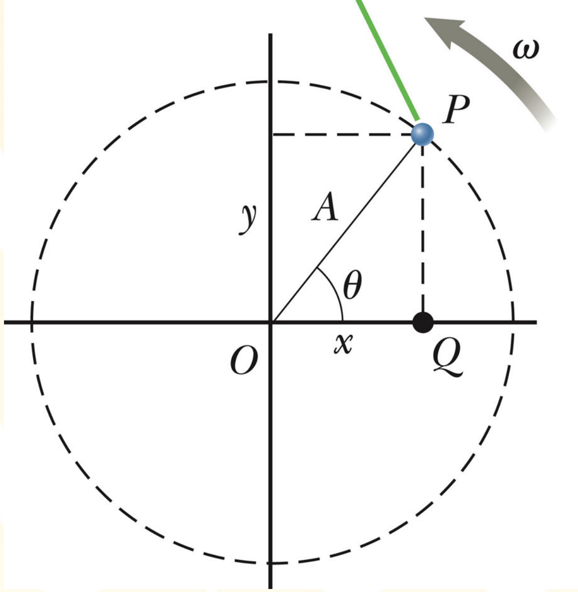
$$x = A\cos(2\pi f t)$$

After some algebra we find similar equations for the velocity and acceleration as a function of time:

$$v = -A\omega\sin(2\pi f t)$$

$$a = -A\omega^2\cos(2\pi f t)$$

As the ball at *P* rotates in a circle with uniform angular speed, its projection *Q* along the *x*-axis moves with simple harmonic motion.

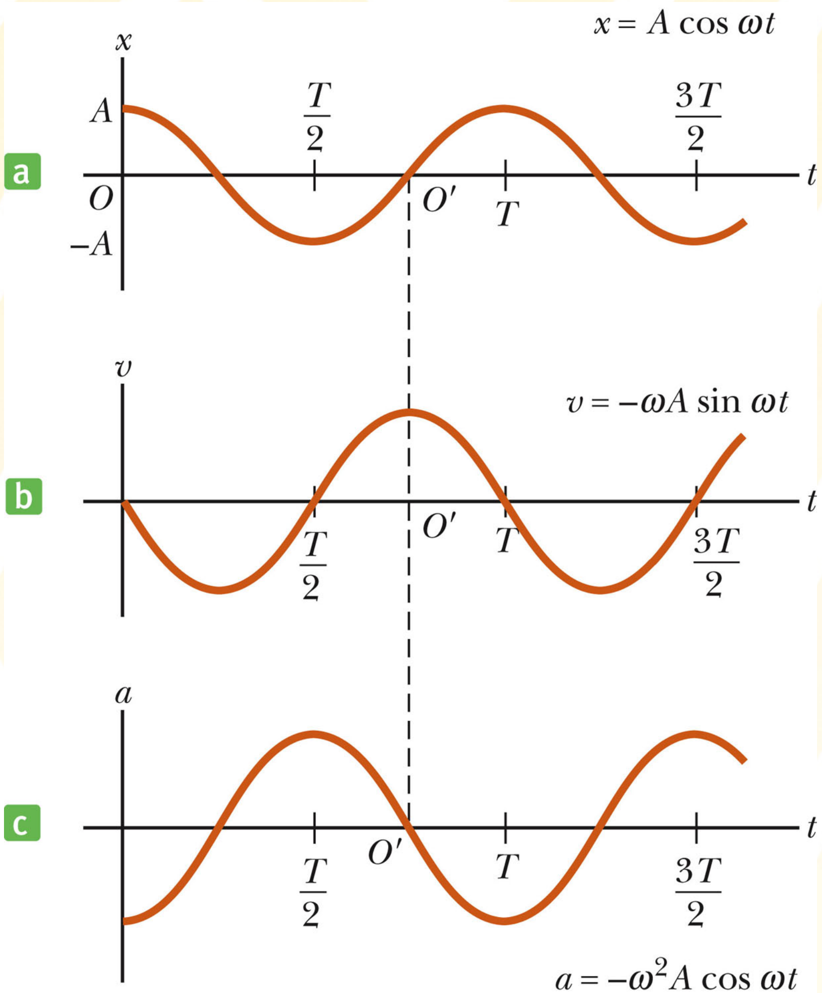


Position, velocity, Acceleration

There is a phase shift of 90 degree between position, velocity, and acceleration in simple harmonic motion.

When x = 0 m (equilibrium point), the velocity is maximum and the acceleration is zero.

When x = A, the velocity is zero and the acceleration is at its negative extremum.



The pendulum

A pendulum of length L oscillates because of the force of gravity acting on it.

The restoring force is the tangential component of F_g:

$$F_t = -mg\sin\theta = -mg\sin\left(\frac{s}{L}\right)$$

For small angles $\sin\theta \approx \theta = s/L$:

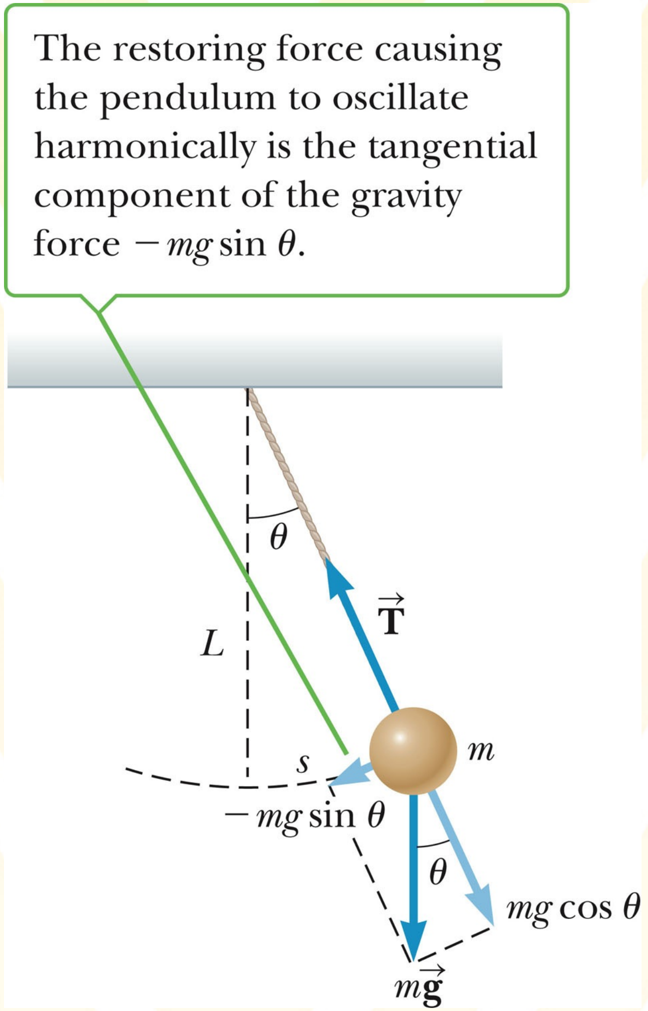
$$F_t = -\left(\frac{mg}{L}\right)s$$

This equation has the form of Hooke's Law ($F = -kx$) with $k = mg/L$.

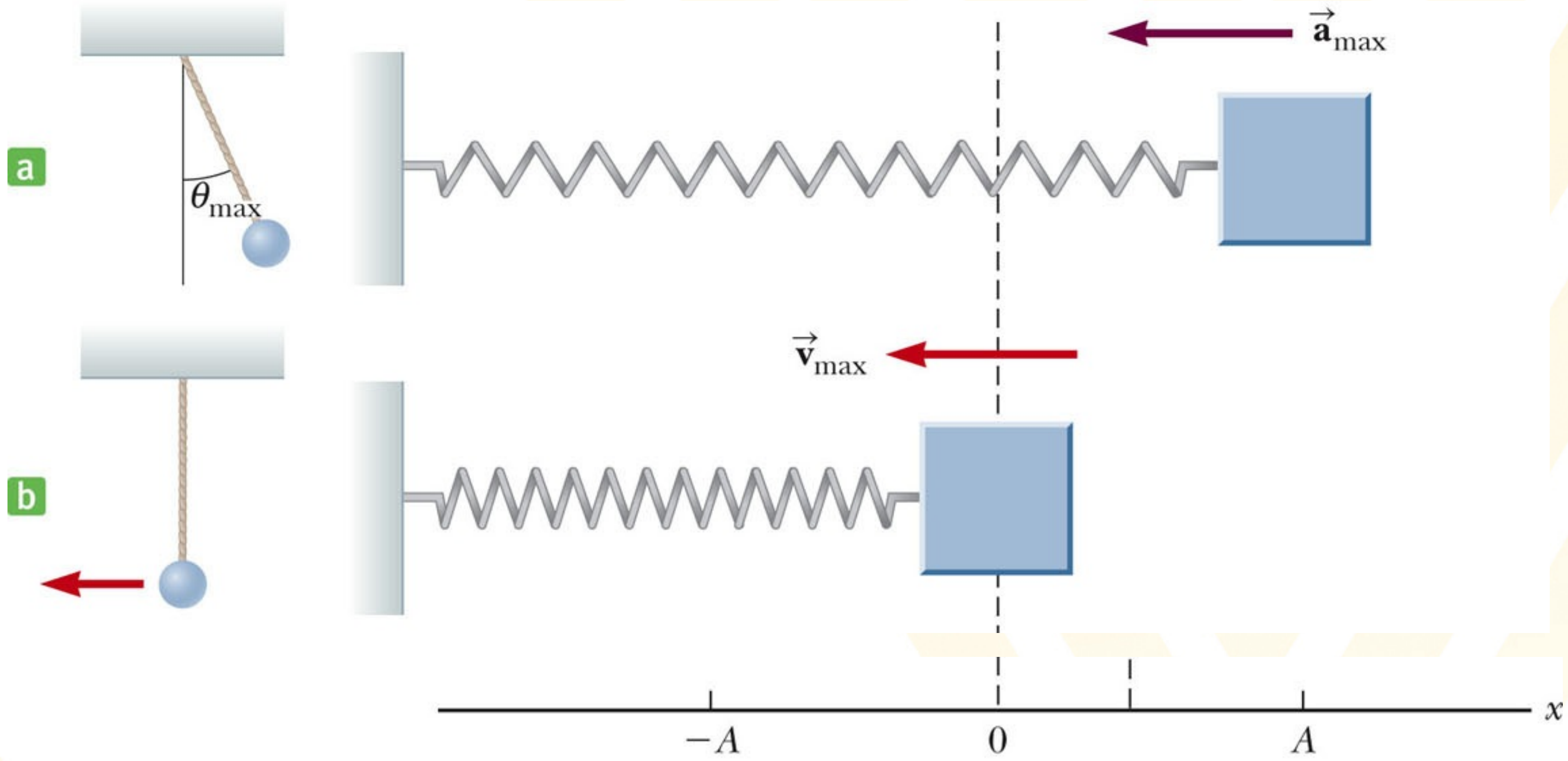
Using $\omega = \sqrt{\frac{k}{m}}$ yields: $\omega = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$

$$\rightarrow \quad T = 2\pi\sqrt{\frac{L}{g}}$$

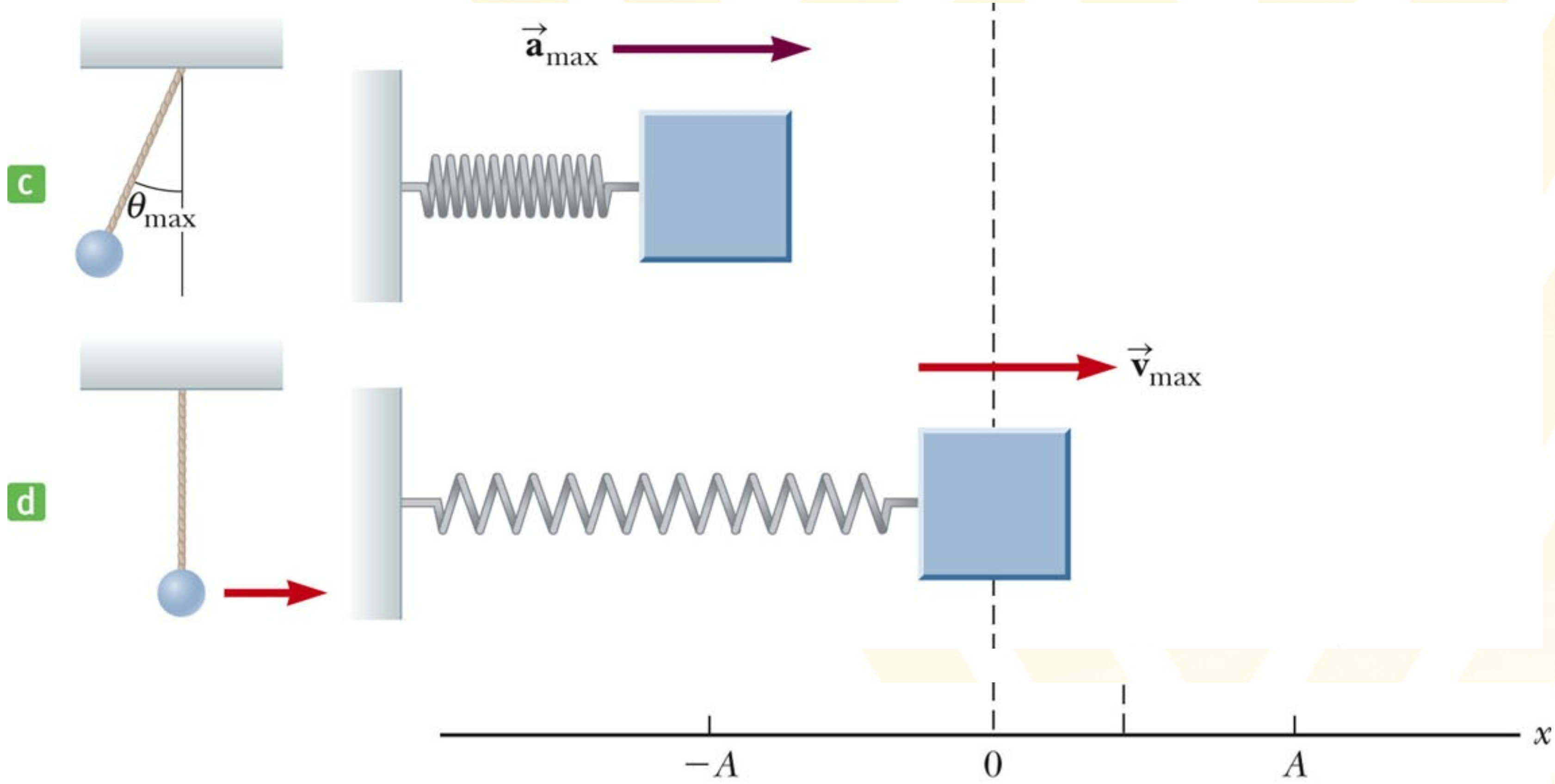
The period does not depend on the mass and amplitude.



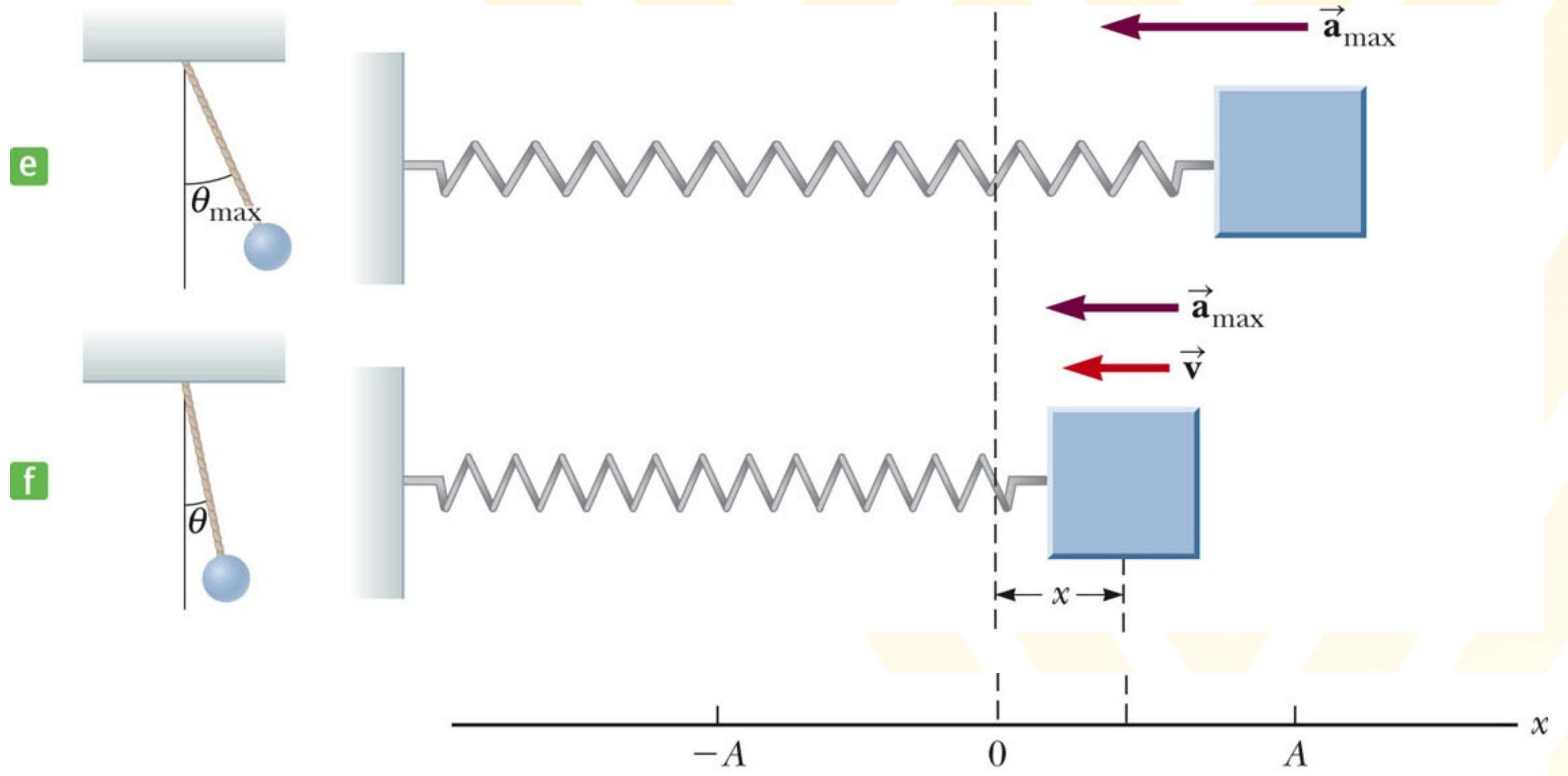
Analogy: Spring - pendulum



Analogy: Spring - pendulum



Analogy: Spring - pendulum



What is a wave?

Until now we discussed oscillations, i.e. simple harmonic motion around a *static* equilibrium point.

Waves are moving oscillations, i.e. the equilibrium point moves and is no longer static.

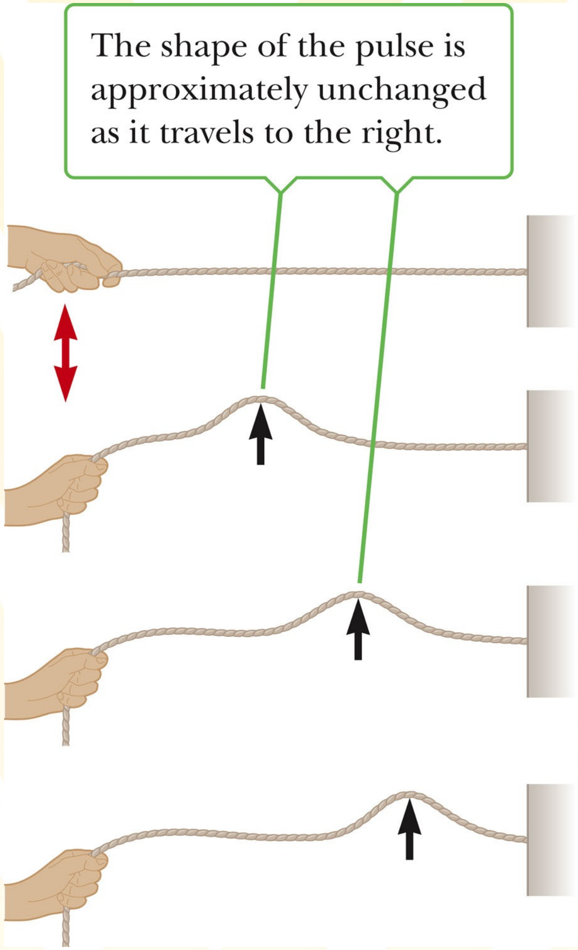
Examples for important wave phenomena:

- Sound waves
- Light waves
- Seismic waves
- radio waves
- water waves

A mechanical wave requires:

(i) A source of disturbance
(ii) A medium that can be disturbed

The medium itself does not move - the disturbance moves.



Different types of waves

Transverse waves:

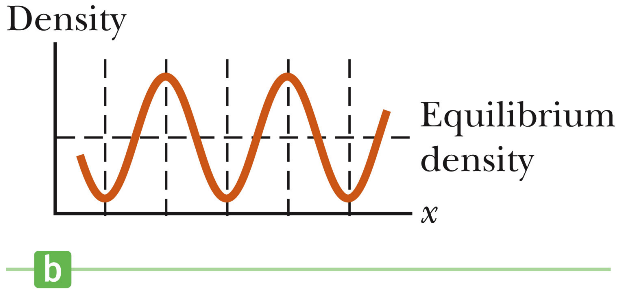
Each element that is disturbed moves in a direction *perpendicular to the wave motion.*

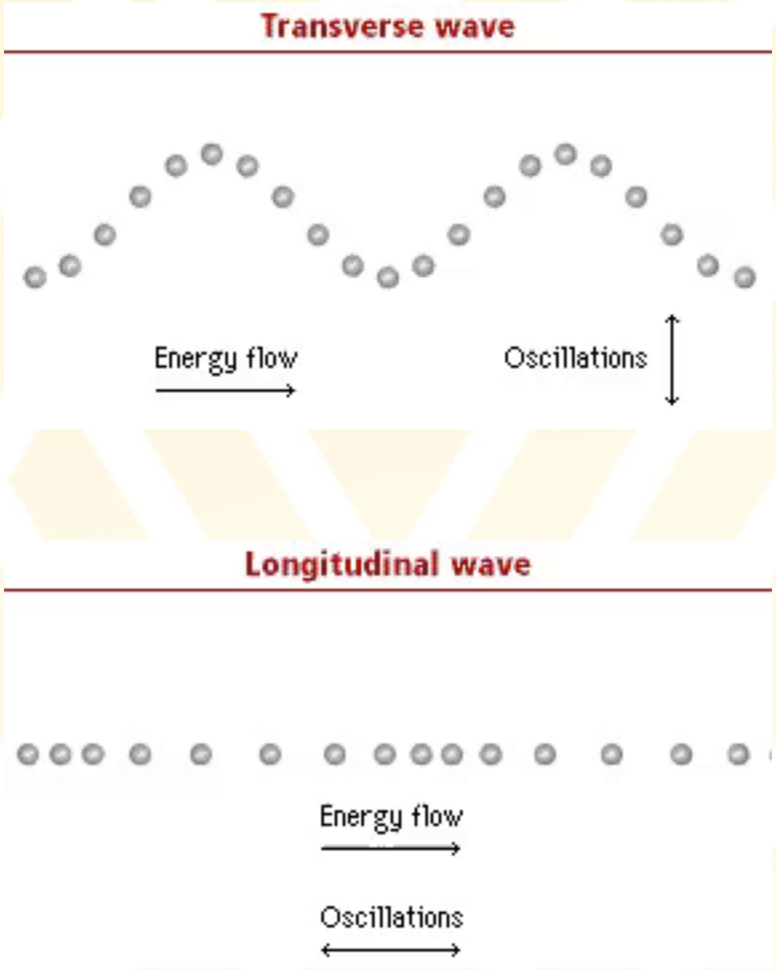
Example: Wave on a rope.

Longitudinal waves:

The elements of the medium undergo displacements *parallel to the motion of the wave.*

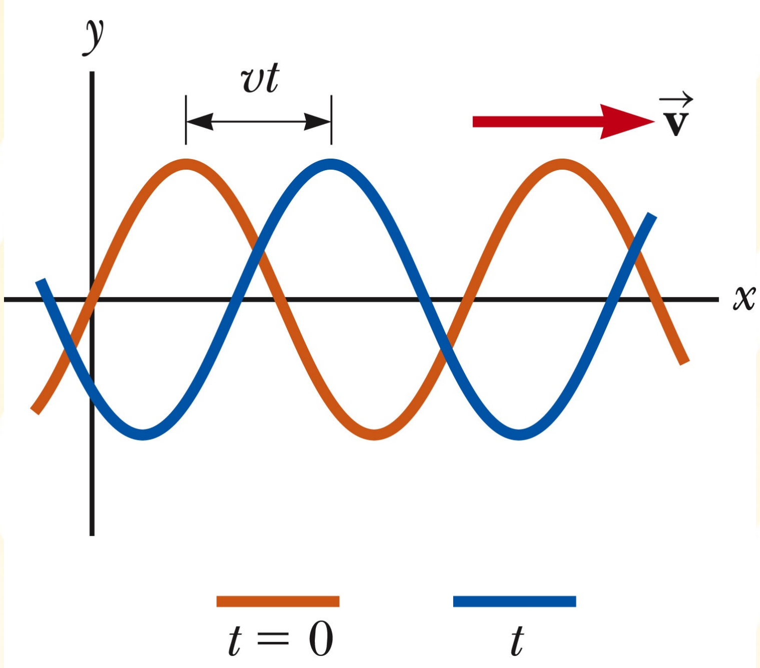
Example: Sound (density wave)





Wave motion

- The brown curve is a "snapshot" of the wave *at some instant in time*
- The blue curve is *later* in time
- The high points are <u>crests</u> of the wave
- The low points are <u>troughs</u> of the wave



Amplitude, Period, Frequency

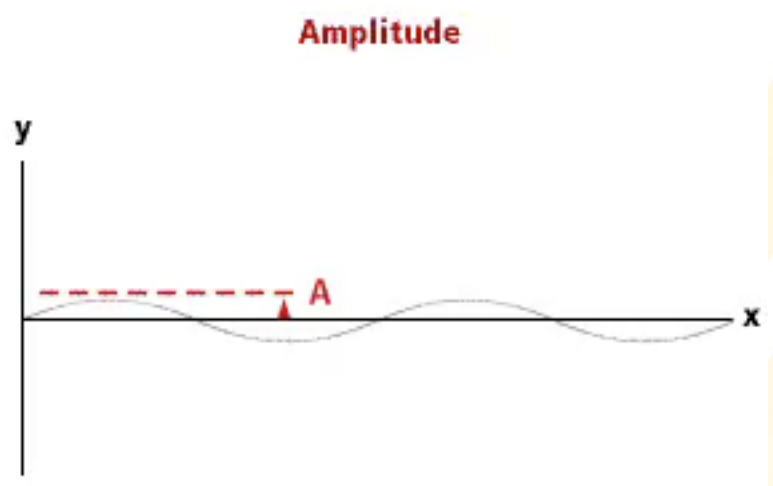
Example - Wave on a string:

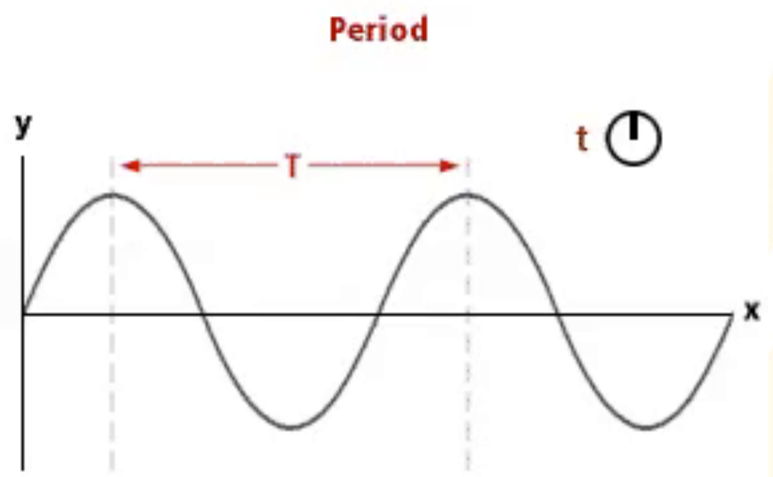
The maximum distance the string moves above or below the equilibrium position is called the amplitude, A, of the wave.

The time interval it takes two consecutive crests to pass a given position is called the wave period, T.

The inverse of the period is the frequency:

$$f = \frac{1}{T}$$



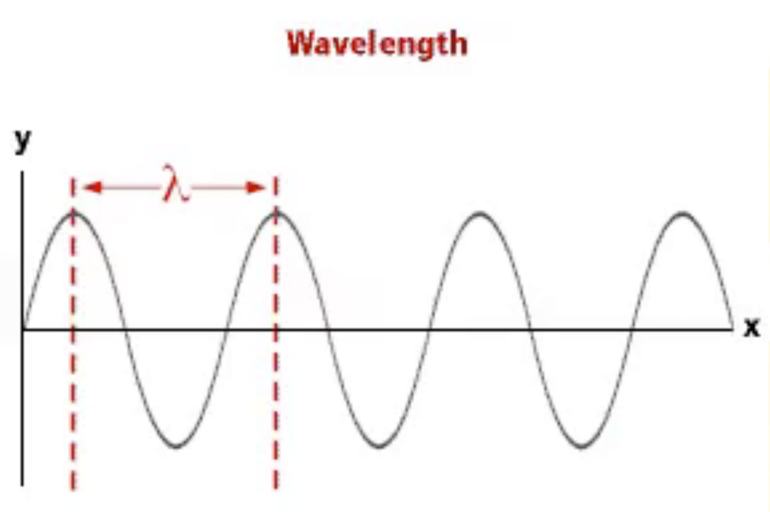


Wavelength and wave speed

The distance between two successive points that behave identically is called the wavelength, λ.

A wave advances a distance of one wavelength during one period:

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f$$



The speed of a wave propagating on a string stretched under some **tension, F** is:

$$v = \sqrt{\frac{F}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}$$

Here, μ is called the *linear density.*

The speed depends only upon the properties of the medium through which the disturbance travels