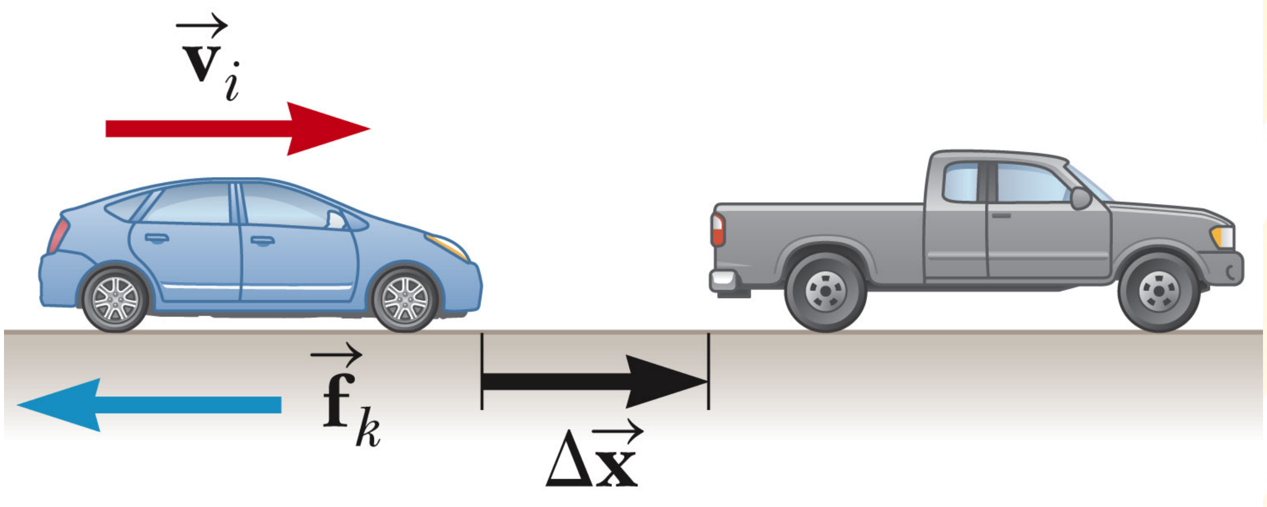
Example Problem

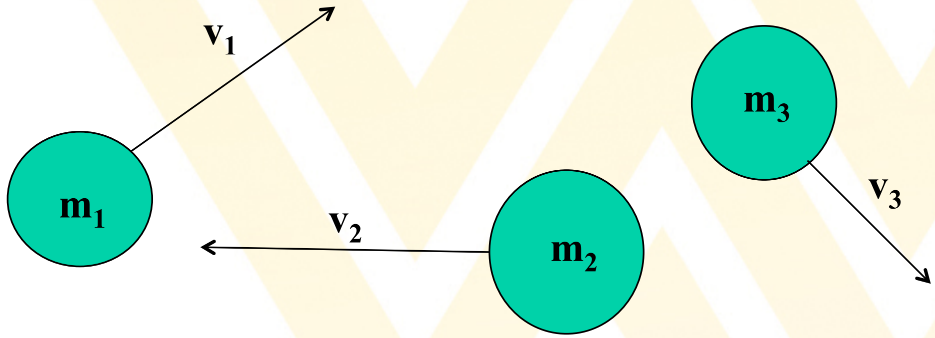


The driver of a car (m = 1000 kg) traveling on the interstate at 35 m/s slams on his brakes to avoid hitting a second vehicle in front of him, which had come to rest because of congestion ahead. After the brakes are applied, a constant friction force of 8000 N acts on the car.

A. At what minimum distance should the brakes be applied to avoid a collision?

B. If the distance between the vehicles is initially 30 m, at what speed would the collision occur?

A system of multiple objects



$$KE_{system} = \tfrac{1}{2} m_1 v_1^{\,2} + \tfrac{1}{2} m_2 v_2^{\,2} + \tfrac{1}{2} m_3 v_3^{\,2}$$

In a system of multiple objects, the total kinetic energy is the sum of the objects' individual kinetic energies.

Energy is never lost: It can be transferred from one object to the other or transformed from one kind of energy, e.g. kinetic energy, to another type of energy, e.g. potential energy.

Conservative and non-conservative forces

There are 2 fundamentally different types of forces in nature:

1. A force is called conservative, if the work it does moving an object between two points is the same independent of the path taken.

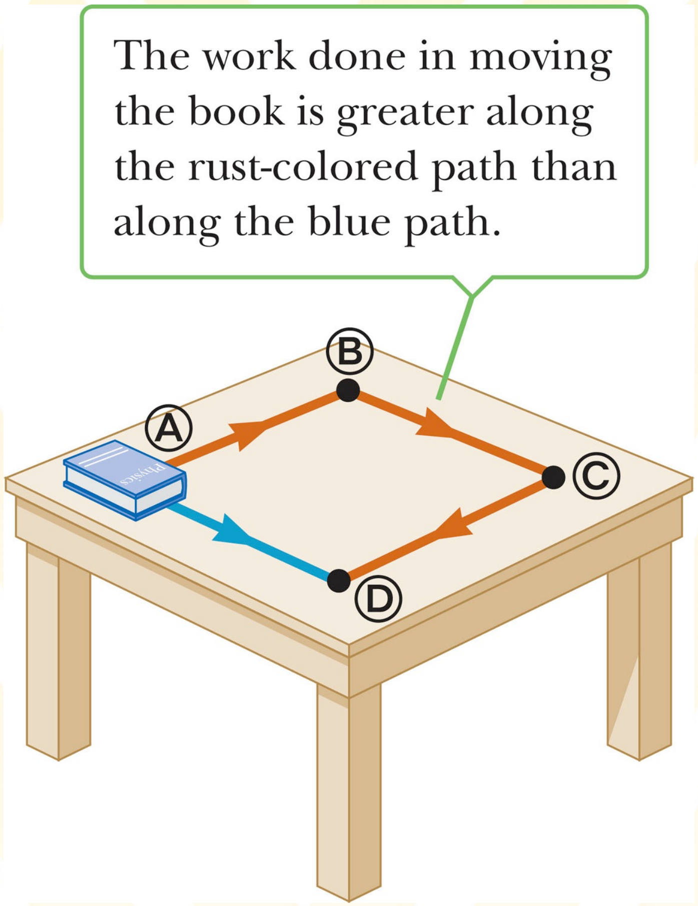
   Example: Gravity

2. A force is called non-conservative, if this work depends on the path taken.

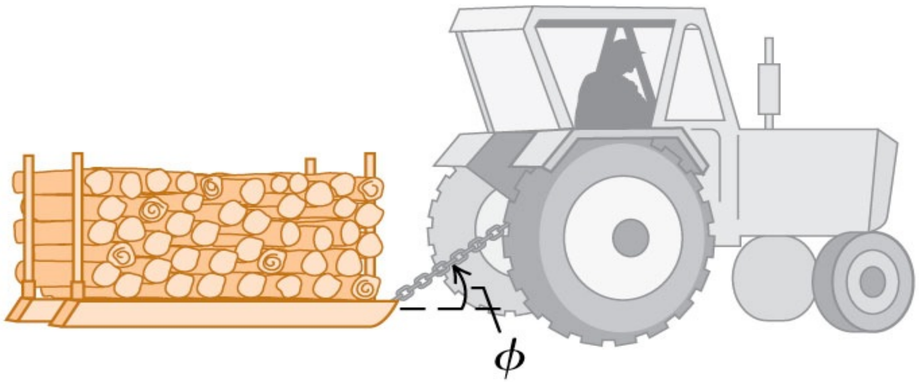
   Example: Friction

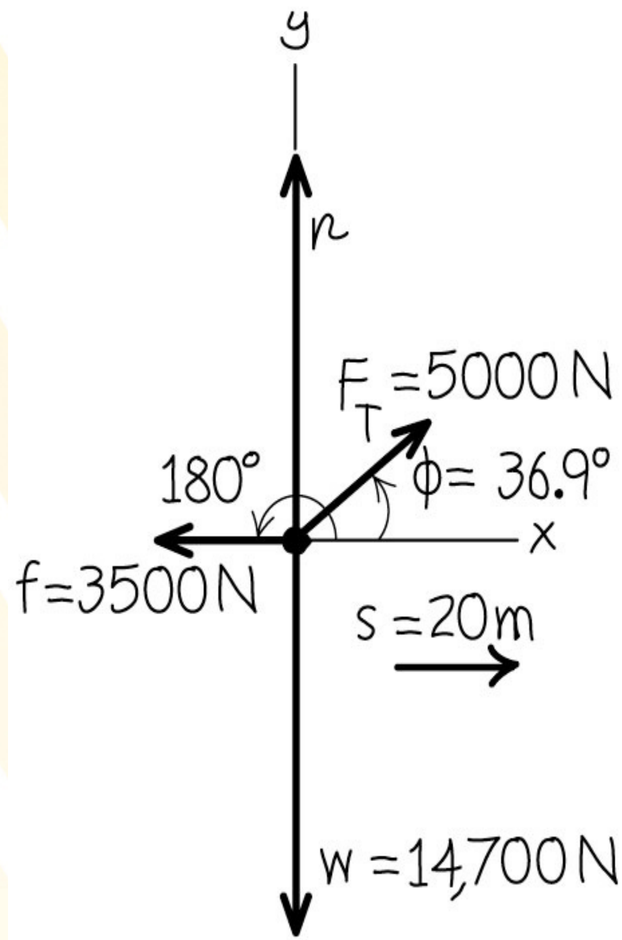
The Work-Energy Theorem can be written as the sum of conservative, W_c, and non-conservative parts, W_{nc}:

$$W_{net} = W_{nc} + W_c = \Delta KE$$



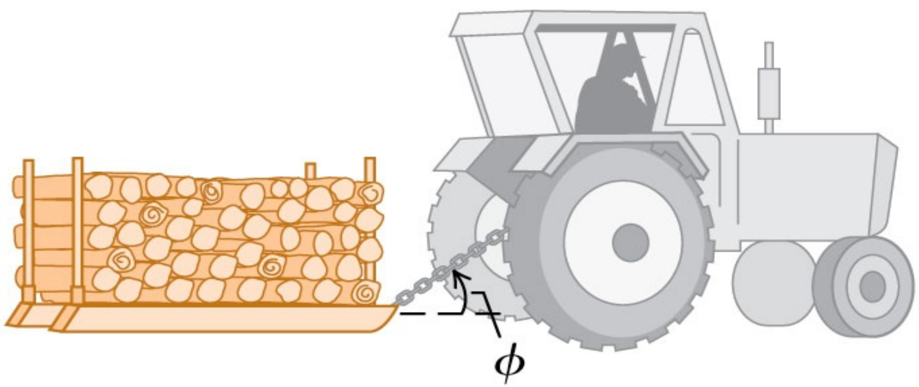
Work done by multiple forces





A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 6.7a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9° above the horizontal. A 3500-N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.

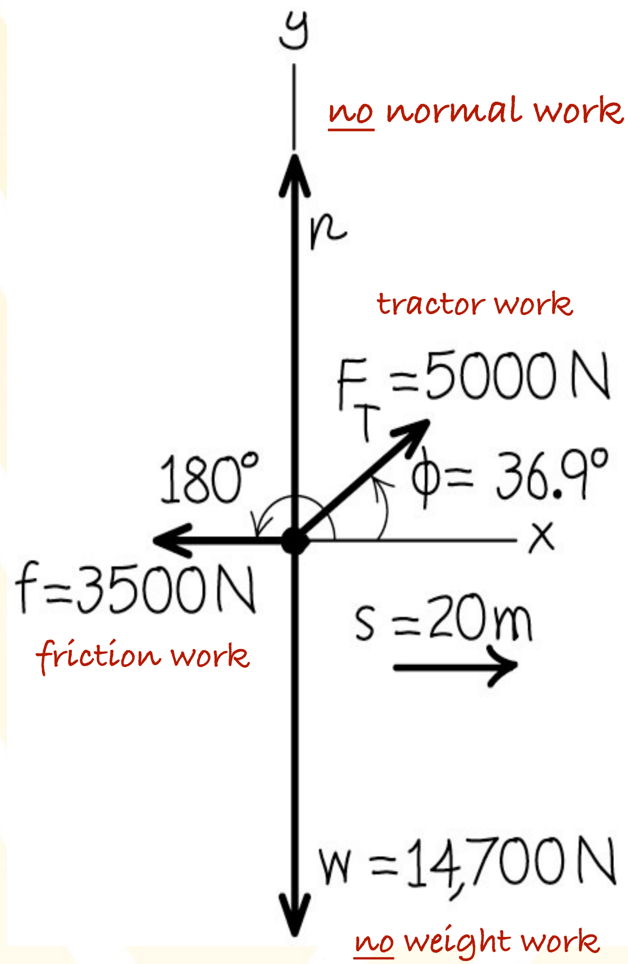
Work done by multiple forces



$$W_T = F_T s\cos\phi = (5000\ \mathrm{N})(20\ \mathrm{m})(0.800) = 80\ \mathrm{kJ}$$

$$W_f = f s\cos 180^\circ = (3500\ \mathrm{N})(20\ \mathrm{m})(-1) = -70\ \mathrm{kJ}$$

total work $= W_T + W_f = +10$ kJ

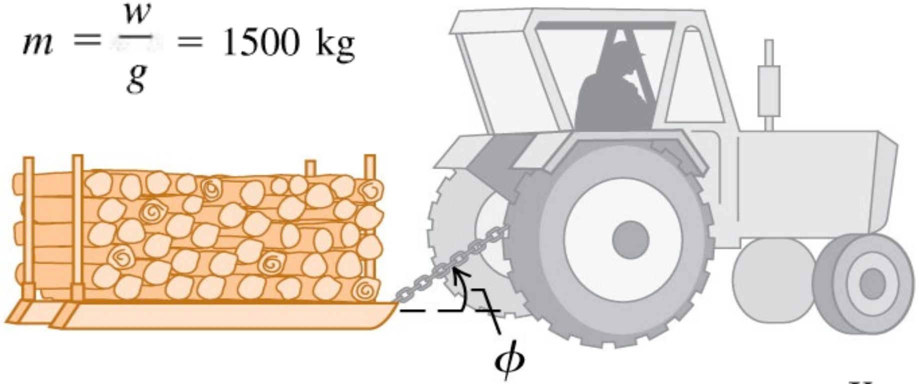


Work-Energy: Finding the Speed

total work $= W_T + W_f = +10$ kJ

$$\boxed{W_{tot} = K_2 - K_1}$$

$$m = \frac{w}{g} = 1500 \text{ kg}$$

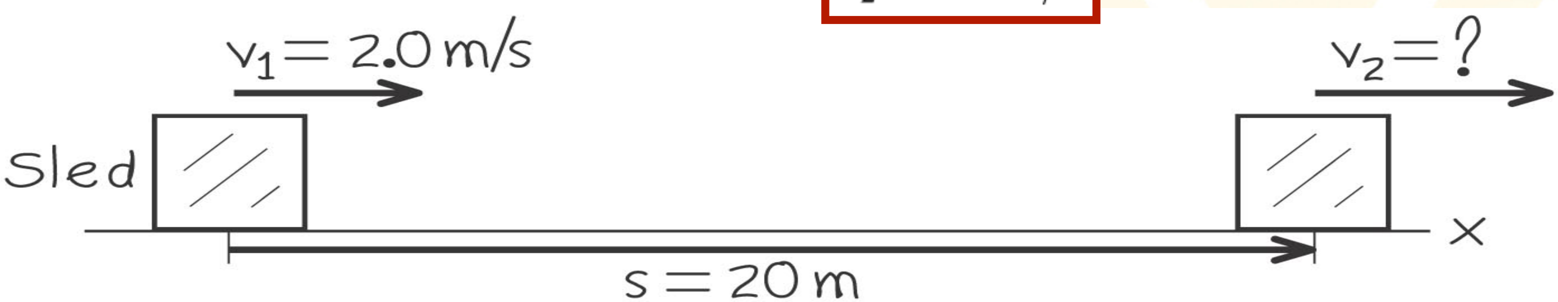


$$K_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2}(1500 \text{ kg})(2.0 \text{ m/s})^2$$
$$= 3000 \text{ J}$$

$$K_2 = K_1 + W_{tot} = 3000 \text{ J} + 10{,}000 \text{ J} = 13{,}000 \text{ J}$$

$$K_2 = \tfrac{1}{2} m v_2^2 = \tfrac{1}{2}(1500 \text{ kg}) v_2^2$$

$$\boxed{v_2 = 4.2 \text{ m/s}}$$



Work-Energy Theorem

$$W_{net} = W_{nc} + W_c = \Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The total work done is the sum of the work done by conservative and non-conservative forces.

Let's look at the work done by conservative forces, e.g. gravitation: How much work will be done, if a book falls down from y_i to y_f?

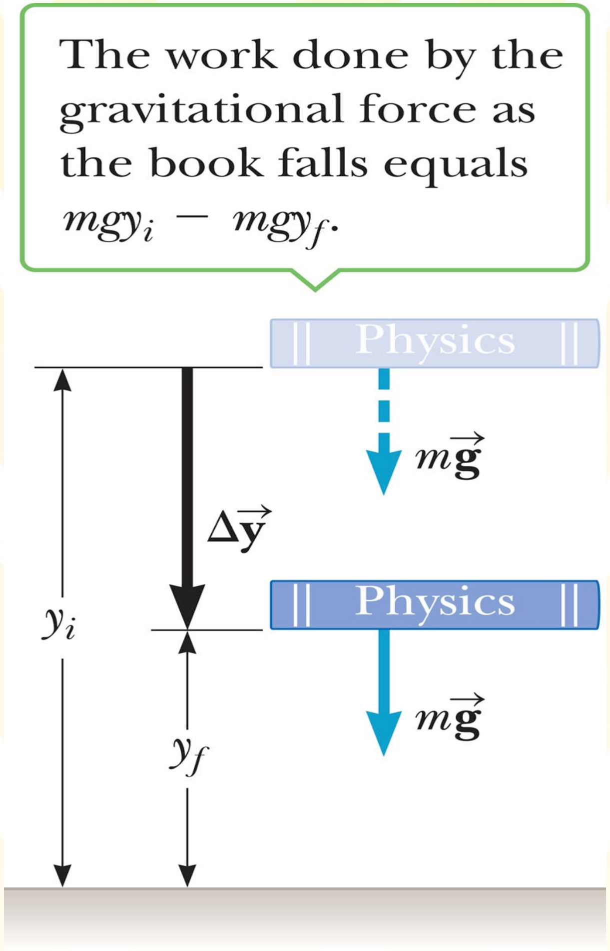
$$W_c = F_g \cos(\theta)\Delta y = mg(y_i - y_f)\cos(0^\circ) = -mg(y_f - y_i)$$

Definition of gravitational potential energy: $\boxed{PE_g = mgy}$

$$\rightarrow \quad W_c = PE_{gi} - PE_{gf}$$

Substitute this into the Work-Energy Theorem:

$$W_{nc} = (KE_f - KE_i) + (PE_{gf} - PE_{gi})$$



The meaning of potential energy

$\boxed{PE_g = mgy}$ y ≜ height above reference level

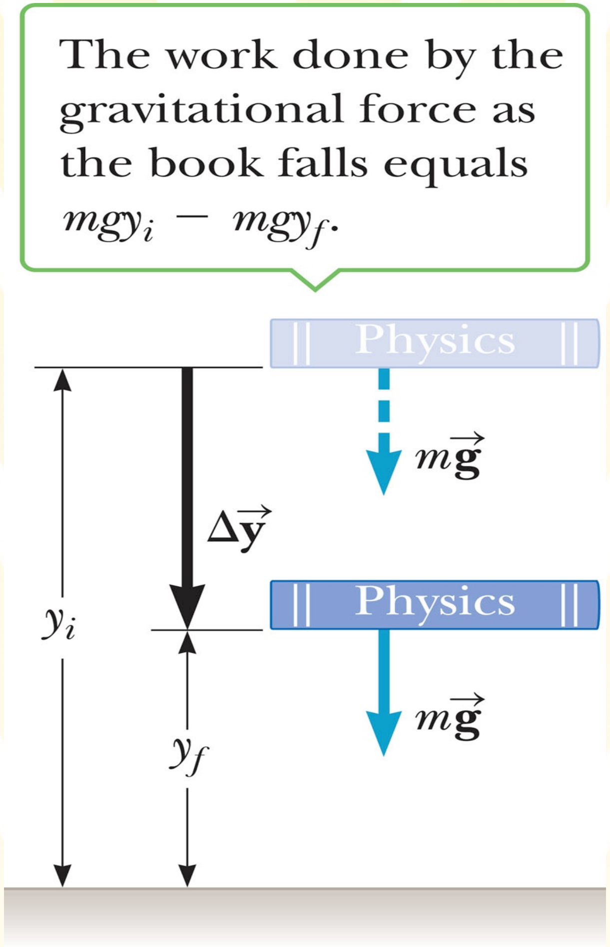
An object has *potential* energy, because due to its location it can *potentially* do work.

The choice of the reference level is arbitrary, since only differences of potential energies matter in the work-energy theorem:

$$W_{nc} = (KE_f - KE_i) + (PE_{gf} - PE_{gi})$$

Once chosen the reference level must remain the same.

Every conservative force can be associated with a potential energy, since the work done by this force depends only on the displacement and not on the path length.

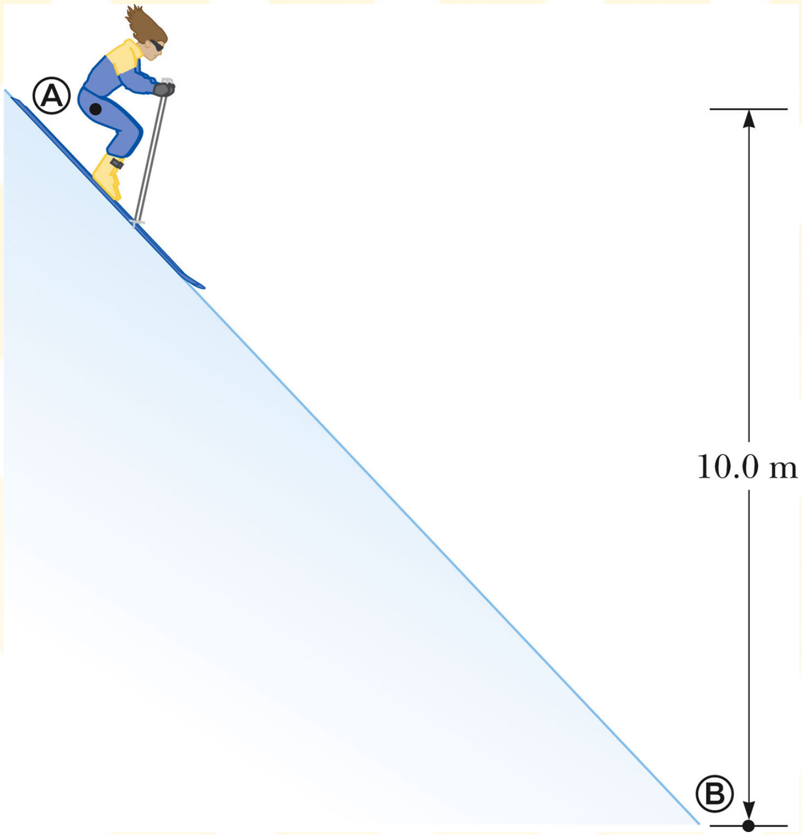


Example: The reference height is arbitrary

A 60 kg skier is at the top of a slope. At position A she is 10 m above point B.

A. Setting the zero level for the potential energy at B, find the gravitational potential energy, when the skier is at A.

B. Repeat this problem with the zero level at point A.



Clicker question

A piece of fruit falls straight down. As it falls,

A. the gravitational force does positive work on it and the gravitational potential energy increases.

B. the gravitational force does positive work on it and the gravitational potential energy decreases.

C. the gravitational force does negative work on it and the gravitational potential energy increases.

D. the gravitational force does negative work on it and the gravitational potential energy decreases.

$$W = (F\cos\theta)\Delta x \qquad PE_g = mgy$$

Conservation of energy

Work-Energy Theorem:

$$W_{nc} = (KE_f - KE_i) + (PE_{gf} - PE_{gi})$$

Let's look at a situation, where non-conservative forces, e.g. friction, can be neglected:

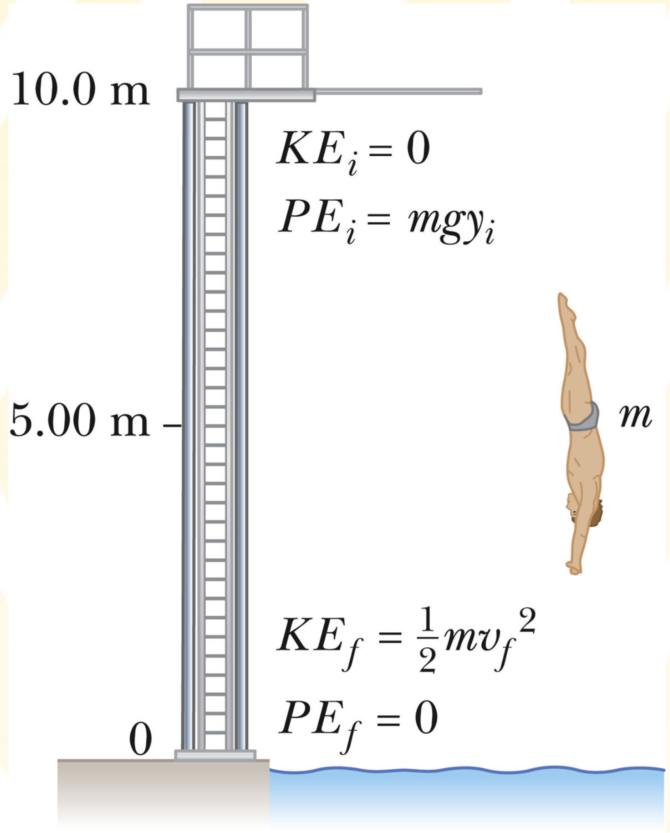
$$0 = (KE_f - KE_i) + (PE_{gf} - PE_{gi})$$

$$\rightarrow \boxed{KE_i + PE_{gi} = KE_f + PE_{gf}}$$

This equation means that the total energy (sum of KE and PE) is conserved.

Substitution of the equations for the kinetic and potential energy yield:

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$



Example problem: Conservation of energy

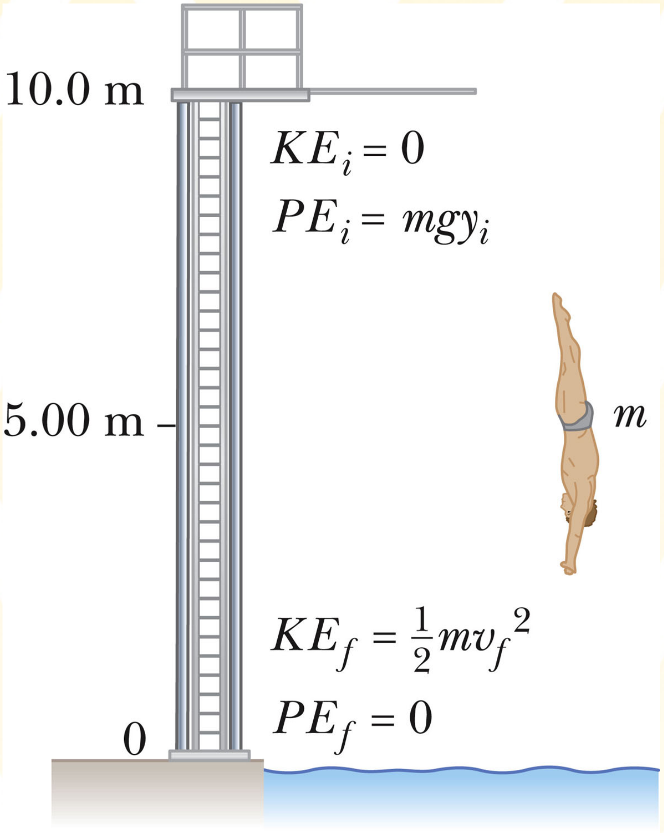
A diver of mass m drops from a board 10 m above the water's surface. Neglect air resistance.

A. What is his speed 5 m above the water surface?

B. Find his speed as he hits the water.

Conservation of energy:

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$



Summary

- Every conservative force can be associated with a potential energy, i.e. an energy that allows the corresponding object to *potentially* do work.

- One example of such a conservative force is gravitation. The gravitational potential energy is:

$$PE_g = mgy$$

- In the absence of non-conservative forces the total energy is conserved:

$$KE_i + PE_{gi} = KE_f + PE_{gf}$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$