Displacement, Velocity, Acceleration in 2d

In 2d problems, the position of an object is determined by its position vector.

If an object moves from an initial to a final position, the displacement will be a vector, too (unit: m):

\[ \Delta \vec{r} = \vec{r}_f - \vec{r}_i \]

Velocity (unit: m/s) and acceleration (unit: m/s\(^2\)) are also vectors:

\[ \vec{v}_{\text{av}} = \frac{\Delta \vec{r}}{\Delta t} \]
\[ \vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t} \]

An object can accelerate in different ways:

1. The magnitude of the velocity changes with time (the same as 1d problems)
2. The direction of the velocity may change with time, e.g. circular motion, at constant speed.
3. Magnitude and direction change in parallel.
A projectile is any body, given an initial velocity, that then follows a path determined by the effects of gravity and air resistance. We neglect air resistance.
The physics of Wiley E Coyote
How would Coyote E. Wiley fall down a cliff in reality?

- As Mr. Coyote runs off the cliff, he has horizontal velocity.

- A change in velocity is acceleration, in this case horizontal acceleration, which must come from a force in the horizontal direction. \( v_{fx} = v_{ox} + a_x t \)

- If we ignore air resistance (horizontal force = 0), then there is no horizontal force to slow him down horizontally. \( v_{fx} = v_{ox} \)

- Thus, Mr. Coyote will travel **horizontally at the same speed** the whole time until he hits the ground!
How would Coyote E. Wiley fall down a cliff in reality?

• Vertical motion is treated separately.

• As soon as the coyote leaves the cliff he will experience a vertical force due to gravity.

• This force will cause him to start to accelerate in the vertical direction. As he falls he will be going faster and faster in the vertical direction.

• The horizontal and vertical components of the motion of an object going off a cliff are separate from each other, and **can not affect each other**.
Mathematical description of projectile motion

\[ \vec{v}_o \equiv \text{initial velocity vector} \]
\[ \theta_0 \equiv \text{initial direction of velocity vector} \]
\[ v_y = 0 \text{ at top of trajectory} \]
\[ v_x = v_{x0} \text{ remains the same throughout trajectory because there is no acceleration along the } x\text{-direction.} \]
\[ v_{x0} = v_0 \cdot \cos \theta \]
\[ v_{y0} = v_0 \cdot \sin \theta \]

We can use the equations to describe 1d motion with constant acceleration for the horizontal and vertical direction separately:

\[ x(t) = x_0 + v_{x0}t + \frac{1}{2}at^2 \quad \rightarrow \quad x(t) = v_{x0}t \]
\[ v_x(t) = v_{x0} + at \quad \rightarrow \quad v_x(t) = v_{x0} \]

\[ y(t) = y_0 + v_{y0}t + \frac{1}{2}at^2 \quad \rightarrow \quad y(t) = v_{y0}t - \frac{1}{2}gt^2 \]
\[ v_y(t) = v_{y0} + at \quad \rightarrow \quad v_y(t) = v_{y0} - gt \]
Example problem: Projectile motion

An alaskan rescue plane drops a package of emergency rations to stranded hikers. The plane is traveling horizontally at 40 m/s at a height of 100 m above the ground.

(a) Where does the package strike the ground relative to the point at which it was released?

(b) What are the vertical and horizontal components of the velocity of the package just before it hits the ground?

(c) Find the angle of impact.
Summary

- In 2d problems, displacement, velocity, and acceleration are vectors:

\[ \Delta \vec{r} = \vec{r}_f - \vec{r}_i \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} \quad \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} \]

An acceleration will happen, if the magnitude and/or direction of the velocity vector is changed as a function of time. Circular motion at constant speed is an accelerated motion!

- A projectile is an object, given an initial velocity, that moves under the influence of gravitation.

- In the frame of projectile motion, the motions in horizontal and vertical direction are completely independent. Without air resistance, the velocity in horizontal direction is constant. In vertical direction the projectile experiences a constant acceleration (-g).

- We can use the equations for 1d motion for each direction separately:

\[ x(t) = x_0 + v_{x0}t + \frac{1}{2}at^2 \rightarrow x(t) = v_{x0}t \quad y(t) = y_0 + v_{y0}t + \frac{1}{2}at^2 \rightarrow y(t) = v_{y0}t - \frac{1}{2}gt^2 \]

\[ v_x(t) = v_{x0} + at \rightarrow v_x(t) = v_{x0} \quad v_y(t) = v_{y0} + at \rightarrow v_y(t) = v_{y0} - gt \]
Average Speed

A person travels by car from one city to another with different constant speeds between pairs of cities. She drives for 35 min at 75 km/h, 12 min at 50 km/h, and 30 min at 50 km/h and spends 25 min eating lunch and buying gas.

(a) Determine the average speed for the trip.

(b) Determine the distance between the initial and final cities along the route.
Free Fall example problem

A baseball is thrown up in the air at an initial velocity of 22.0 m/s.

(a) How high up does it go?
(b) How long is it in the air if you catch it at the same height you initially let go of the ball?

Some helpful equations:

\[ v(t) = v_0 - gt \]
\[ y(t) = y_0 + v_0 t - \frac{1}{2}gt^2 \]
Projectile motion example problem

A fireman $d = 57$ m away from a burning building directs a stream of water from a ground-level fire hose at an angle of $23^\circ$ above the horizontal.

If the speed of the stream as it leaves the hose is $v_i = 40$ m/s, at what height will the stream of water strike the building?
Relative velocities

Velocities depend on the frame of reference. Something moves at velocity $\mathbf{v}$ relative to what? Think of moving walkways…

Since $\mathbf{v}_{av} \equiv \frac{\Delta \mathbf{r}}{\Delta t}$, relating velocities is the same as relating positions (since each position can be divided by $\Delta t$).
Test review: first, the rules

• Leave one seat empty between students.
• No programmable calculator.
• No notes.
• The answer options include rounded results. Choose the option that is closest to your calculated result.
• Write your name on the first page.
• Show your student ID, when you submit your test.
Conversion of units

Every physical quantity has a unit. Make sure not to forget these in the exam!

Typical problem: Convert an acceleration of 25 m/s$^2$ to km/min$^2$. 
Scientific notation

What would be the most accurate method for writing the following numbers in scientific notation:

13,000
560,000
0.0003
0.01
Trigonometry

\[
\sin (\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}
\]

\[
\cos (\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}
\]

\[
\tan (\theta) = \frac{\text{opposite side}}{\text{adjacent side}}
\]

Pythagorean theorem:

\[
x^2 + y^2 = r^2
\]

In a rectangular triangle: \(x = 5\, \text{cm}, \theta = 45^\circ\). What are \(y\) and \(r\)?
velocity and acceleration (1d)

• the average and instantaneous velocities are defined as:

\[
\mathbf{v}_{\text{average}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}
\]

\[
\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}
\]

the instantaneous velocity at time, t, corresponds to the slope of the tangent in a position-time diagram at this time.

• the average and instantaneous accelerations are defined as:

\[
\mathbf{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}
\]

\[
a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}
\]

the instantaneous acceleration at time, t, corresponds to the slope of the tangent in a velocity-time diagram at this time.

typical problem: a car is moving at 50 km/h for 2 hours. how many kilometers did the car travel?
Average and instantaneous velocity

What is the average velocity in the time interval between 1s and 3s?

What is the instantaneous velocity at $t = 1s$?
Velocity is the rate of change of position.

Acceleration is the rate of change of velocity.

Typical problem: What are the signs of the velocity and acceleration in plot c?
1d motion with constant acceleration

An object moves in one direction with constant acceleration \([a(t) = \text{const.}]\).

\(a(t) = \text{const.}\) results in a horizontal line in the a-t-graph.

How can we calculate \(v(t)\) and \(x(t)\)?

\[
\begin{align*}
    v(t) &= v_0 + at \\
x(t) &= x_0 + v_0 t + \frac{1}{2}at^2
\end{align*}
\]

- \(v(t)\) is a linear function and results in a straight line in the v-t-graph.
- \(x(t)\) is a quadratic function and results in a parabola in the x-t-graph.
Example problem: 1d motion with constant acceleration

A car starts from rest and accelerates at $10 \text{ m/s}^2$ in a straight line on a level street for a distance of 100 m.

What is the velocity of the car at the end of this distance?
Free Fall

• Free Fall is the motion of an object under the influence of gravity alone (no other forces/accelerations).

\[ a = \frac{\Delta v}{\Delta t} = -g = -9.81 \text{m/s}^2 \]

• Under the influence of gravity alone (no air drag), all objects experience the same acceleration independent of their mass. Thus, a feather and a coin hit the ground at the same time, if dropped from the same height.

• On earth, there is typically an air drag, that is different for different objects. Thus, a feather falls more slowly compared to a coin.

• For most problems (1d motion with constant acceleration) the following equations are sufficient:

\[ v(t) = v_0 - gt \quad y(t) = y_0 + v_0 t - \frac{1}{2} gt^2 \]

• It can be useful to divide the motion of an object into several sections.
A person drops a ball from a height of 3 m above the surface of the earth.

How long will it take the ball to hit the ground?
Vectors

- A vector is an arrow characterized by its **magnitude** and **direction**.
  \[ \vec{A} = \left( \begin{array}{c} A_x \\ A_y \end{array} \right) \]
  Magnitude: \[ A = \sqrt{A_x^2 + A_y^2} \]
  Direction: \[ \tan(\theta) = \frac{A_y}{A_x} \]

- Vectors are required to describe the position of objects and their motion in more than one direction.

- Two vectors are identical, if their magnitudes and directions are the same.

- Multiplying a vector with a scalar, \( k \), changes its magnitude by a factor of \( k \): \[ k\vec{A} = \left( \begin{array}{c} kA_x \\ kA_y \end{array} \right) \]

- Two vectors can be added geometrically or algebraically:
  \[ \vec{A} = \left( \begin{array}{c} A_x \\ A_y \end{array} \right), \quad \vec{B} = \left( \begin{array}{c} B_x \\ B_y \end{array} \right) \rightarrow \vec{R} = \left( \begin{array}{c} R_x \\ R_y \end{array} \right) = \left( \begin{array}{c} A_x + B_x \\ A_y + B_y \end{array} \right) \]

  A minus sign reverses a vector’s direction.
  Subtracting 2 vectors is the same as adding a negative vector.

- A vector’s components can be determined: \( A_x = A \cdot \cos(\theta), \quad A_y = A \cdot \sin(\theta) \)
Projectile motion

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Example problem: Projectile motion

A student stands at the edge of a cliff and throws a stone horizontally over the edge with a speed of 18 m/s. The cliff is 50 m above a flat, horizontal beach.

(a) What are the coordinates of the initial position of the stone?
(b) What are the components of the initial velocity?
(c) Write the equations for the x- and y-components of the velocity of the stone with time.
(d) Write the equations for the position of the stone with time.
(e) When does the stone strike the beach?
(f) With what speed and angle does the stone land?