# Today's lecture

Units, Estimations, Graphs, Trigonometry:

- Units - Standards of Length, Mass, and Time
- Dimensional Analysis
- Uncertainty and significant digits
- Order of magnitude estimations
- Coordinate Systems
- Trigonometry

Motion in one dimension:

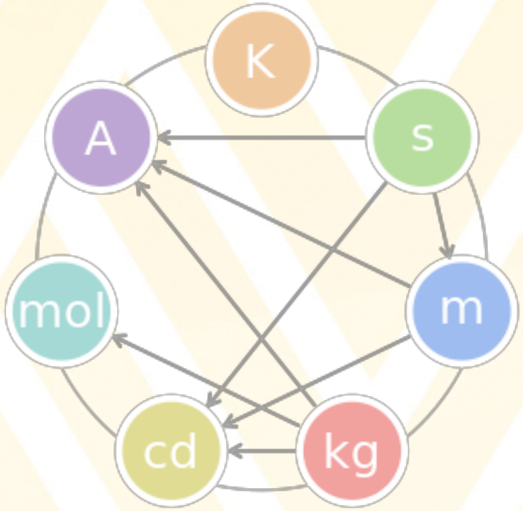
- Frame of reference and Displacement
- Velocity and Speed
- Acceleration

# Chapter 1: Introduction

# Fundamental Quantities & Dimensions

- At the human-sized scale, there are only three (3!) fundamental quantities in **mechanics**:

| | |
|---|---|
| mass | [M] |
| length | [L] |
| time | [T] |




- *All other mechanical units* are derived from these three. For example, velocity is length / time = [L] / [T] and acceleration is velocity / time = [L] / [T]$^2$

# Units

- Measurements of these physical quantities are made in certain **units**.

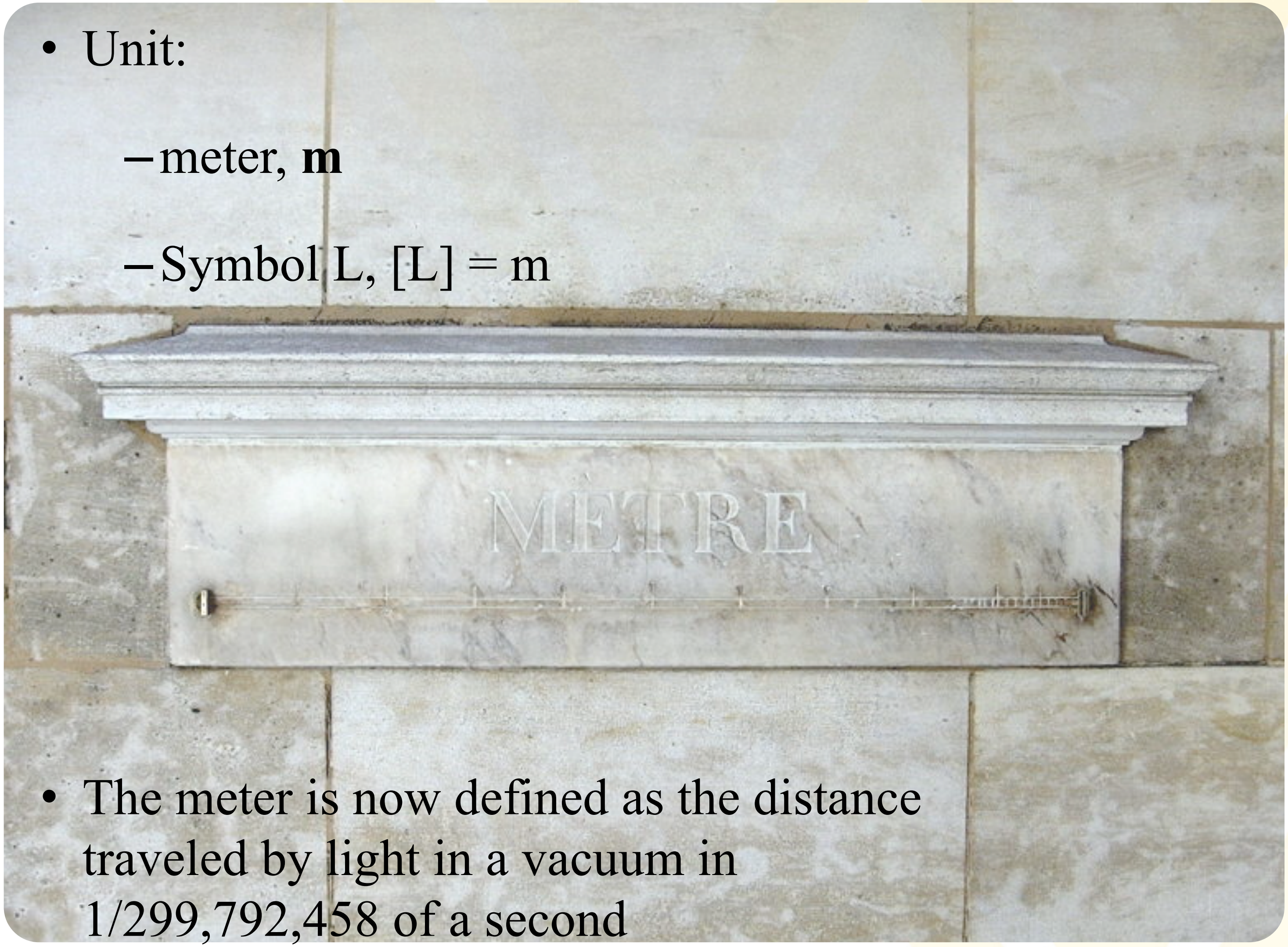
- Providing a physical quantity without a unit is meaningless and wrong.

- SI means ***Systéme International***

- Agreed to in 1960 by a worldwide committee, the SI is the main system used in our textbook, and by virtually 100% of scientists everywhere.

# Length

- Unit:
  - meter, **m**
  - Symbol L, [L] = m



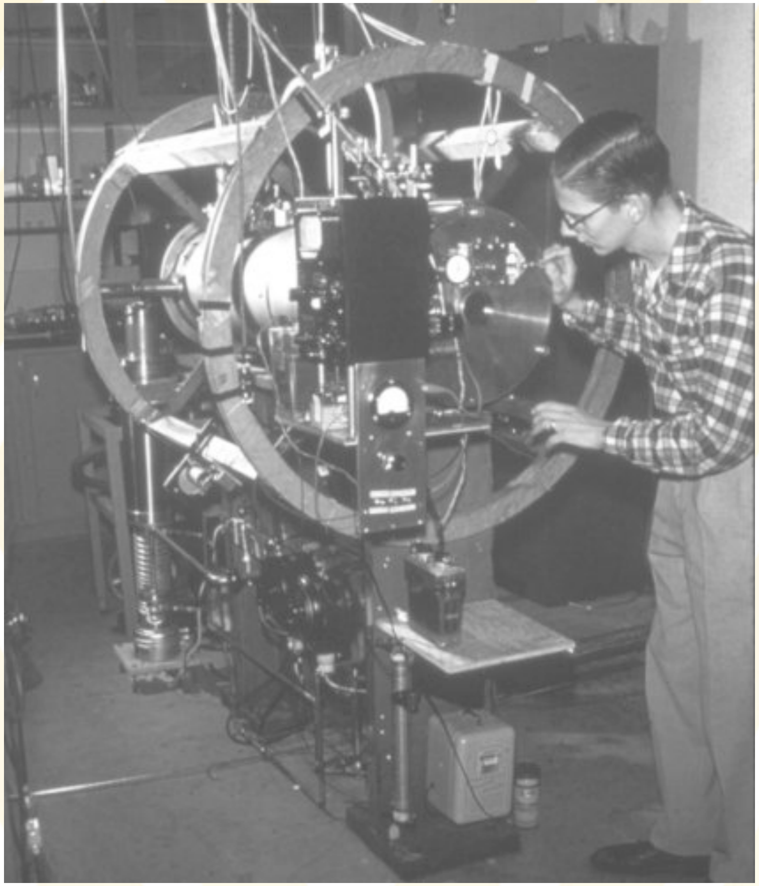

- The meter is now defined as the distance traveled by light in a vacuum in 1/299,792,458 of a second

# Mass

- Unit:
  - kilogram, **kg**
- Now defined as the mass of this cylinder at the International Bureau of Weights and Measures
- Symbol: m, [m] = kg

# Time

- Unit:
  - second, **s**
- Now defined by the oscillation of radiation from a cesium atom
- Symbol: T, [T] = s



*1952: one of the earliest atomic clocks*

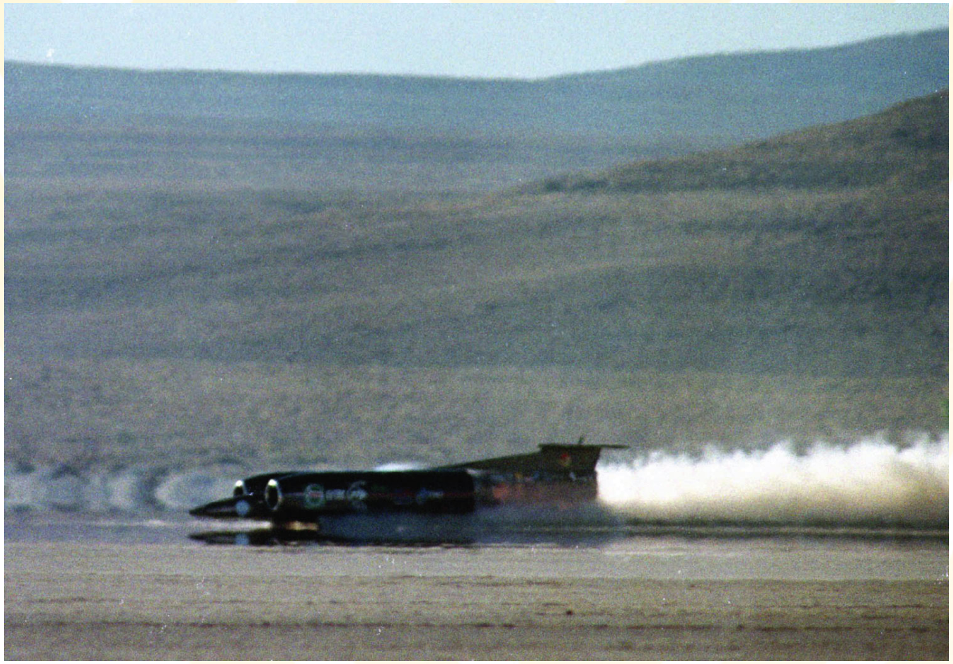
# Other important physical quantities

- Velocity

  $[v] = [L/T] = m/s$

- Acceleration

  $[a] = [L/T^2] = m/s^2$



# Other unit systems

- **cgs** – sometimes called "Gaussian" after

  – Named for the first letters of the units it uses for fundamental quantities (**c**entimeter, **g**ram, **s**econd)

- US Customary
  – "everyday" units; often uses weight in pounds (instead of mass) as a fundamental quantity
  – important not to confuse these! https://www.youtube.com/watch?v=urcQAKKAAl0

# Conversion of units

| System | Length | Mass | Time |
|---|---|---|---|
| SI | meter | kilogram | second |
| cgs | centimeter | gram | second |
| US Customary | foot | slug | second |

**For example:**

1 kilogram (kg) = 0.0011 ton (t)
1 kilogram (kg) = 2.2046 pounds (lb)
1 gram (g) = 0.035 ounce (oz)
1 milligram (mg) = 0.000035 ounces (oz)
1 ounce (oz) = 28,350 milligrams (mg)
1 ounce (oz) = 28.350 grams (g)
1 pound (lb) = 0.454 kilogram (kg)

# Scientific notation

- Prefixes correspond to **powers of ten**
- Each prefix has a name and an abbreviation
- This is how we make *really big* or *really small* numbers easier to think about (and easier to manipulate mathematically)!

See Appendix A.2 for more details

**Table 1.4** Some Prefixes for Powers of Ten Used with "Metric" (SI and cgs) Units

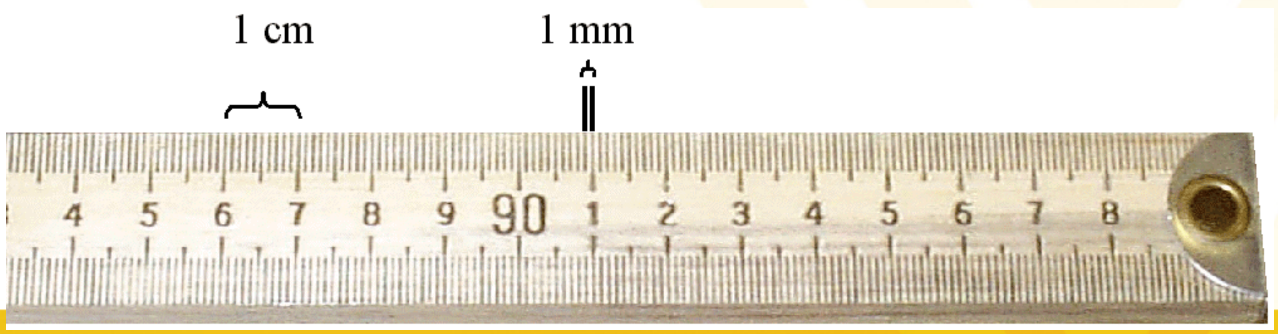
| **Power** | **Prefix** | **Abbreviation** |
|---|---|---|
| $10^{-18}$ | atto- | a |
| $10^{-15}$ | femto- | f |
| $10^{-12}$ | pico- | p |
| $10^{-9}$ | nano- | n |
| $10^{-6}$ | micro- | $\mu$ |
| $10^{-3}$ | milli- | m |
| $10^{-2}$ | centi- | c |
| $10^{-1}$ | deci- | d |
| $10^{1}$ | deka- | da |
| $10^{3}$ | kilo- | k |
| $10^{6}$ | mega- | M |
| $10^{9}$ | giga- | G |
| $10^{12}$ | tera- | T |
| $10^{15}$ | peta- | P |
| $10^{18}$ | exa- | E |

# Dimensional Analysis

- This is how we check the consistency of an equation, and also how we can convert between units without screwing up.
- Dimensions (length, mass, time, etc.) can be treated as algebraic entities that we can add, subtract, multiply, divide.
- Both sides of equation must have the same dimensions!
- However, a correct dimensional analysis does not necessarily mean that the equation is correct.

# Uncertainty and Significant Figures

- There is uncertainty in every measurement.
  - We have to keep track of it, to get an honest result that reflects our level of precision.
- One easy way to do this is with "*significant figures*".
- If we measure the distance between two objects with a meter stick, we will only be able to measure it with an accuracy corresponding to the division of the stick.
- This determines the "error" and the number of significant figures.



# Significant Figures - Rules

In multiplying/dividing two or more quantities, the number of significant figures in the final result is the same as the number of significant figures in the least accurate of the factors being combined.

Example: $$1.39 \times 2.1 = 2.919 \approx 2.9$$

When numbers are added/subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum or difference.

Example: $$1.39 + 2.1 = 3.49 \approx 3.5$$

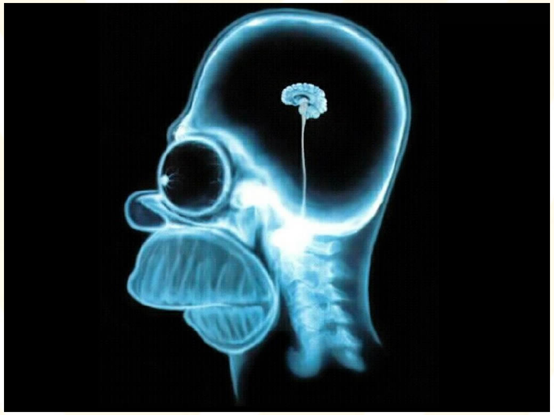
# Announcements

- You get **5** attempts per question on WebAssign (this is always the default).
- Check the weather forecasts and the local school closings, we will follow Monongalia County Schools when deciding whether to cancel.

# Order of magnitude estimations

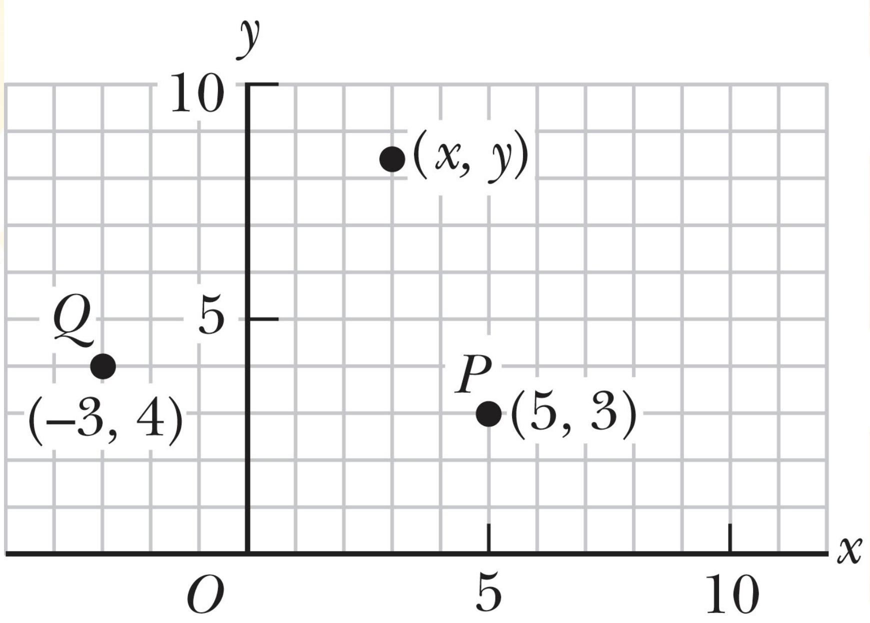
- Can yield useful approximate answers.
  - An exact answer may be difficult or impossible.
    - Mathematical reasons
    - Limited information available
- Can serve as a partial check for exact calculations.

How many cells are located in the human brain?



# Cartesian coordinate systems

- Used to describe the position of a point in space
- Coordinate system consists of
  - A fixed reference point called the origin, **O**
  - Specified axes with scales and labels
- x- and y- axes
- Points are labeled (x,y)
- Positive x is usually selected to be to the right of the origin
- Positive y is usually selected to be to upward from the origin



# Trigonometry I

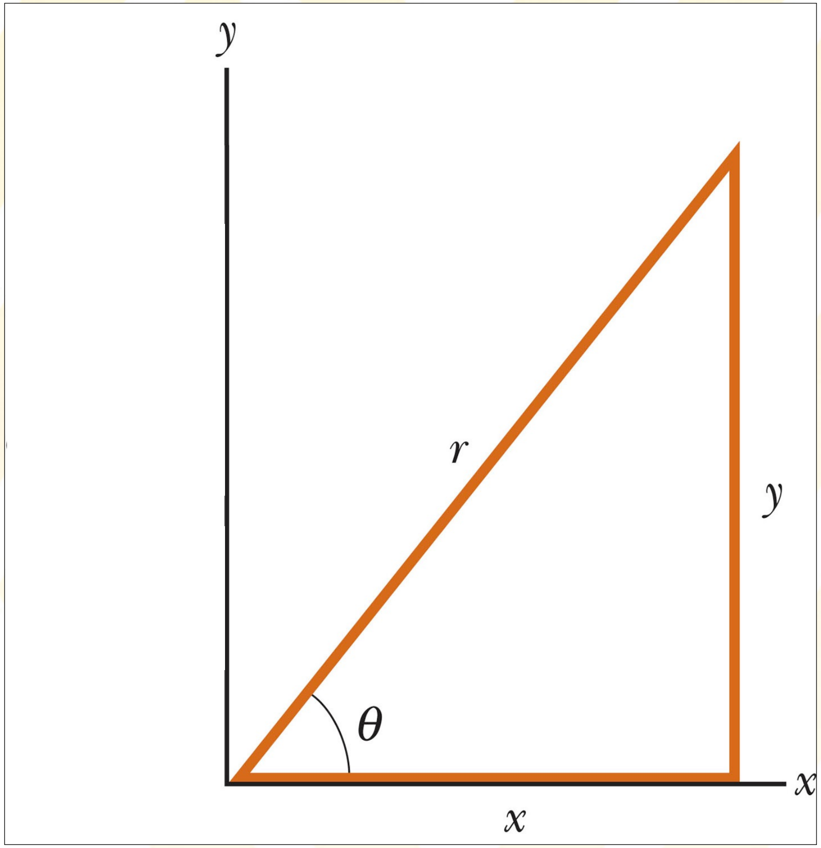
$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

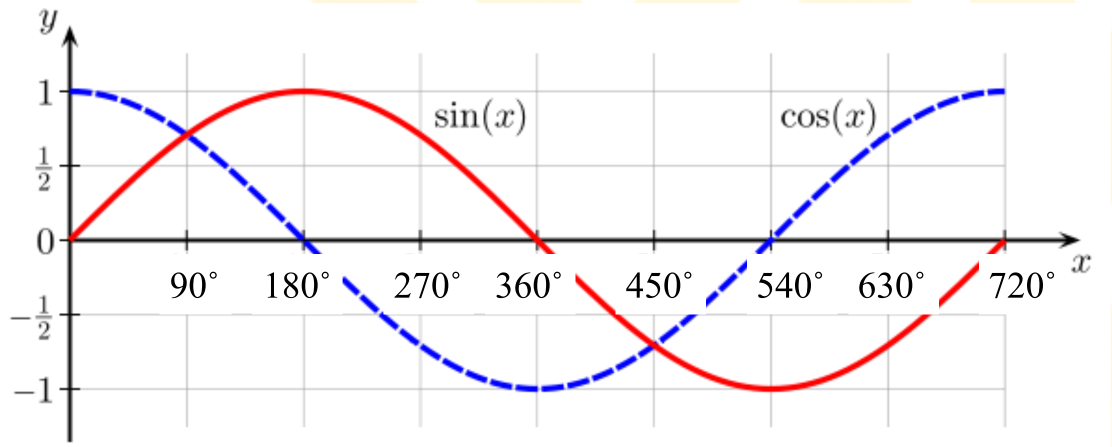
$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

Pythagorean theorem:

$$x^2 + y^2 = r^2$$



# Trigonometry II




sin(x) and cos(x) are mathematical functions that describe oscillations.

This will be important later, when we talk about oscillations and waves.

# Trigonometry III

- To find an angle, you need the inverse trig function.
    - For example, $\theta = \sin^{-1} 0.707 = 45°$
- Be sure your calculator is set for the appropriate angular units for the problem!
    - For example:

$$\tan^{-1} 0.5774 = 30.0°$$

$$\tan^{-1} 0.5774 = 0.5236 \text{ rad}$$

---

A detailed recap of mathematics required for this class can be found in Appendix A of our textbook.

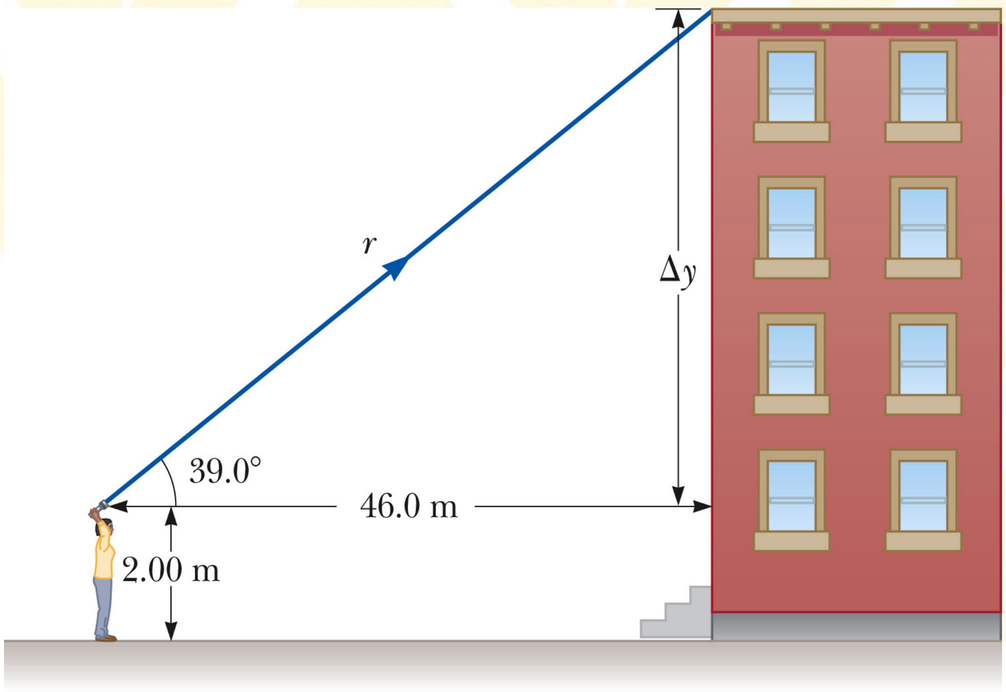
---

# Typical trigonometry problem

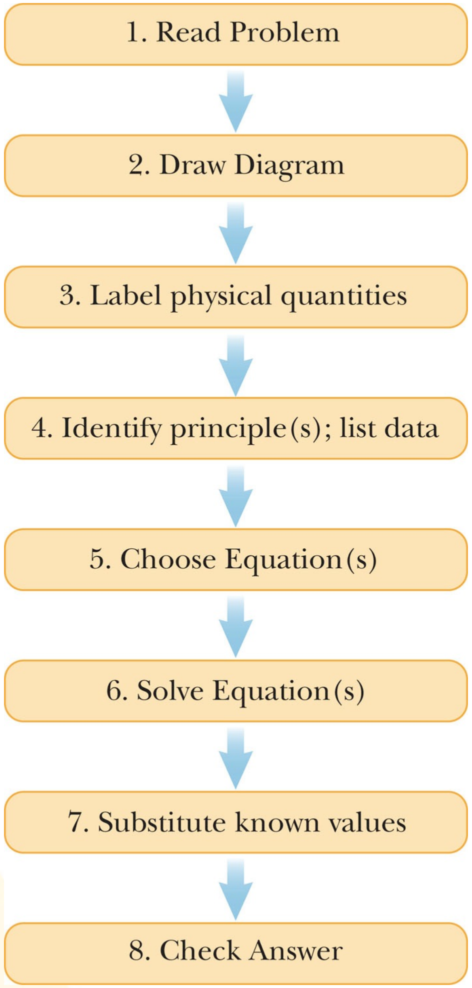
A person measures the height of a building by walking out a distance of 46.0 m from its base and shining a flashlight beam to its top. When the beam is elevated at an angle of 39.0° with respect to the horizontal, the beam just strikes the top of the building.

(a) If the flashlight is held at a height of 2.0 m, find the height of the building.

(b) Calculate the length of the light beam.



# Problem Solving Strategy




- read the problem twice! maybe three times!
  - can you estimate the answer's order of magnitude?
- identify what *type* of problem it is
- label your diagram with the given information: variables, values, coordinates...
- do algebraic steps carefully
- check your answer for *sense* and *units*
  - should it be *positive* or *negative*?
- is it consistent with your initial estimate?

# Problem Solving Strategy

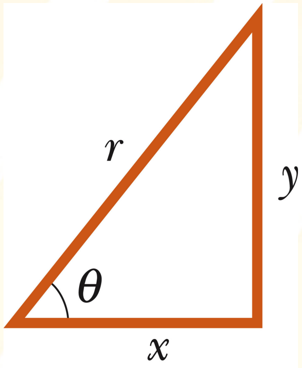
1. Read Problem
2. Draw Diagram
3. Label physical quantities
4. Identify principle(s); list data
5. Choose Equation(s)
6. Solve Equation(s)
7. Substitute known values
8. Check Answer

- Equations are the tools of physics
  - Understand what the equations mean and how to use them
- Carry through the algebra as far as possible
  - Substitute numbers at the end
- *Be organized*!

# Summary

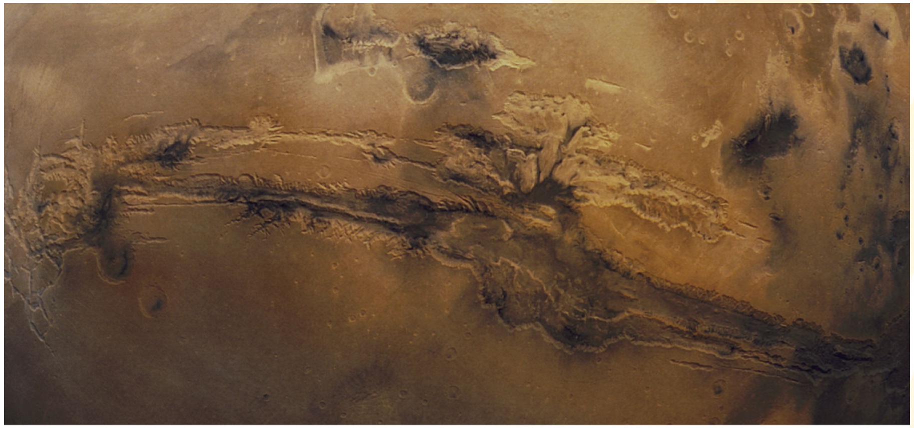
- All physical quantities in mechanics can be expressed by meters (m), kilograms (kg), and seconds (s).
- Every physical quantity must have a unit. Units must be consistent. We use SI-units.
- Dimensional analysis can be used to check equations.
- No physical quantity can be determined without an uncertainty, that determines its significant figures.
- An order of magnitude estimation can be useful to check the result of a calculation.
- The cartesian coordinate system consists of two perpendicular and labeled axes (x- and y-axis). Points are located by specifying their x- and y-values.
- Trigonometry: $\sin(\theta) = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$ $\tan(\theta) = \frac{y}{x}$

$$x^2 + y^2 = r^2 \qquad \text{Pythagorean theorem}$$



# Chapter 2: Motion in one Dimension

# Frame of Reference I

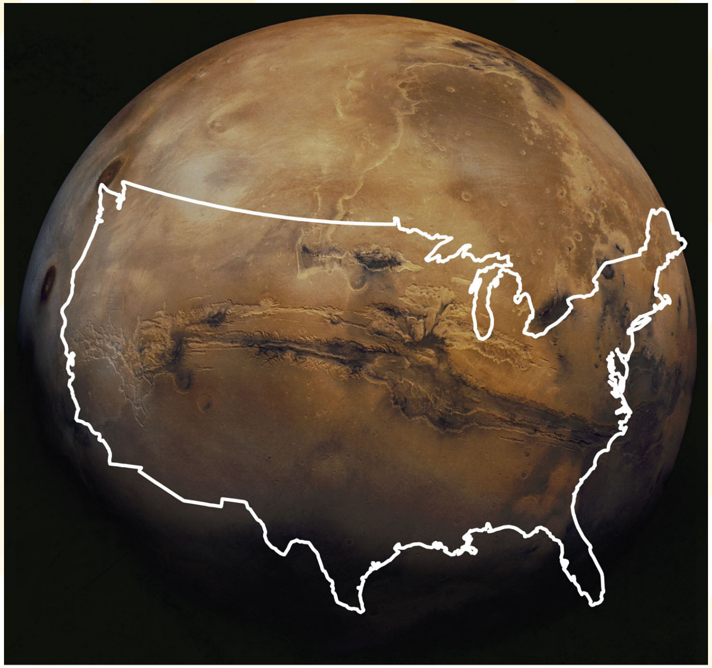

a

This picture shows a canyon on Mars.

If you just look at the picture, you will not have any idea how long it is.

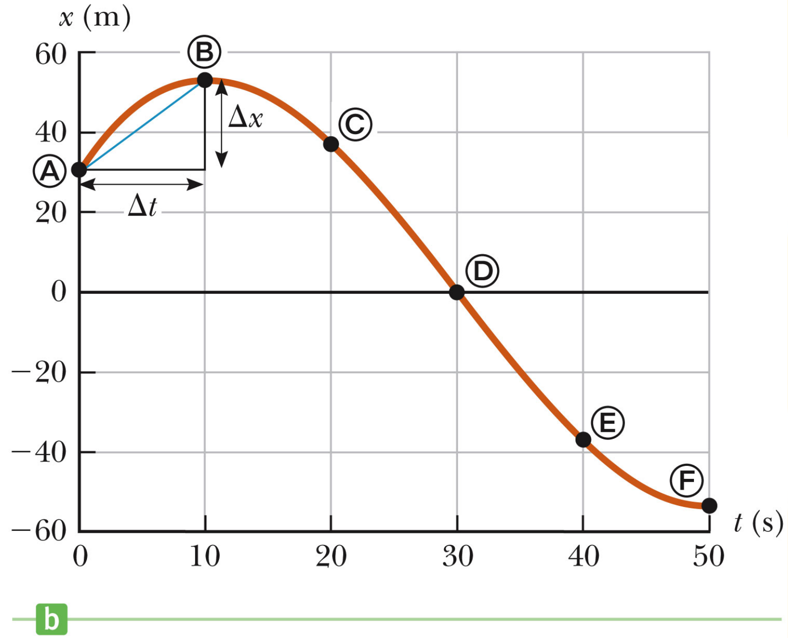
It could be 100 m or 10000 km.

We need a Reference Frame!


b

The reference frame shows that the canyon is about 2000 km long.

# Frame of Reference II



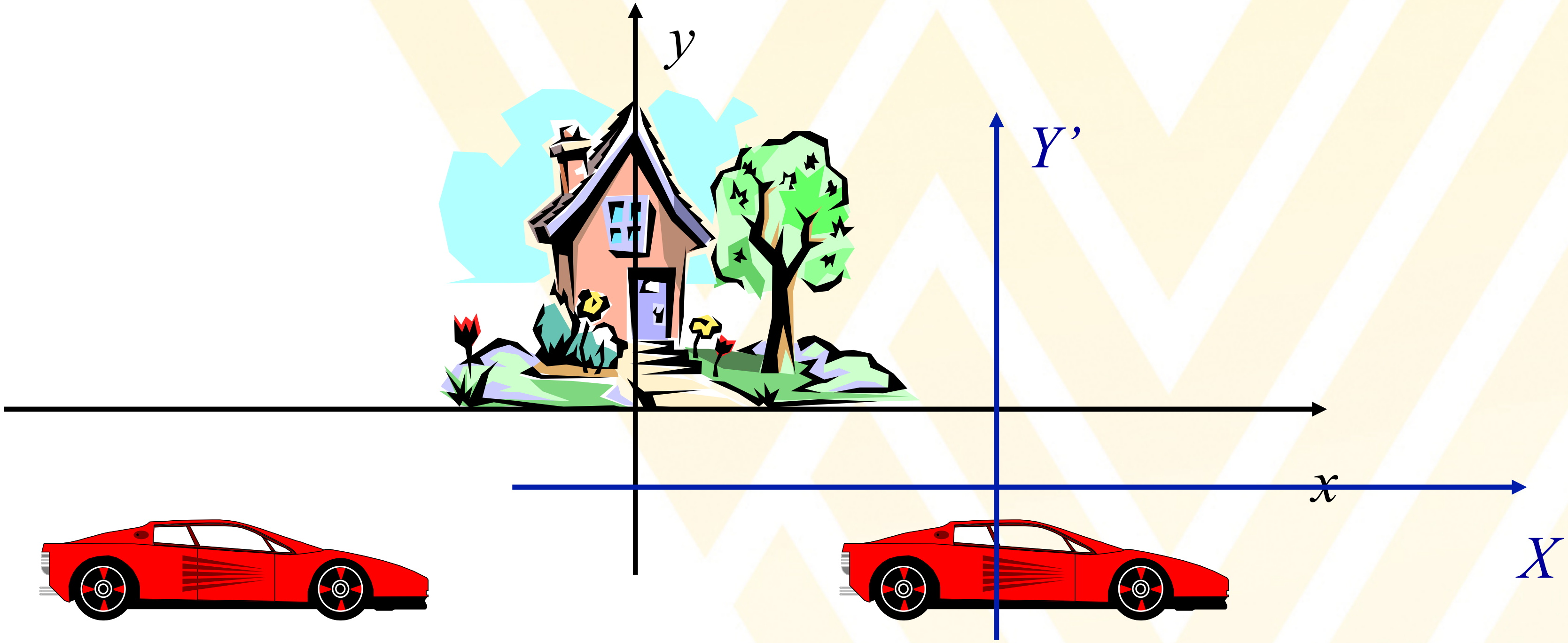

In science, a frame of reference is a coordinate system, whose origin is placed at a particular position.

In this reference system the displacement of an object is defined as:

$$\Delta x \equiv x_f - x_i$$ (*i* for initial, *f* for final, Δ for difference)

# Displacement - Example

Change of the initial position of an object




$x_f$ = -5.0 m

$x'_f$ = -8.0 m

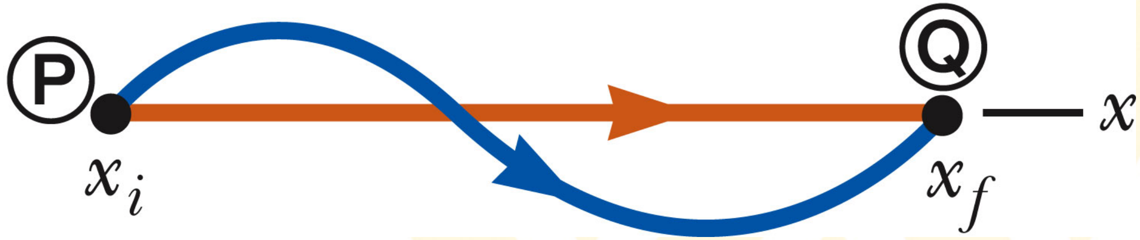
$x_i$ = +3.0 m

$x'_i$ = 0.0 m

Displacement: $\Delta x = x_f - x_i$ $\Delta x$ = -5.0 m - (+3.0 m) = -8.0 m

Other reference frame: $\Delta x = x_f - x_i$ $\Delta x$ = -8.0 m – (0.0 m) = -8.0 m

# Displacement vs. Path Length




Displacement is not the same as Path Length!

Displacement, $\Delta x$, depends only on the endpoints of the path, while the path length, $l$, depends on the actual route taken.

Thus: $l \gtrsim \Delta x$

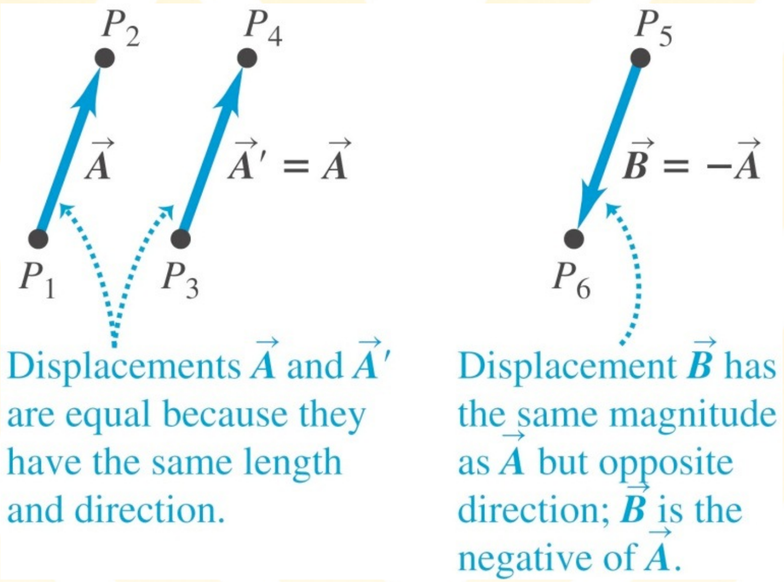
# Vectors vs. Scalars

There are two fundamentally different types of physical quantities:

1) Vectors
2) Scalars

A scalar quantity is fully characterized by its magnitude.

A vector must be characterized by magnitude and direction. It is represented by an arrow.




Do you know any examples for scalar and vector quantities?

Scalars: Mass, time, volume, distance

Vectors: Displacement, velocity, acceleration, flux

In 1d we do not need vectors, since there is only 1 direction.

More on vectors on Jan 28th!

# Velocity and Speed

The average velocity during a time interval, $\Delta t$, is defined as the displacement, $\Delta x$, during this time interval divided by $\Delta t$:

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Velocity is a vector quantity, since it has a magnitude and a direction!

In contrast to velocity, the average speed is a scalar and defined as:

$$\text{Average speed} = \frac{\text{path length}}{\text{elapsed time}}$$

Speed is always a positive number. Velocity can be negative.

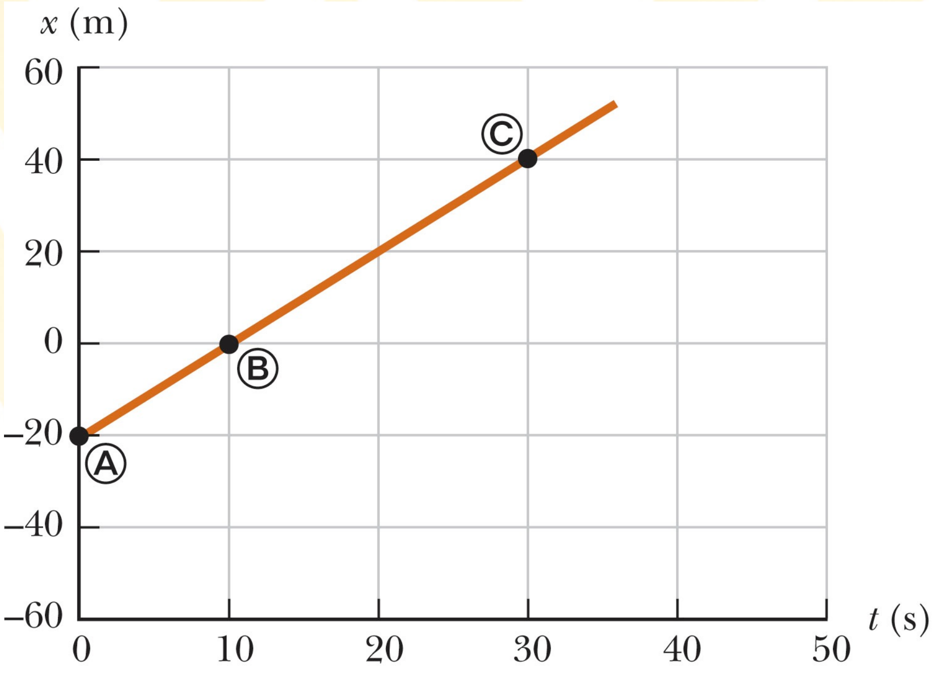
The unit of velocity and speed is m/s.

# Graphical interpretation of velocity

- The average velocity between times $t_1$ and $t_2$ can be determined from a position-time graph.
- It equals the slope of the line joining the initial and final positions.

$$slope = \frac{\text{change in vertical axis}}{\text{change in horizontal axis}}$$

- Objects moving at constant velocity will result in straight line in such a plot.
- Slopes have units!

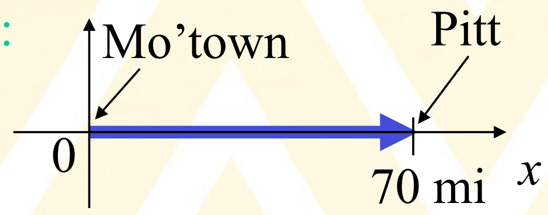


# Average velocity - Example

Go from Morgantown to Pittsburgh in 2h and return back to Morgantown 3h after leaving.

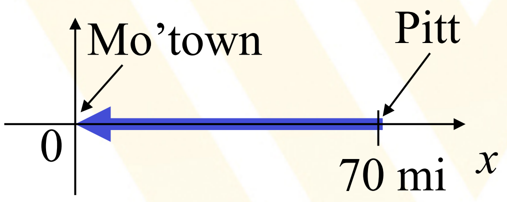
Average velocity going to Pitt:

$x_i = 0 \quad t_i = 0$
$x_f = +70\text{ mi} \quad t_f = 2\text{ hrs}$




Average velocity coming back from Pitt:

$x_i = +70\text{ mi} \quad t_i = 2\text{ hrs}$
$x_f = 0\text{ mi} \quad t_f = 3\text{ hrs}$



# Instantaneous velocity

The instantaneous velocity is the velocity at a particular moment in time, while the average velocity is the velocity averaged over a time interval.

There can be a huge difference between average and instantaneous velocity.

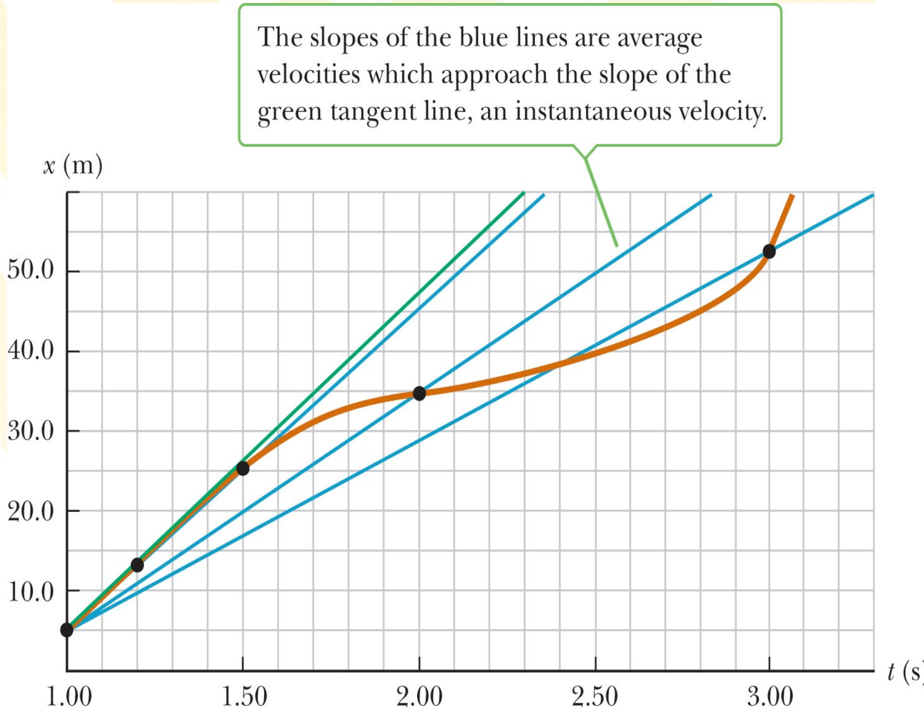

You can go from Morgantown to Pittsburgh by car in 2 h in many different ways:

1. You travel at 35 mi/h for 2 hours. In this case your average and instantaneous velocities are both 35 mi/h

2. You travel at 10 mi/h for 1.5 hours and at 110 mi/h during the remaining 0.5 hours. Your average velocity will also be 35 mi/h, but your instantaneous velocity will be 10 mi/h at every moment during the first 1.5 hours and 110 mi/h during the last 0.5 h.

# Mathematical and graphical determination

- This graph illustrates the motion of an object whose velocity is changing in time (not a straight line).
- The slopes of the blue lines correspond to the average velocities during the respective time intervals.
- By reducing the time interval as much as possible around t = 1s, the instantaneous velocity at this time is determined.
- The instantaneous velocity corresponds to the slope of the tangent at t = 1 s.




$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

# Acceleration

Acceleration is defined as the change of velocity during a time interval, Δt.

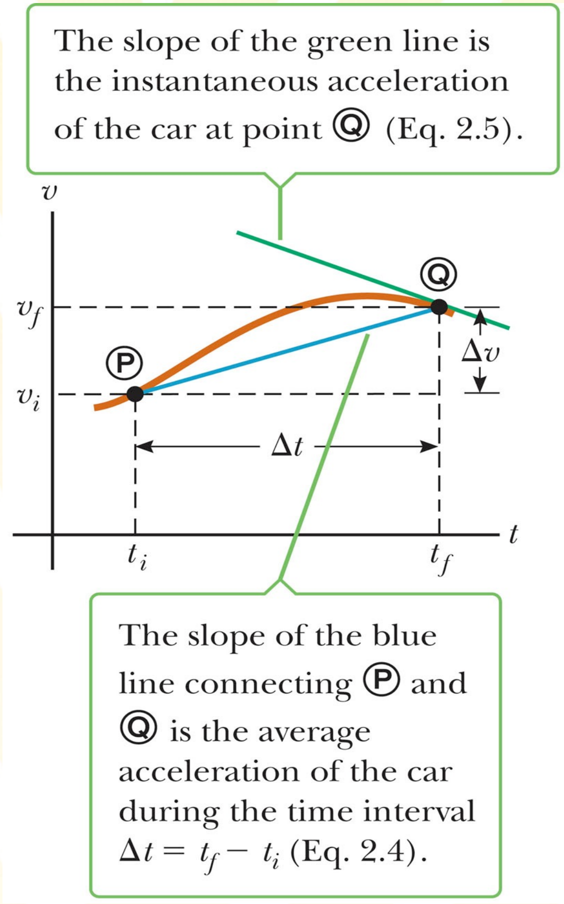
Remember: Velocity is the change of position during Δt.

Again, there is an average and an instantaneous acceleration:

$$\bar{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Acceleration is a vector quantity, since it has a magnitude and direction. Its unit is m/s$^2$.

The instantaneous acceleration of an object at time, $t_f$, is the slope of the tangent of the velocity-time graph at this time.



# Summary

- Discussing an object's motion requires a reference frame.
- Displacement is defined as the change of an object's position: $\Delta x \equiv x_f - x_i$
- A vector has a direction and a magnitude, while scalars only have a magnitude.
- The average and instantaneous velocities are defined as:

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The instantaneous velocity at time, t, corresponds to the slope of the tangent in a position-time diagram at this time.

- The average and instantaneous accelerations are defined as:

$$\bar{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

They instantaneous acceleration at time, t, corresponds to the slope of the tangent in a velocity-time diagram at this time.