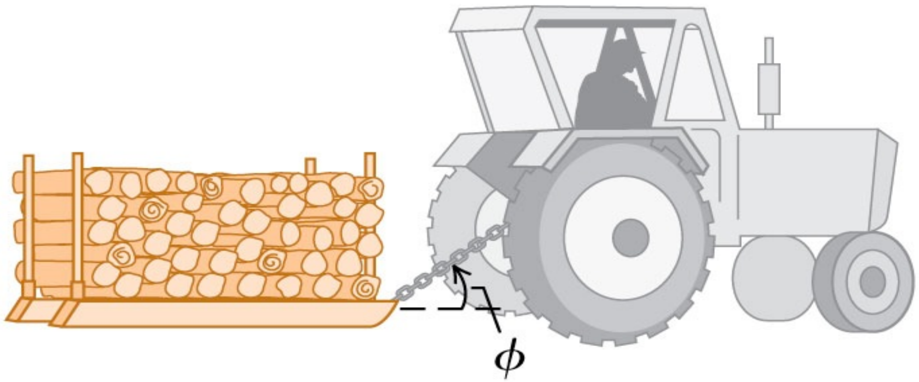
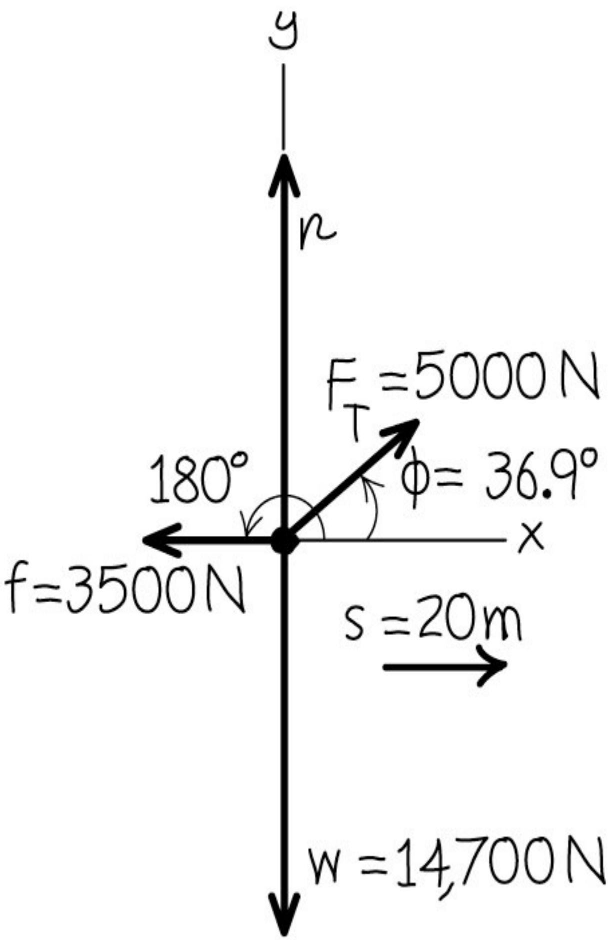
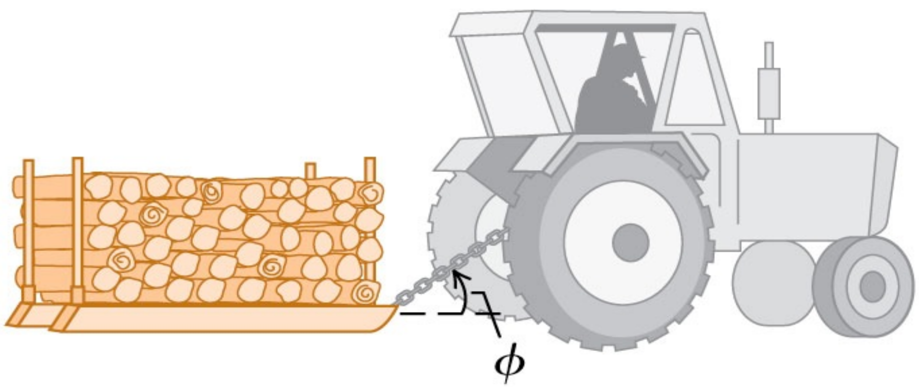
Work done by multiple forces

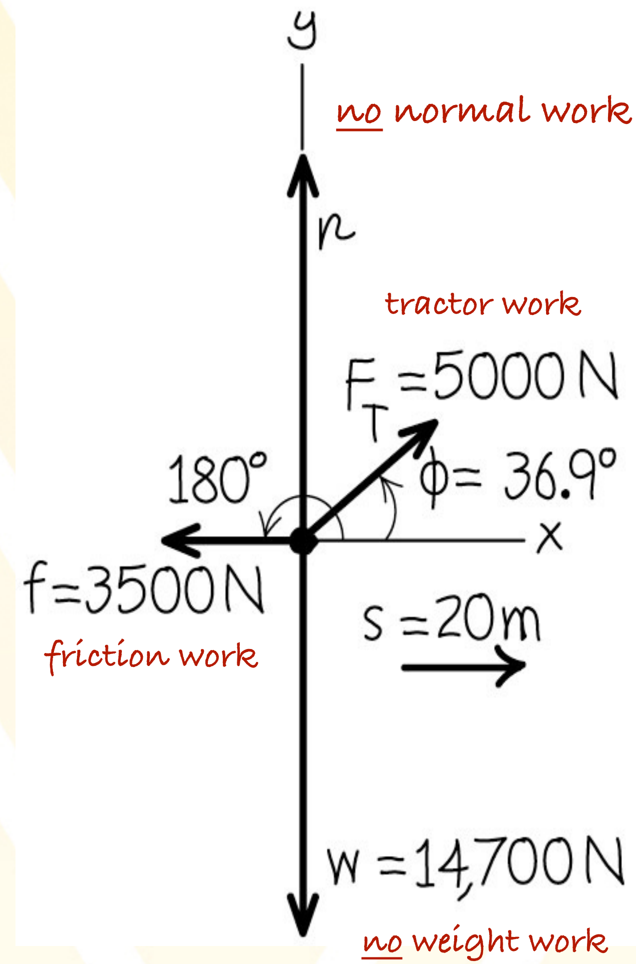


A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 6.7a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9° above the horizontal. A 3500-N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.



Work done by multiple forces





$$W_T = F_T s\cos\phi = (5000\ \text{N})(20\ \text{m})(0.800)$$
$$= 80\ \text{kJ}$$

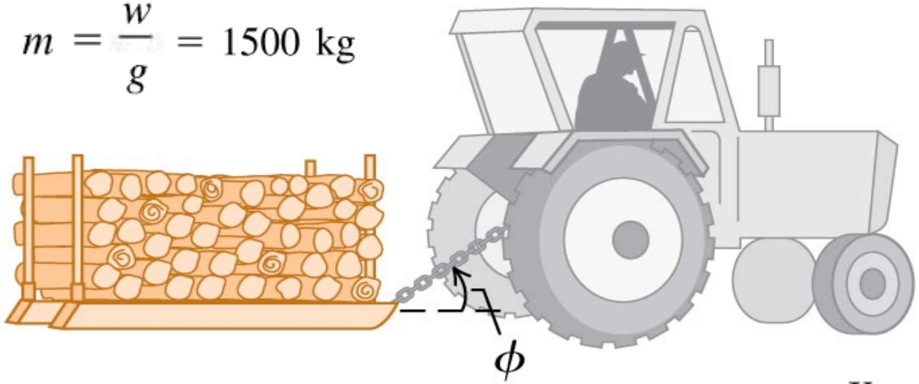
$$W_f = fs\cos 180° = (3500\ \text{N})(20\ \text{m})(-1)$$
$$= -70\ \text{kJ}$$

total work $= W_T + W_f = +10$ kJ

Work-Energy: Finding the Speed

total work = $W_T + W_f = +10$ kJ

$$m = \frac{w}{g} = 1500 \text{ kg}$$



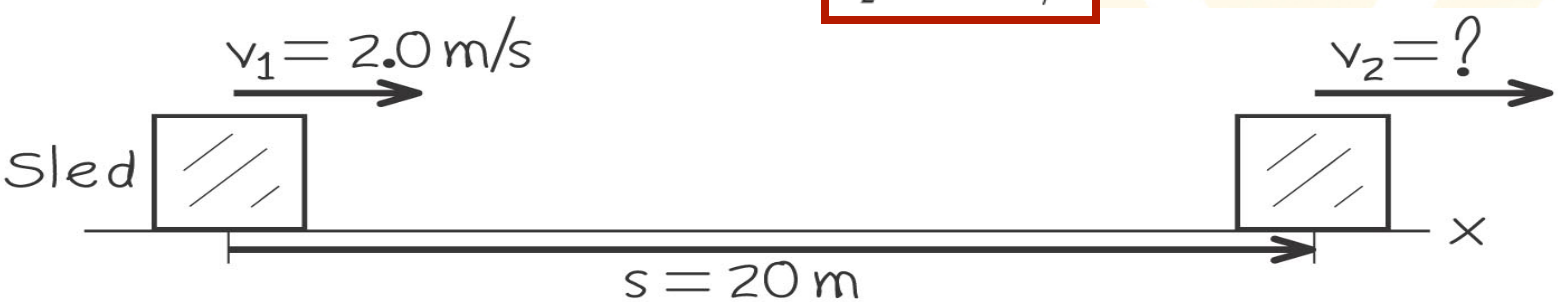
$$\boxed{W_{tot} = K_2 - K_1}$$

$$K_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2}(1500 \text{ kg})(2.0 \text{ m/s})^2$$
$$= 3000 \text{ J}$$

$$K_2 = K_1 + W_{tot} = 3000 \text{ J} + 10{,}000 \text{ J} = 13{,}000 \text{ J}$$

$$K_2 = \tfrac{1}{2} m v_2^2 = \tfrac{1}{2}(1500 \text{ kg}) v_2^2$$

$$\boxed{v_2 = 4.2 \text{ m/s}}$$



Clicker question

A piece of fruit falls straight down. As it falls,

A. the gravitational force does positive work on it and the gravitational potential energy increases.

B. the gravitational force does positive work on it and the gravitational potential energy decreases.

C. the gravitational force does negative work on it and the gravitational potential energy increases.

D. the gravitational force does negative work on it and the gravitational potential energy decreases.

$$W = (F\cos\theta)\Delta x \qquad PE_g = mgy$$

Conservation of energy

Work-Energy Theorem:

$$W_{nc} = (KE_f - KE_i) + (PE_{gf} - PE_{gi})$$

Let's look at a situation, where non-conservative forces, e.g. friction, can be neglected:

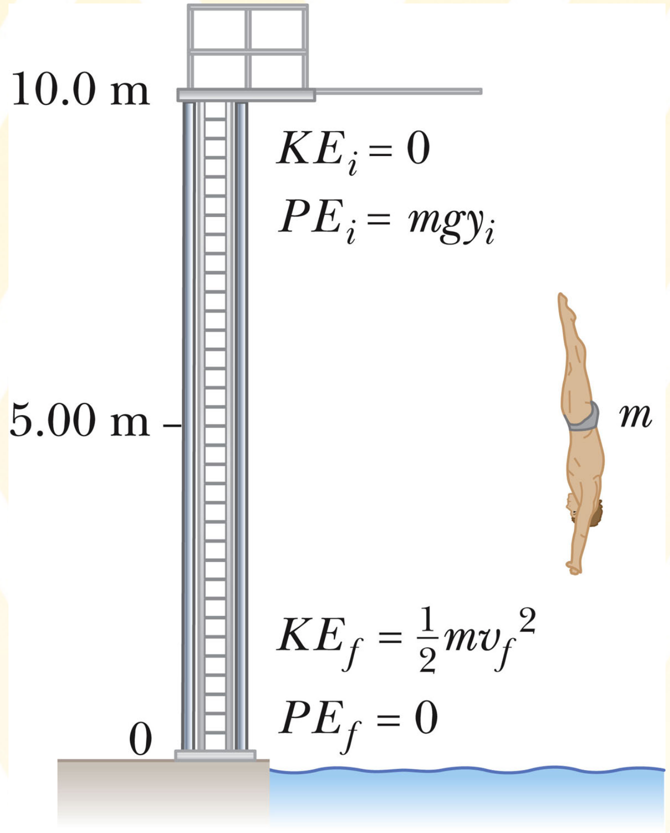
$$0 = (KE_f - KE_i) + (PE_{gf} - PE_{gi})$$

$$\rightarrow \quad \boxed{KE_i + PE_{gi} = KE_f + PE_{gf}}$$

This equation means that the total energy (sum of KE and PE) is conserved.

Substitution of the equations for the kinetic and potential energy yield:

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$



Example problem: Conservation of energy

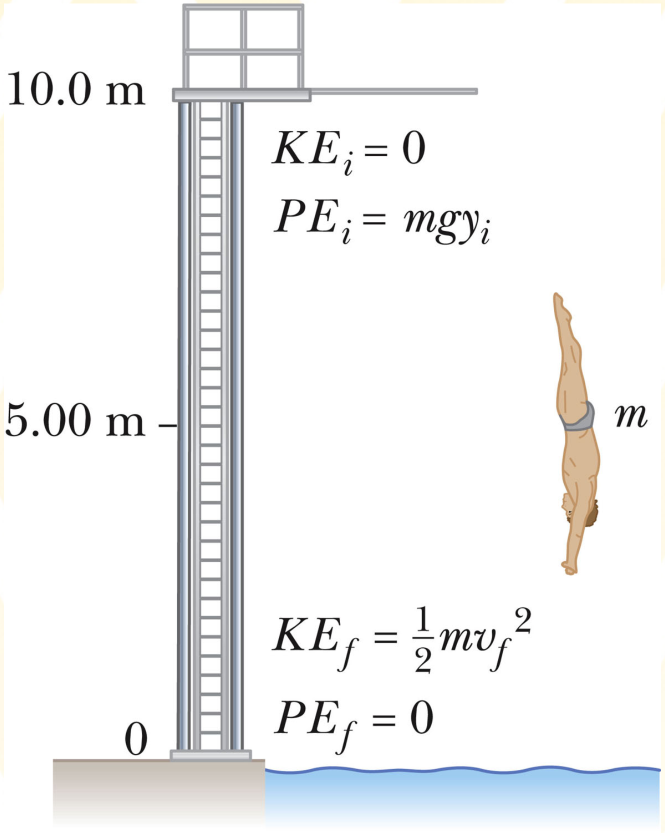
A diver of mass m drops from a board 10 m above the water's surface. Neglect air resistance.

A. What is his speed 5 m above the water surface?

B. Find his speed as he hits the water.

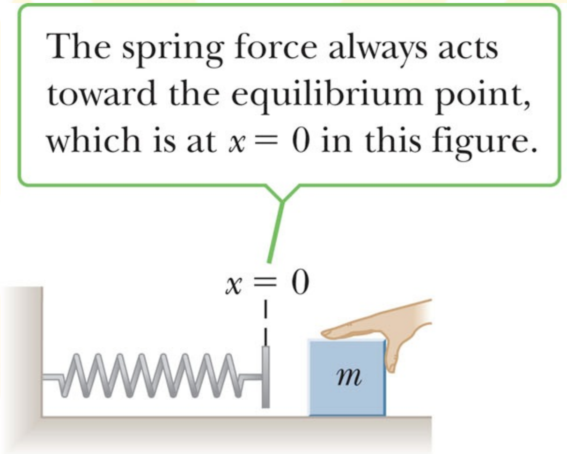
Conservation of energy:

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$



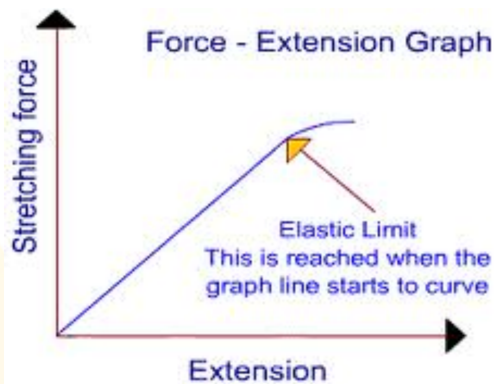
Springs: Hooke's law

The force a spring provides to an attached object is proportional to the amount that the spring is stretched or compressed from its equilibrium position. The force pulls/pushes the object back towards the equilibrium position (minus sign).



Hooke' law: $F_s = -kx$

k is the spring constant (unit: N/m). It is small for flexible spring and large for stiff springs.



Hooke's law is not always valid:

If you stretch a spring too much (elastic limit), the restoring force will no longer be linearly proportional to the extension, x.

Spring potential energy

$\boxed{\text{Hooke's law: } F_s = -kx}$

The spring force is conservative.

Thus, a potential energy - the spring potential energy, PE_s - can be associated with it.

In order to calculate PE_s we have to determine the work done by the spring:

$$W = F\Delta x$$

This equation is only valid for constant forces, but F_s depends on x, i.e. is not constant.

Therefore, we have to calculate the *average* force, when stretching the spring from its equilibrium position to x. This force can be treated as effectively constant.

$$\bar{F}_s = \frac{F_0 + F_1}{2} = \frac{0 - kx}{2} = -\frac{kx}{2}$$

With $\Delta x = x$ this yields:

$$W_s = -\frac{1}{2}kx^2 \quad \rightarrow \quad \boxed{PE_s = \frac{1}{2}kx^2}$$

Work-Energy Theorem

Including the spring potential energy the Work-Energy Theorem is:

$$W_{nc} = (KE_f - KE_i) + (PE_{gf} - PE_{gi}) + (PE_{sf} - PE_{si})$$

Change of kinetic energy

Change of gravitational potential energy

Change of spring potential energy

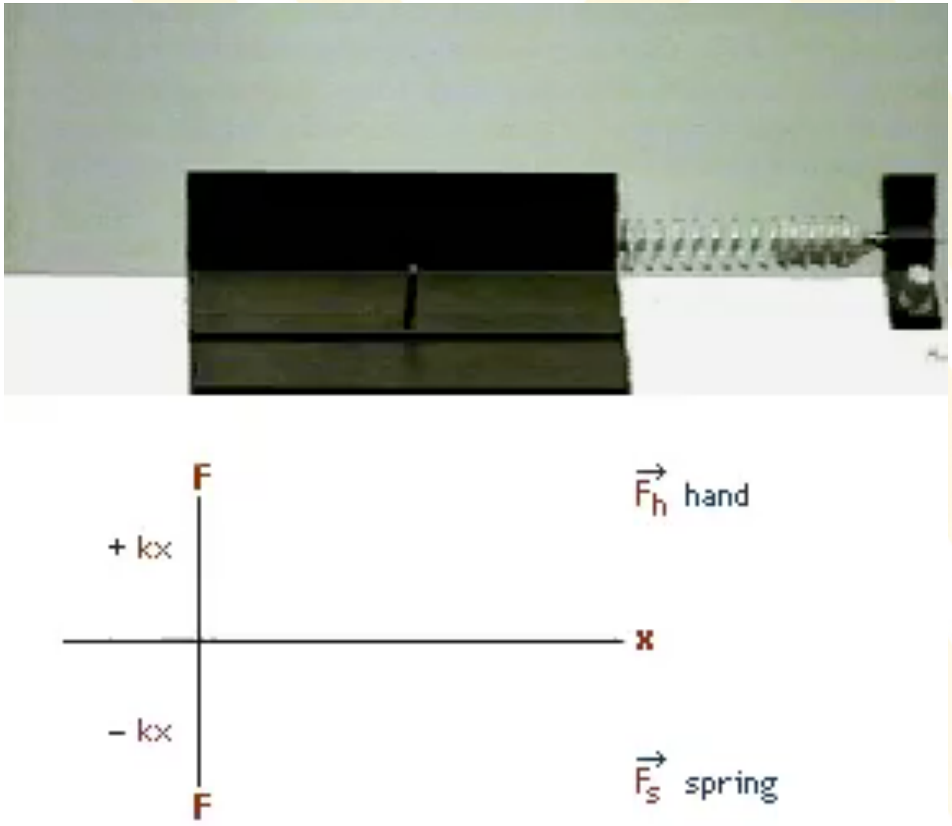
If non-conservative forces, e.g. friction, can be neglected, the mechanical energy is conserved:

$$0 = (KE_f - KE_i) + (PE_{gf} - PE_{gi}) + (PE_{sf} - PE_{si})$$

$$\rightarrow (KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$

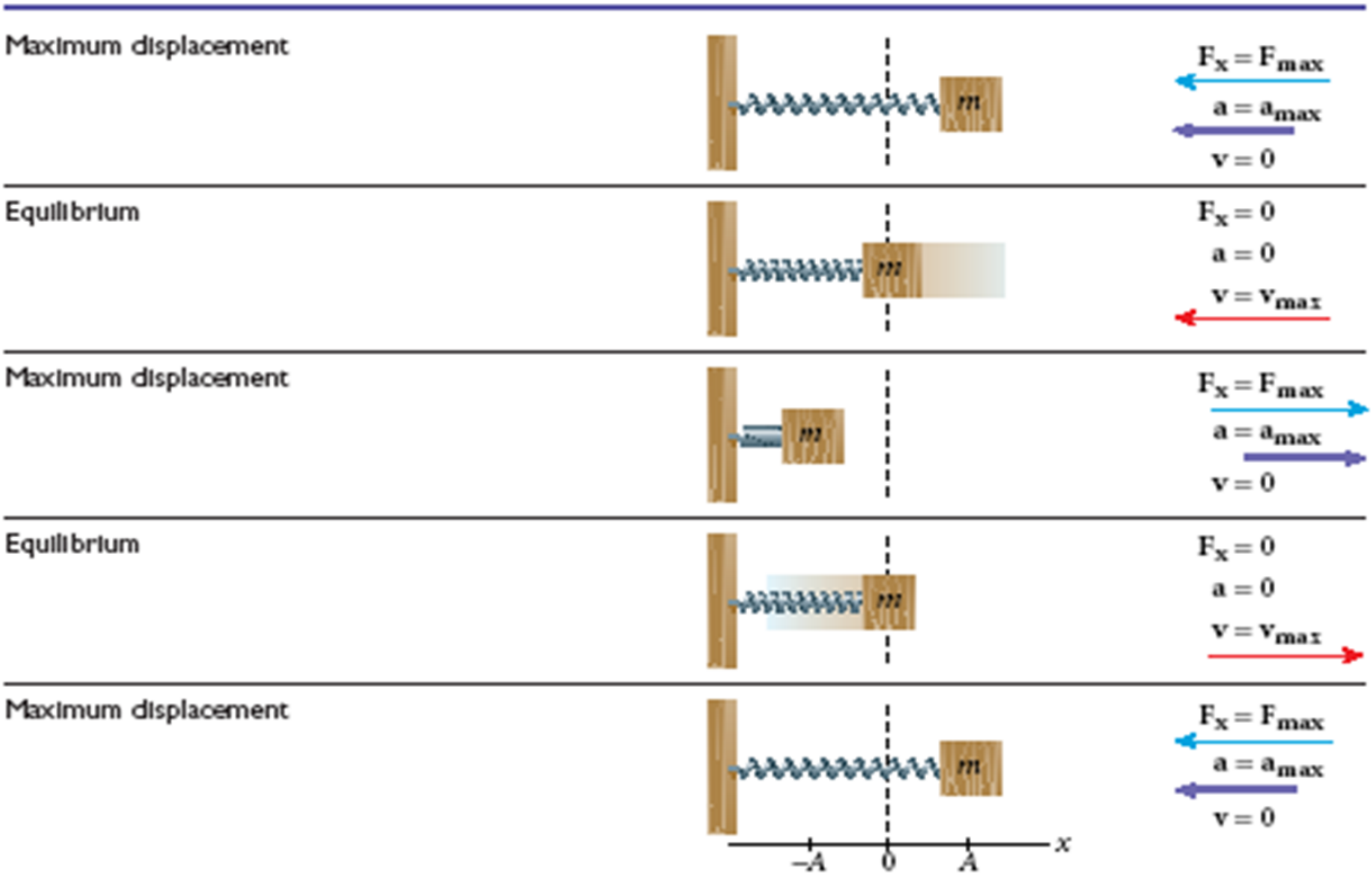
Generally, the total energy of a given system is always conserved. Energy is only transformed from one form to another. However, if non-conservative forces matter, energy will be transformed to e.g. heat, which cannot be easily transformed back to kinetic or potential energy.

Illustration - Hooke's law



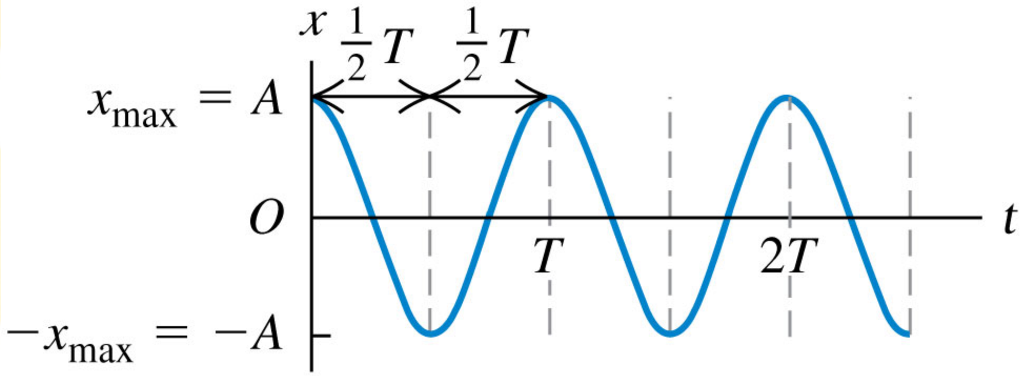
$$F_s = -kx$$

One oscillation cycle



Clicker question

This is an x-t diagram for an object attached to an oscillating spring. Friction is neglected, i.e. there are no non-conservative forces.



At which of the following times does the object have the most negative velocity, v_x?

A. t = T/4

B. t = T/2

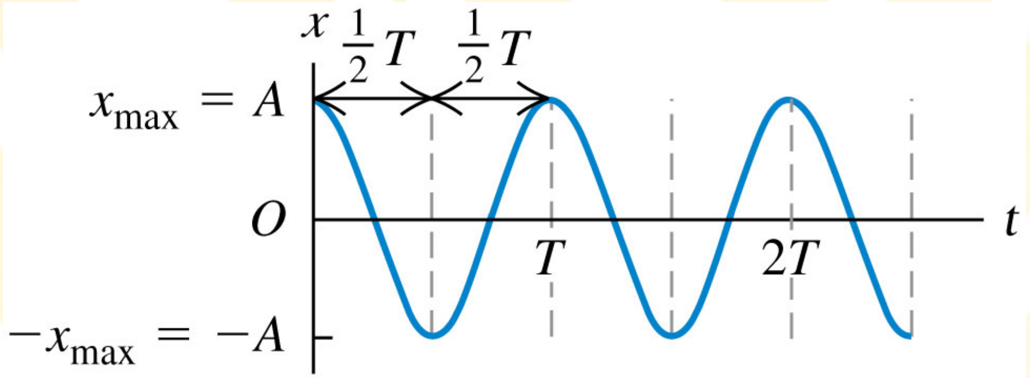
C. t = 3/4 T

D. t = T

Clicker question

This is an x-t diagram for an object attached to an oscillating spring. Friction is neglected, i.e. there are no non-conservative forces.

$$PE_s = \frac{1}{2}kx^2$$



At which of the following times is the potential energy of the spring the greatest?

A. t = T/8

B. t = T/4

C. t = 3/8 T

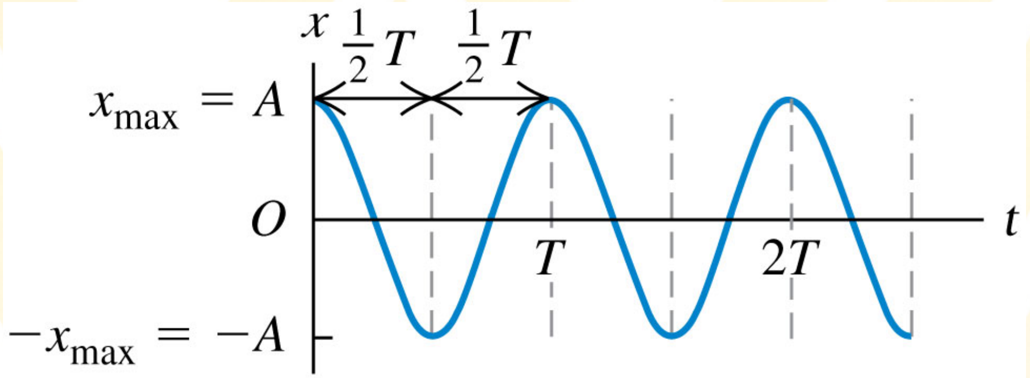
D. t = T/2

E. More than one of the above.

Clicker question

This is an x-t diagram for an object attached to an oscillating spring. Friction is neglected, i.e. there are no non-conservative forces.

$$KE = \frac{1}{2}mv^2$$



At which of the following times is the kinetic energy of the object the greatest?

A. t = T/8

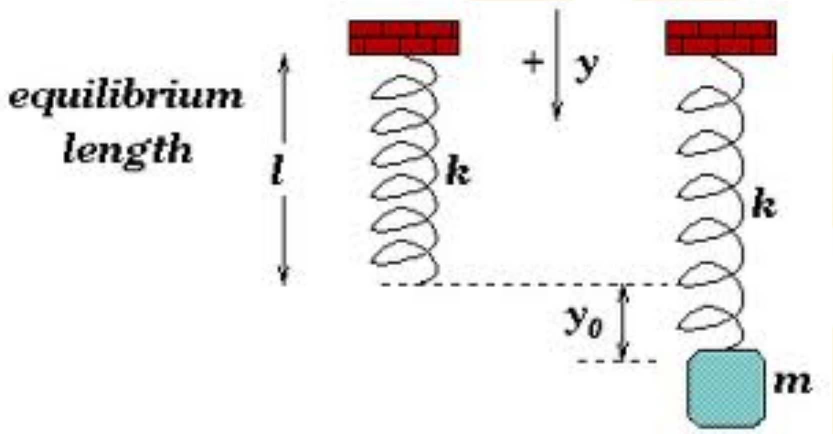
B. t = T/4

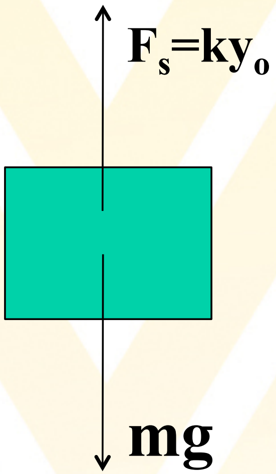
C. t = 3/8 T

D. t = T/2

E. More than one of the above.

Example problem: Vertical Springs





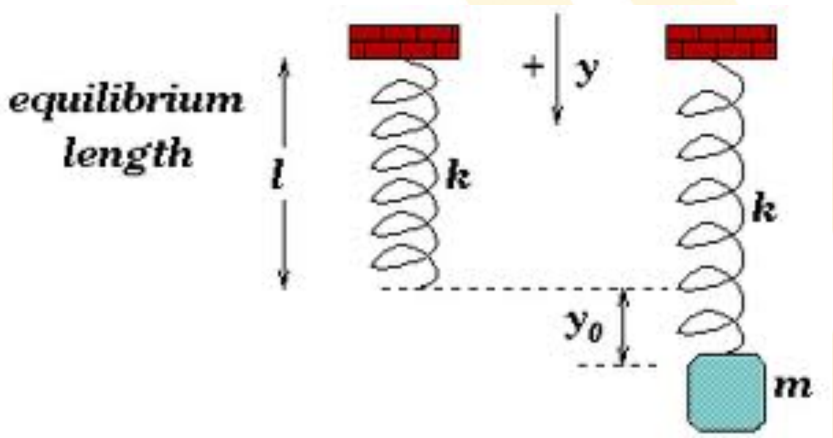
Free Body Diagram

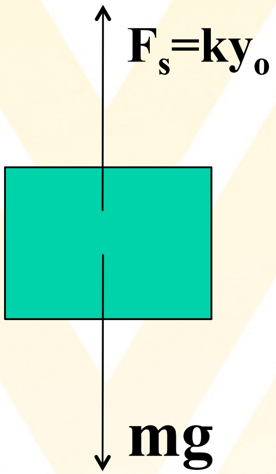
When a 2.5 kg object is hung vertically on a certain light spring, the spring stretches to a distance y_o. What **force** does the spring apply to the object?

If the string stretches 2.76 cm, what is the force constant of the spring?

What is the force if you stretch it 8 cm?

Example problem: Vertical Springs





Free Body Diagram

A 2.5 kg object is hung vertically on a certain light spring with spring constant k=888N/m.

How much work must an external agent do to stretch the same spring 8.00 cm from its unstretched position?

Summary

- Every conservative force can be associated with a potential energy, i.e. an energy that allows the corresponding object to *potentially* do work.

- One example of such a conservative force is gravitation. The gravitational potential energy is:

$$PE_g = mgy$$

- Another important conservative force is the force that springs exert on objects according to Hooke's law:

$$F_s = -kx$$

- The spring potential energy is:

$$PE_s = \frac{1}{2}kx^2$$

- In the absence of non-conservative forces the total mechanical energy is conserved:

$$(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$