Recap - Forces

- Newton's first law:

  An object moves with a velocity that is constant in magnitude and direction unless a non-zero net force acts on it.

- Newton's second law:

$$\boxed{\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}}$$

- Newton's 3rd law:

$$\boxed{\text{Action} = -\ \text{Reaction}}$$

- Friction forces are directed into the opposite direction of an externally applied force.

Static friction: $|\vec{f}_s| \leq \mu_s|\vec{n}|$ Kinetic friction: $|\vec{f}_k| = \mu_k|\vec{n}|$

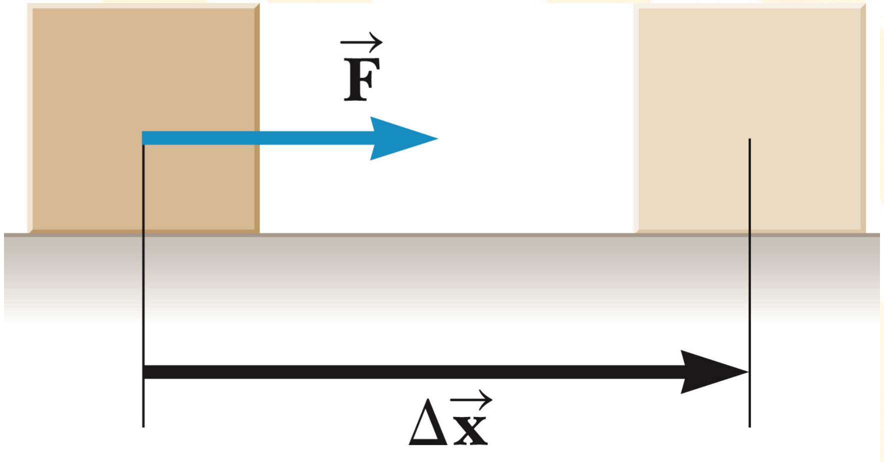
Today's lecture

Energy:

- Work
- Kinetic Energy

Chapter 5: Energy

Work



In physics, work has a different meaning than it does in everyday usage.

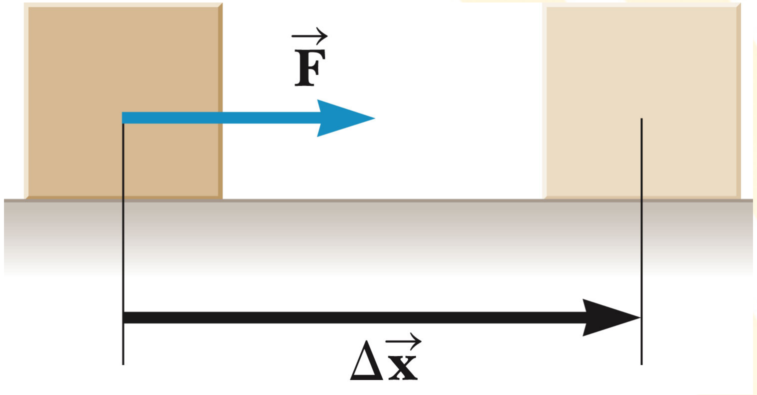
The work, W, done by a *constant* force on an object during a linear displacement along the x-axis is:

$$W = F_x \Delta x$$

F_x is the x-component of the force and Δx is the object's displacement.

Work will later provide a link between forces and energy.

Work II



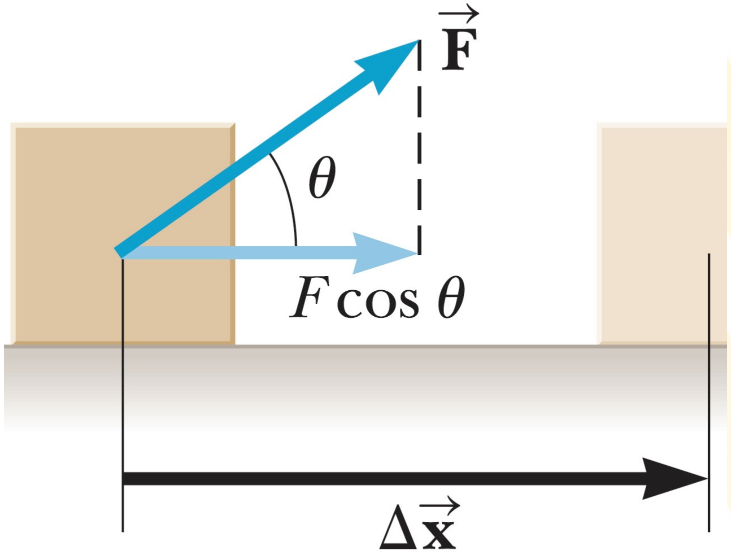
$$W = F_x \Delta x$$

Unit: 1 Joule (J) = N · m = kg m^2/s^2

- Work is a scalar quantity.
- The above equation will only be valid, if the force and the displacement are in the same direction.

What happens, if this is not the case (e.g. pull under some angle)?

Work III



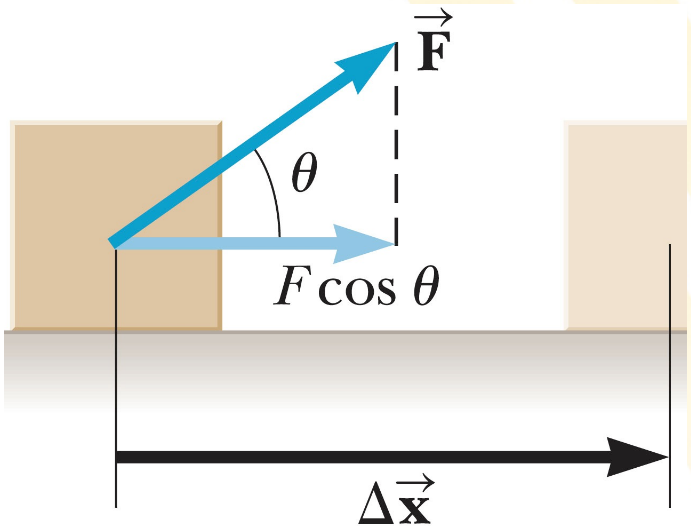
If the force is not in the same direction as the displacement, the component of the force in the direction of the displacement must be calculated:

$$W = (F\cos\theta)\Delta x$$

Component in the direction of Δx

- Only the parallel component of the force does work on the object.
- The vertical component does no work [cos(90°) = 0].

Positive and negative work



General equation for work:

$$W = (F\cos\theta)\Delta x$$

Depending on θ work can be positive, zero, or negative.

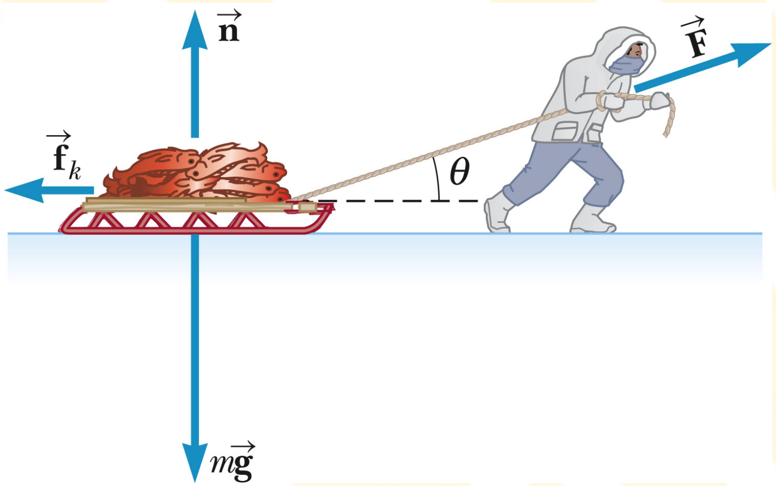
$0° \leq \theta < 90° \rightarrow \cos\theta > 0$: Positive work (force and displacement are in the same direction)

$\theta = 90° \rightarrow \cos\theta = 0$: No work (Force is perpendicular to the displacement)

$90° < \theta \leq 180° \rightarrow \cos\theta < 0$: Negative work (force and displacement are in opposite directions)

If there are multiple forces acting on an object, the total work done is the sum of the amount of work done by each force.

Example problem



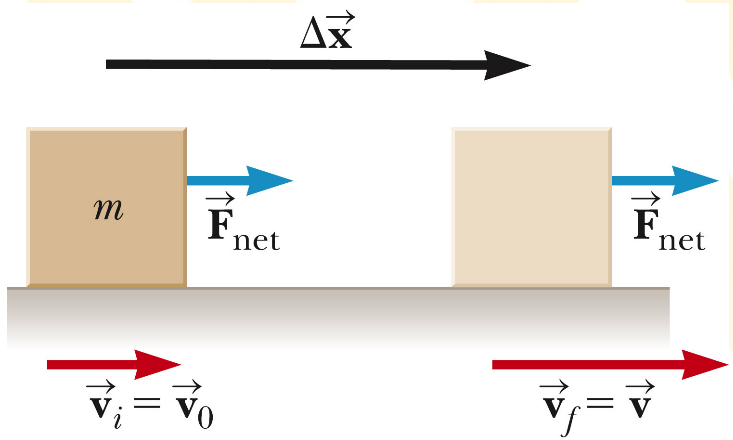
An Inuit pulls a sled loaded with salmon. The total mass of the sled and the salmon is 50 kg. The Inuit exerts a force of magnitude 120 N on the sled by pulling on the rope.

A. How much work does he do on the sled, if the rope is horizontal to the ground and he pulls the sled 5 m?

B. How much work does he do on the sled if $\theta = 30°$ and he pulls the sled the same distance?

C. At a coordinate position of 12.4 m, the Inuit lets up on the applied force. A friction force of 45 N between the ice and the sled brings the sled to rest at 18.2 m. How much work does friction do on the sled?

The Work-Energy Theorem



This theorem relates the net work done on an object to the change in its speed:

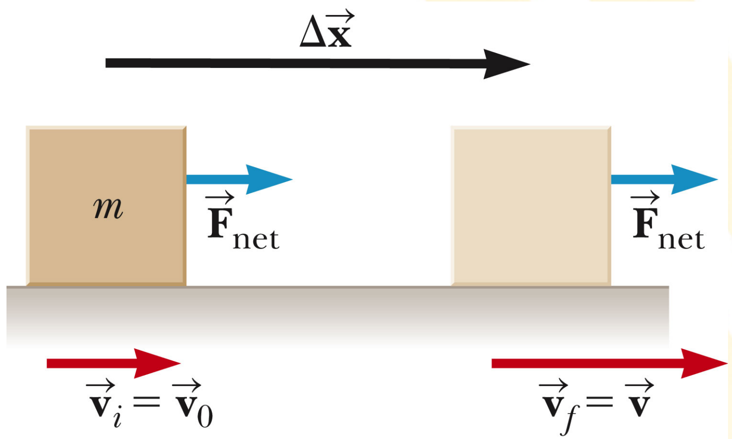
$$W_{net} = F_{net}\Delta x = (ma)\Delta x$$

From chapter 2 (motion with uniform acceleration) we know:

$$v^2 = v_0^2 + 2a\Delta x \quad \rightarrow \quad a\Delta x = \frac{v^2 - v_0^2}{2}$$

This yields: $$W_{net} = m\left(\frac{v^2 - v_0^2}{2}\right) \quad \rightarrow \quad \boxed{W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}$$

Kinetic Energy



$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The net work done on an object can result in a change of a quantity of the form $1/2mv^2$.

This scalar quantity has the unit of an energy and is called kinetic energy:

$$\boxed{KE = W_{kin} = \frac{1}{2}mv^2}$$

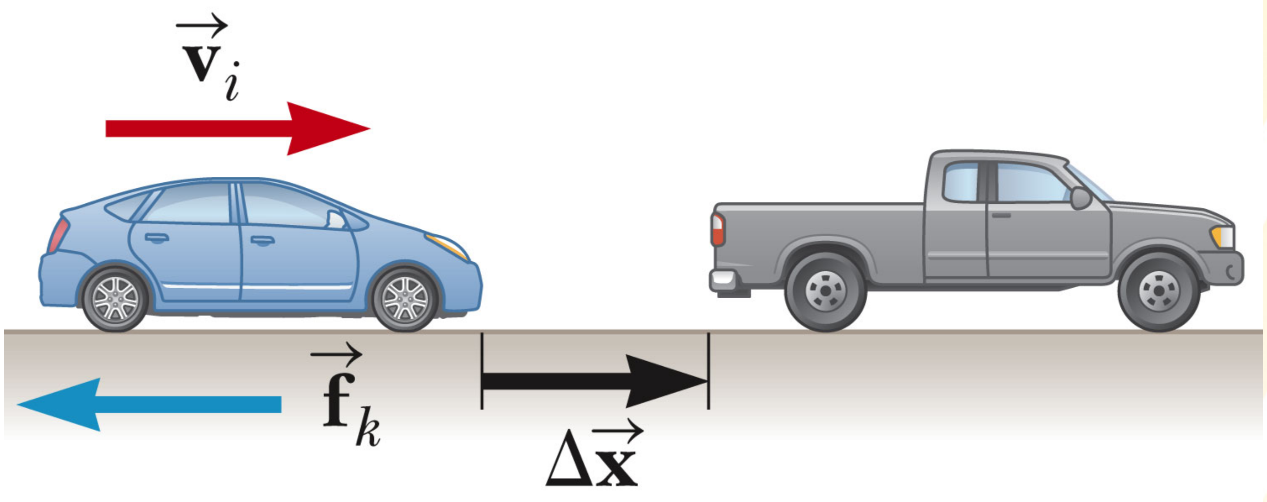
Unit: $J = kg \cdot m^2/s^2$

Work-Energy Theorem: $$W_{net} = KE_f - KE_i = \Delta KE$$

This will only be valid, if the work is completely transferred to kinetic energy.

Positive net work means that the object's speed increases.
Negative net work means that the object's speed decreases.

Example Problem

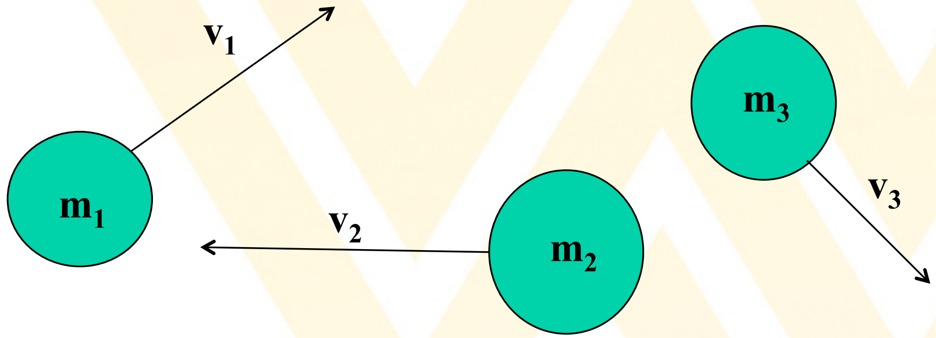


The driver of a car (m = 1000 kg) traveling on the interstate at 35 m/s slams on his brakes to avoid hitting a second vehicle in front of him, which had come to rest because of congestion ahead. After the brakes are applied, a constant friction force of 8000 N acts on the car.

A. At what minimum distance should the brakes be applied to avoid a collision?

B. If the distance between the vehicles is initially 30 m, at what speed would the collision occur?

A system of multiple objects



$$KE_{system} = \tfrac{1}{2}m_1v_1^{\;2} + \tfrac{1}{2}m_2v_2^{\;2} + \tfrac{1}{2}m_3v_3^{\;2}$$

In a system of multiple objects, the total kinetic energy is the sum of the objects' individual kinetic energies.

Energy is never lost: It can be transferred from one object to the other or transformed from one kind of energy, e.g. kinetic energy, to another type of energy, e.g. potential energy.

Conservative and non-conservative forces

There are 2 fundamentally different types of forces in nature:

1. A force is called conservative, if the work it does moving an object between two points is the same independent of the path taken.

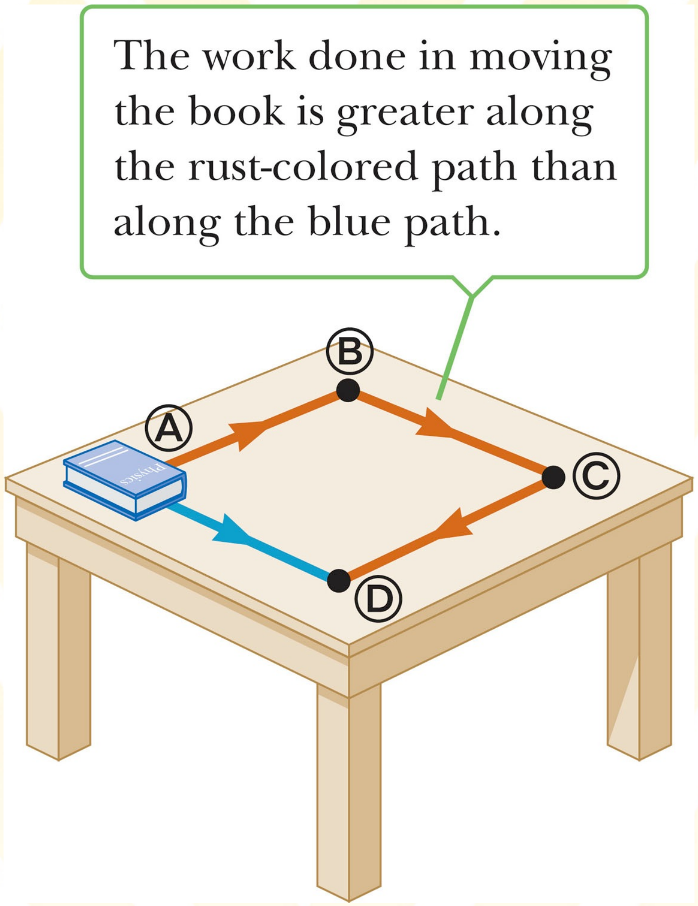
   Example: Gravity

2. A force is called non-conservative, if this work depends on the path taken.

   Example: Friction

The Work-Energy Theorem can be written as the sum of conservative, W_c, and non-conservative parts, W_{nc}:

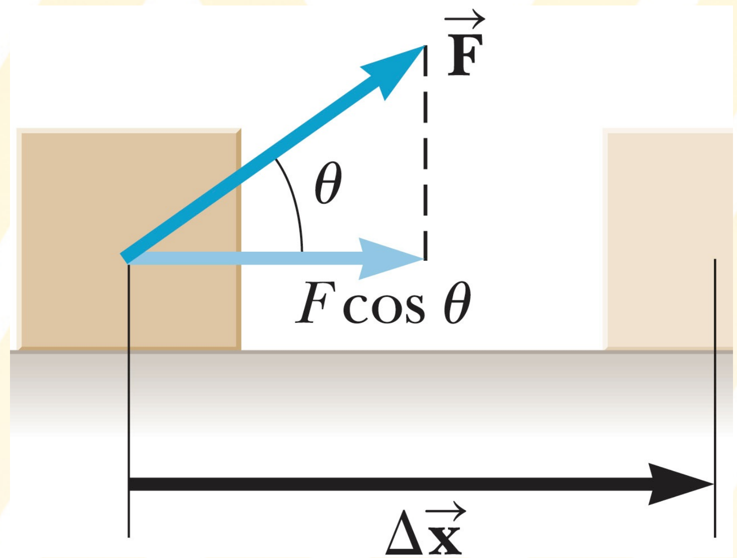
$$W_{net} = W_{nc} + W_c = \Delta KE$$



Summary

- The work, W, done by a *constant* force on an object during a linear displacement along the x-axis is:

$$W = (F\cos\theta)\Delta x$$



- Positive/negative work means that the force and the displacement are in the same/opposite directions.

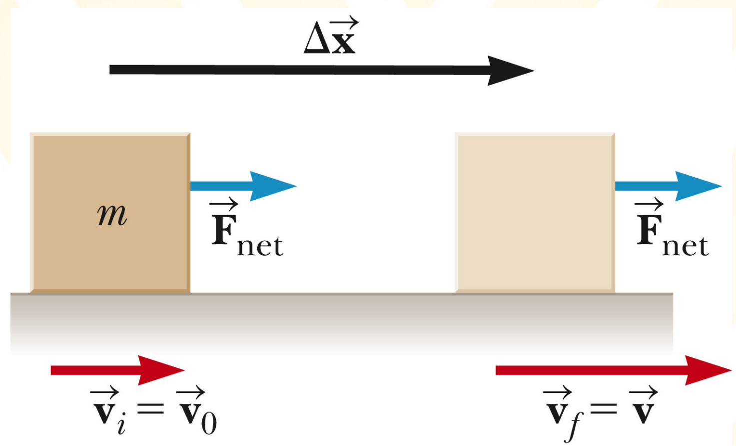
- There are 2 types of forces:

  1. Conservative: Work does not depend on path
  2. Non-conservative: Work depends on path

- Work-Energy Theorem:

$$W_{net} = W_{nc} + W_c = \Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Here, $1/2mv^2$ is the kinetic energy.



- Energy is never lost, but can only be transformed to other kinds of energy.