Vectors

- A vector is an arrow characterized by its magnitude and direction.

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} \quad \text{Magnitude:} \quad A = \sqrt{A_x^2 + A_y^2} \quad \text{Direction:} \quad tan(\theta) = \frac{A_y}{A_x}$$

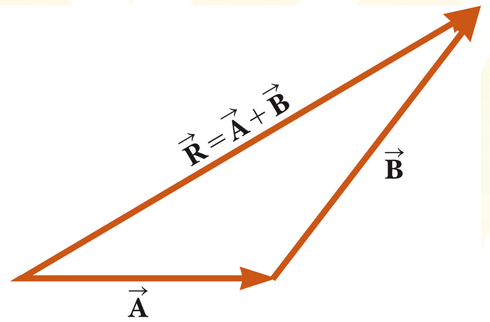
- Vectors are required to describe the position of objects and their motion in more than one direction.

- Two vectors are identical, if their magnitudes and directions are the same.

- Multiplying a vector with a scalar, k, changes its magnitude by a factor of k: $k\vec{A} = \begin{pmatrix} kA_x \\ kA_y \end{pmatrix}$

- Two vectors can be added geometrically or algebraically:

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \end{pmatrix}, \vec{B} = \begin{pmatrix} B_x \\ B_y \end{pmatrix} \rightarrow \vec{R} = \begin{pmatrix} R_x \\ R_y \end{pmatrix} = \begin{pmatrix} A_x + B_x \\ A_y + B_y \end{pmatrix}$$



A minus sign reverses a vector's direction.
Subtracting 2 vectors is the same as adding a negative vector.

- A vector's components can be determined: $A_x = A \cdot cos(\theta), \; A_y = A \cdot sin(\theta)$

What you learn from graphs

Type of graph	Slope gives:	Change of direction
Position vs Time	Velocity	At maximum or minimum
Velocity vs Time	Acceleration	When curve crosses axis
Acceleration vs Time	---	Can't determine

Mathematical description of 1d motion with constant acceleration

An objects moves in one direction with constant acceleration [a(t) = const.].

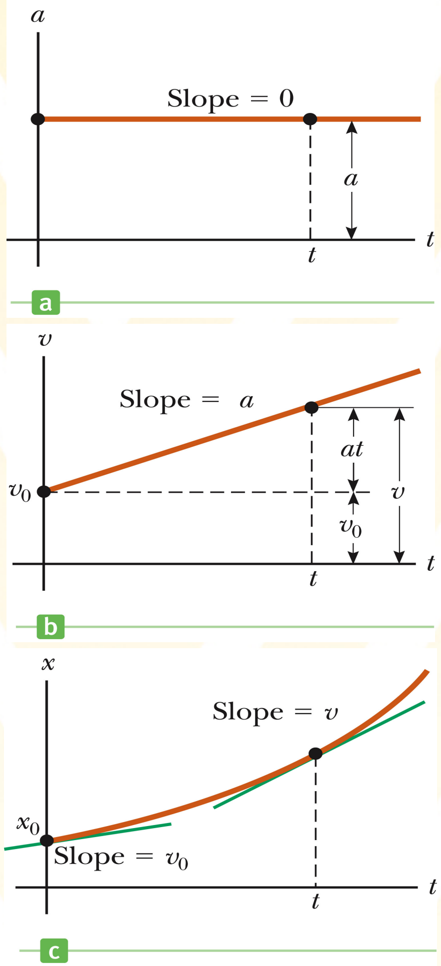
a(t) = const. results in a horizontal line in the a-t-graph.

How can we calculate v(t) and x(t)?

$$v(t) = v_0 + at \qquad x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

Initial velocity at t = 0s   Initial position at t = 0s

- v(t) is a linear function and results in a straight line in the v-t-graph.
- x(t) is a quadratic function and results in a parabola in the x-t-graph.



Mathematical description of 1d motion with constant acceleration

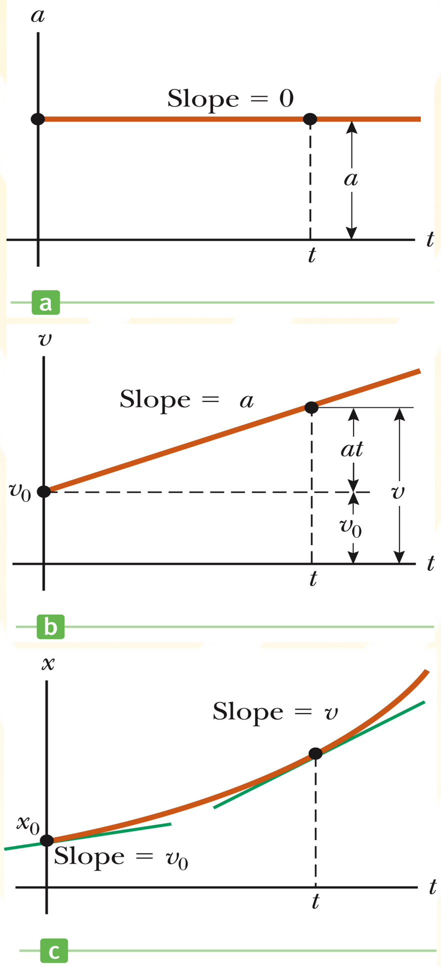
Other useful formula:

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t \qquad \Delta x(t) = x(t) - x_0$$

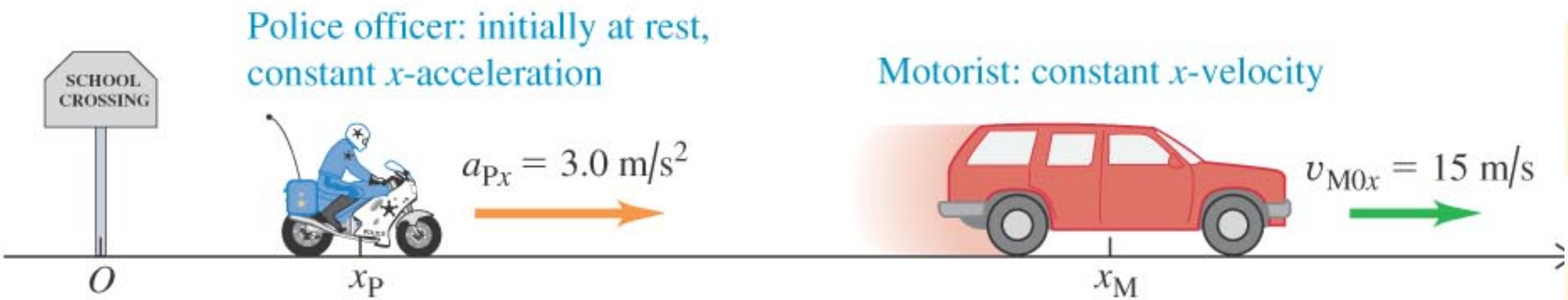
$$v^2(t) = v_0^2 + 2a\Delta x^2(t)$$

These equations (this and previous slide) are only valid for 1d motion with **constant** acceleration.

These formula are extremely important for solving many problems relevant for real life, homework, and exams.



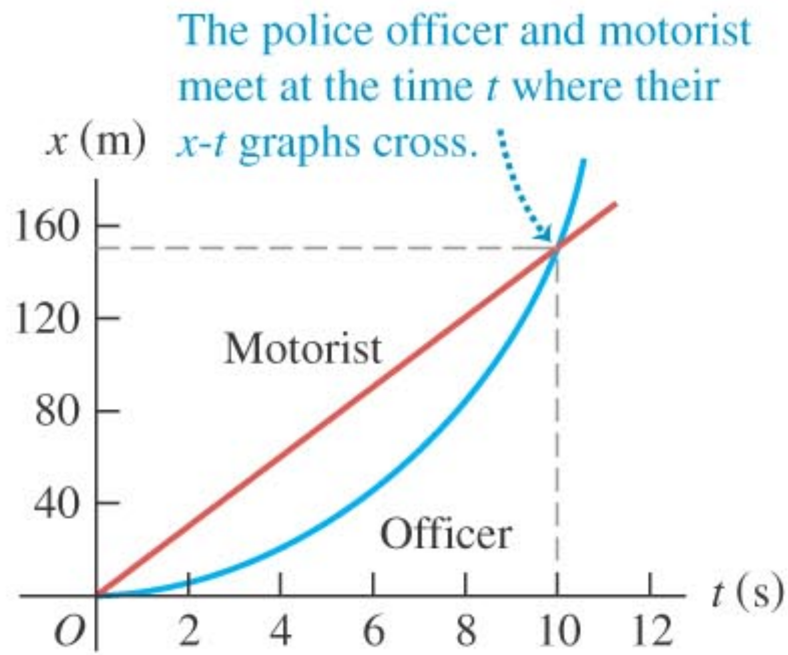
Example problem



A car is traveling at 15 m/s, when it passes a trooper, who does not move. The trooper sets off in chase immediately with a constant acceleration of 3.0 m/s².

(a) How long does it take the trooper to overtake the car?

(b) How fast is the trooper going at that time?



Definition of free Fall

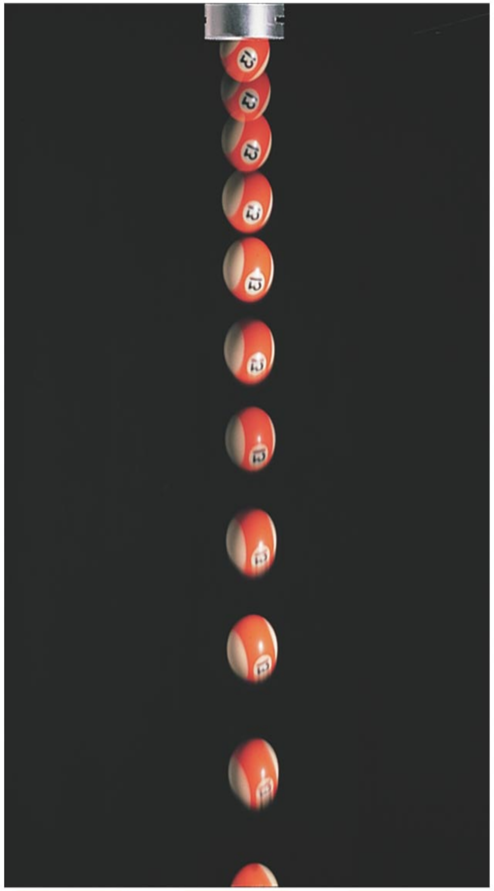
- Free Fall is the motion of an object under the influence of gravity <u>alone</u> (no other forces/accelerations).

- Such objects do not have to start from rest, but can have an initial velocity pointing upwards.

  Example: A ball is thrown upwards. After the ball is thrown (no more acceleration in upward direction), it moves upwards. This is an example of free fall, since only gravity influences the ball's motion.

- The velocity change in each time interval is constant:

$$a = \frac{\Delta v}{\Delta t} = -g = -9.81 m/s^2$$

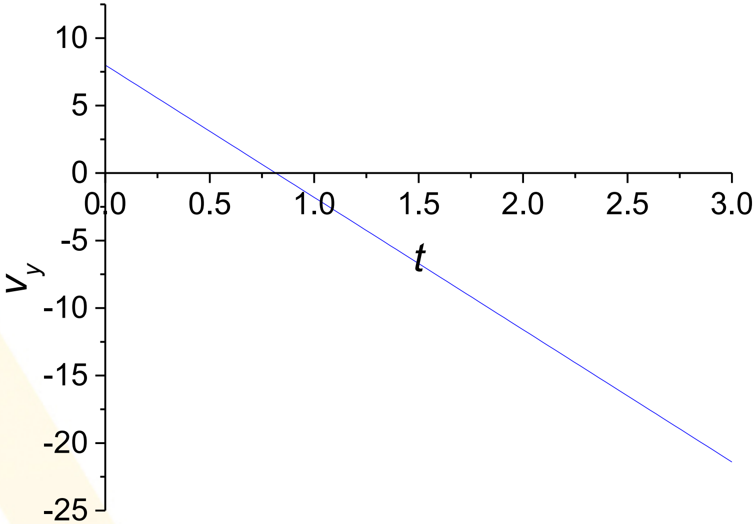


Graphing freely falling bodies (a = -g = const.)

$$v_y = v_0 - gt$$

This is a linear function of t. The slope is negative (-g).

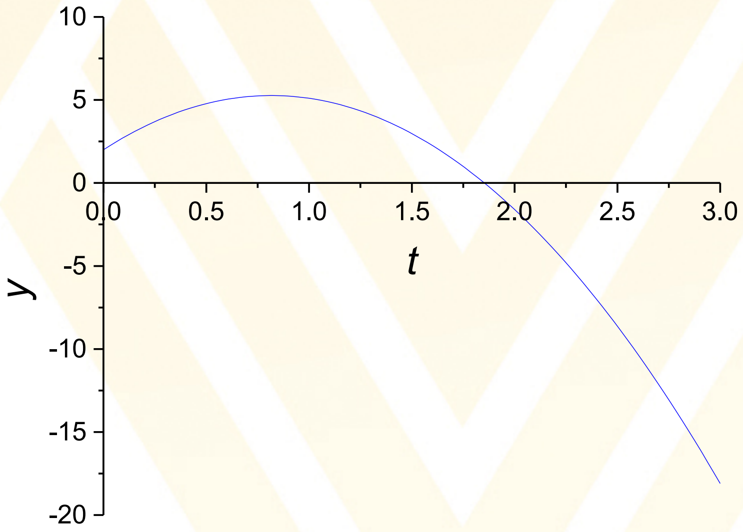
→ Straight line in the v_y - t - diagram.



$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

This is the sum of a linear function and a quadratic function of t.

→ For low values of t, it is a straight line.
→ For high values of t, it is a parabola.



Example problem: Free Fall

A ball is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The ball just misses the edge of the roof on its way down. Determine

A. the time needed for the ball to reach its maximum height.

B. the maximum height itself.

C. the time needed for the ball to return to the height from which it was thrown and the velocity of the ball at that instant.

D. the time needed for the ball to reach the ground.

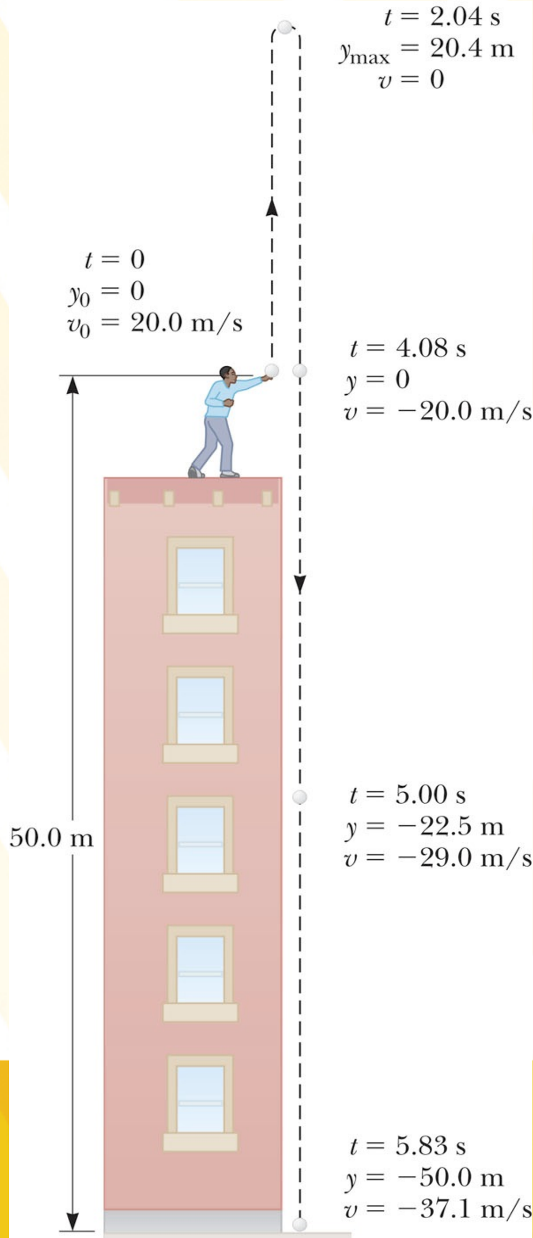
E. the velocity and position of the ball at t = 5s.

Neglect air drag.

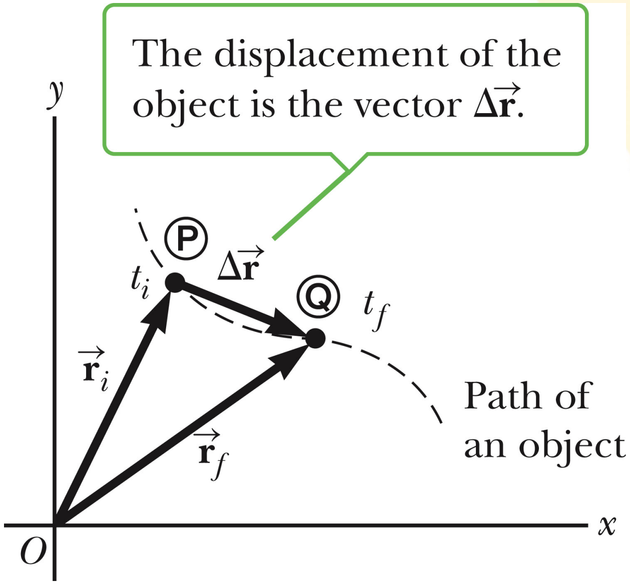
Some helpful equations:

$$v(t) = v_0 - gt$$

$$y(t) = y_0 + v_0 t - \frac{1}{2}gt^2$$



Displacement, Velocity, Acceleration in 2d



In 2d problems, the position of an object is determined by its position vector.

If an object moves from an initial to a final position, the displacement will be a vector, too (unit: m):

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$

Velocity (unit: m/s) and acceleration (unit: m/s^2) are also vectors:

$$\vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad \vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

An object can accelerate in different ways:

1. The magnitude of the velocity changes with time (the same as 1d problems)
2. The direction of the velocity may change with time, e.g. circular motion, at constant speed.
3. Magnitude and direction change in parallel.

Projectile motion

A projectile is any body, given an initial velocity, that then follows a path determined by the effects of gravity and air resistance. We neglect air resistance.

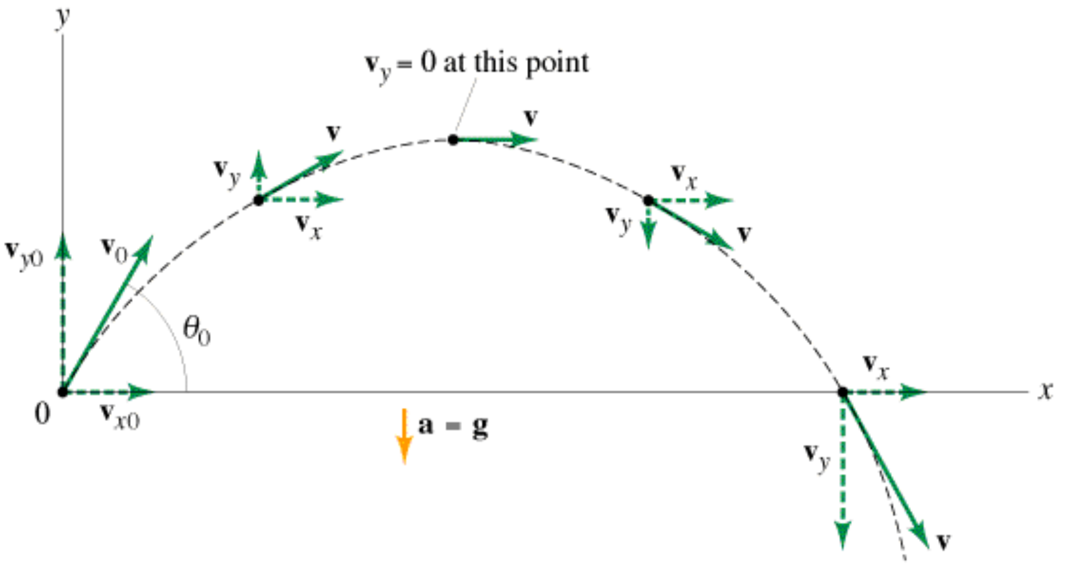
How would Coyote E. Wiley fall down a cliff in reality?

- As Mr. Coyote runs off the cliff, he has horizontal velocity.
- A change in velocity is acceleration, in this case horizontal acceleration, which must come from a force in the horizontal direction. $v_{fx}=v_{ox}+a_x t$
- If we ignore air resistance (horizontal force = 0), then there is no horizontal force to slow him down horizontally. $v_{fx}=v_{ox}$
- Thus, Mr. Coyote will travel ***horizontally at the same speed*** the whole time until he hits the ground!

How would Coyote E. Wiley fall down a cliff in reality?

- Vertical motion is treated separately.
- As soon as the coyote leaves the cliff he will experience a vertical force due to gravity.
- This force will cause him to start to accelerate in the vertical direction. As he falls he will be going faster and faster in the vertical direction.
- The horizontal and vertical components of the motion of an object going off a cliff are separate from each other (if we ignore air resistance), and **<u>cannot affect each other</u>**.

Mathematical description of projectile motion



$\vec{v}_o \equiv$ intial velocity vector

$\theta_o \equiv$ initial direction of velocity vector

$v_y = 0$ at top of trajectory

$v_x = v_{xo}$ remains the same throughout trajectory because there is no acceleration along the x-direction.

$$v_{x0} = v_0 \cdot cos\theta$$

$$v_{y0} = v_0 \cdot sin\theta$$

We can use the equations to describe 1d motion with constant acceleration for the horizontal and vertical direction separately:

$$x(t) = x_0 + v_{x0}t + \frac{1}{2}at^2 \rightarrow x(t) = v_{x0}t$$

$$y(t) = y_0 + v_{y0}t + \frac{1}{2}at^2 \rightarrow y(t) = v_{y0}t - \frac{1}{2}gt^2$$

$$v_x(t) = v_{x0} + at \rightarrow v_x(t) = v_{x0}$$

$$v_y(t) = v_{y0} + at \rightarrow v_y(t) = v_{y0} - gt$$

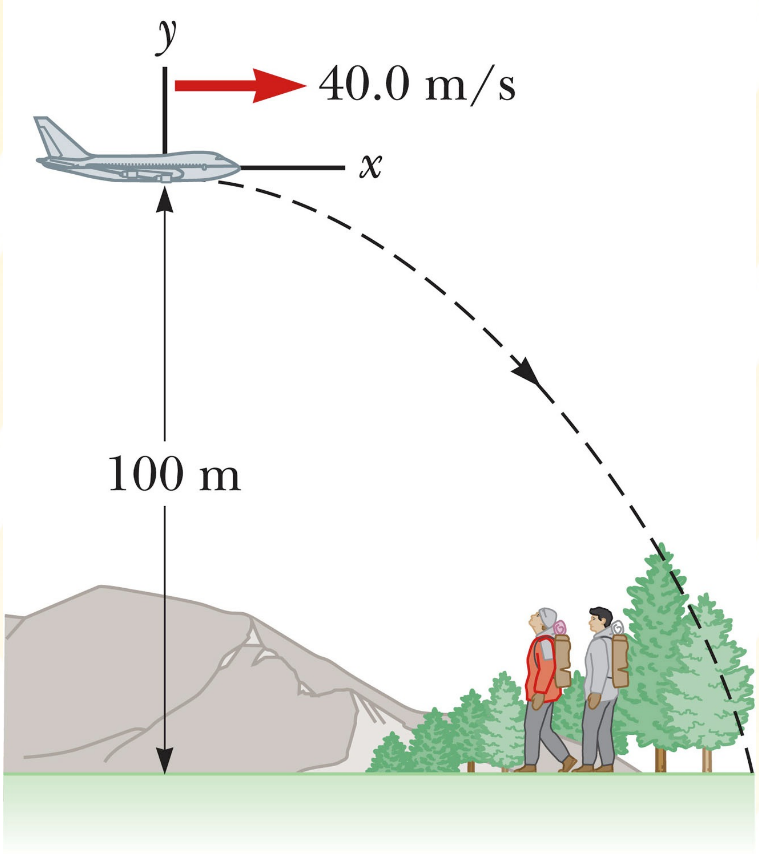
Example problem: Projectile motion

An alaskan rescue plane drops a package of emergency rations to stranded hikers. The plane is traveling horizontally at 40 m/s at a height of 100 m above the ground.

(a) Where does the package strike the ground relative to the point at which it was released?

(b) What are the vertical and horizontal components of the velocity of the package just before it hits the ground?

(c) Find the angle of impact.



Average Speed

A person travels by car from one city to another with different constant speeds between pairs of cities. She drives for 35 min at 75 km/h, 12 min at 50 km/h, and 30 min at 50 km/h and spends 25 min eating lunch and buying gas.

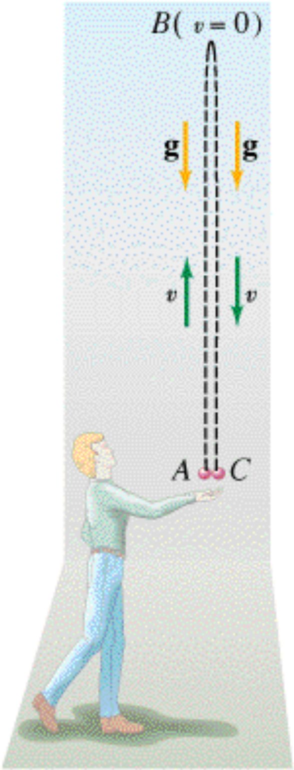
(a) Determine the average speed for the trip.

(b) Determine the distance between the initial and final cities along the route.

Free Fall example problem

A baseball is thrown up in the air at an initial velocity of 22.0 m/s.

(a) How high up does it go?
(b) How long is it in the air if you catch it at the same height you initially let go of the ball?



Some helpful equations:

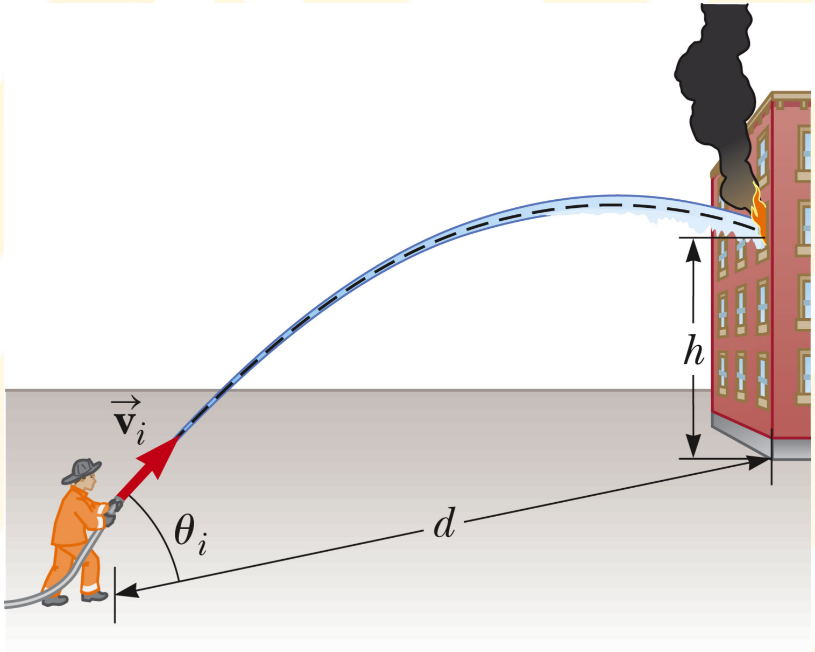
$$v(t) = v_0 - gt$$

$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$

Projectile motion example problem

A fireman d = 57 m away from a burning building directs a stream of water from a ground-level fire hose at an angle of 23°above the horizontal.

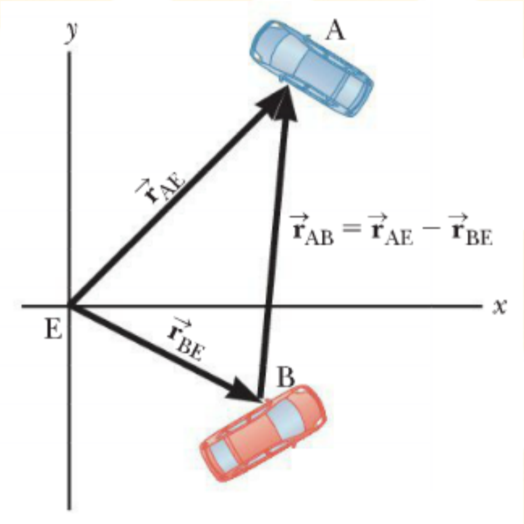
If the speed of the stream as it leaves the hose is v_i = 40 m/s, at what height will the stream of water strike the building?



Relative velocities

Velocities depend on the **frame of reference**. Something moves at velocity **v** *relative to what?* Think of moving walkways…

Since $\vec{\mathbf{v}}_{av} \equiv \dfrac{\Delta\vec{\mathbf{r}}}{\Delta t}$, relating velocities is the same as relating positions (since each position can be divided by Δt).



$$\vec{\mathbf{r}}_{AB} = \vec{\mathbf{r}}_{AE} - \vec{\mathbf{r}}_{BE}$$

$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} - \vec{\mathbf{v}}_{BE}$$

Conversion of units

Every physical quantity has a unit. Make sure not to forget these in the exam!

Typical problem: Convert an acceleration of 25 m/s^2 to km/min^2.

Scientific notation

What would be the most accurate method for writing the following numbers in scientific notation:

13,000

560,000

0.0003

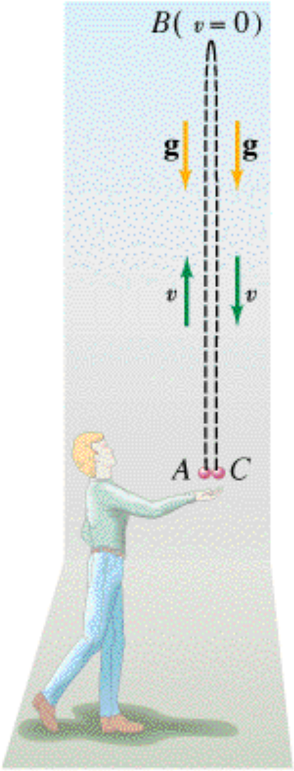
0.01

Example problem: 1d motion with constant acceleration

A car starts from rest and accelerates at 10 m/s^2 in a straight line on a level street for a distance of 100 m.

What is the velocity of the car at the end of this distance?

Free Fall problem



A person drops a ball from a height of 3 m above the surface of the earth.

How long will it take the ball to hit the ground?

Example problem: Projectile motion

A student stands at the edge of a cliff and throws a stone horizontally over the edge with a speed of 18 m/s. The cliff is 50 m above a flat, horizontal beach.

(a) What are the coordinates of the initial position of the stone?

(b) What are the components of the initial velocity?

(c) Write the equations for the x- and y-components of the velocity of the stone with time.

(d) Write the equations for the position of the stone with time.

(e) When does the stone strike the beach?

(f) With what speed and angle does the stone land?