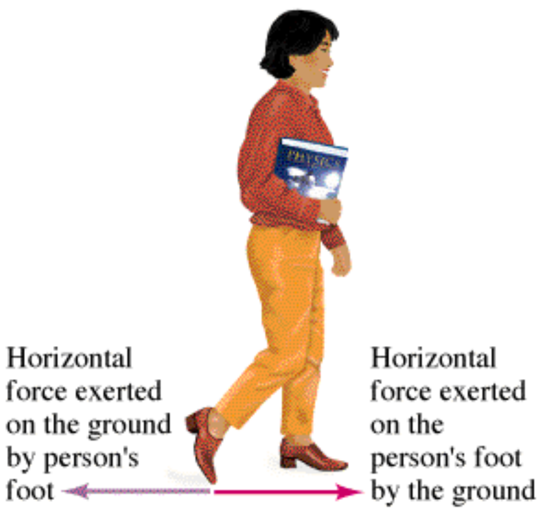
Announcements

- The second midterm exam is March 8, 5-7 PM in White B51 (this room).
- The makeup exam is March 5, 5-7 PM in Clark 317.
- All exam info, including this, is at the class webpage, http://community.wvu.edu/~stmcwilliams/Sean_McWilliams/SP19_PHYS_101.html
- The exam will cover what we covered in class and was listed in the syllabus, from chapters 5 - 6, including all material through Monday's class.
- The questions will be multiple choice.
- Formula sheets will again be provided.

Today's lecture

Momentum and Collisions: Rocket Propulsion

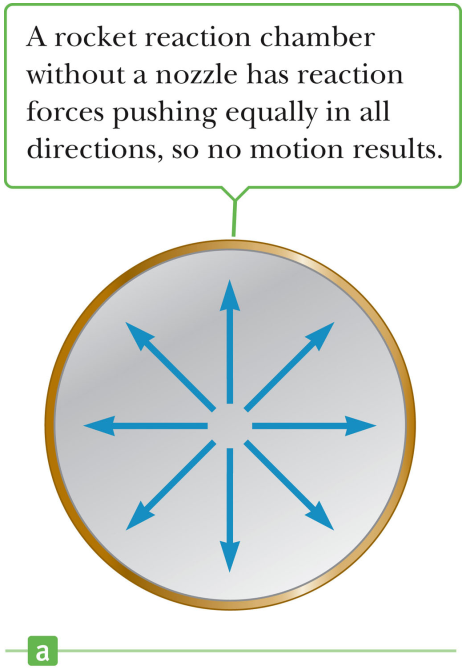
How do we move forward on earth?



When we walk, we exert a force on earth. This force pushes earth, but not us. Due to Newton's third law earth exerts the same, but opposite force on us. This reaction force pushes us forward!

In space, there is nothing to push against. How can an object be accelerated in space?

Rocket propulsion - The idea

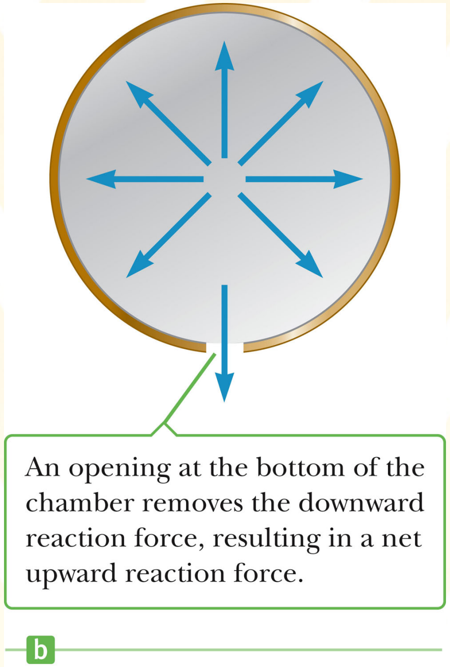


The main part of a rocket is a chamber filled with combustible gas.

When an explosion occurs, the gas expands and pushes against the chamber walls in all directions.

If there is no hole in the chamber, all forces will compensate and the rocket will not move.

However, if there is a hole, there will be an effective force pushing the rocket.



The hot gas does not push on anything external, but on the rocket itself → Therefore, this technique works in space!

Rocket propulsion - the physics

Initially, the rocket moves at velocity v and has a total mass of (M + Δm), where Δm is the mass of the fuel.

The rocket ejects the fuel of mass Δm and speeds up to a velocity v + Δv, where Δv is the increase in velocity due to the ejection of the fuel with velocity v_e. The fuel (Δm) then moves at a velocity v - v_e relative to earth, which is at rest.

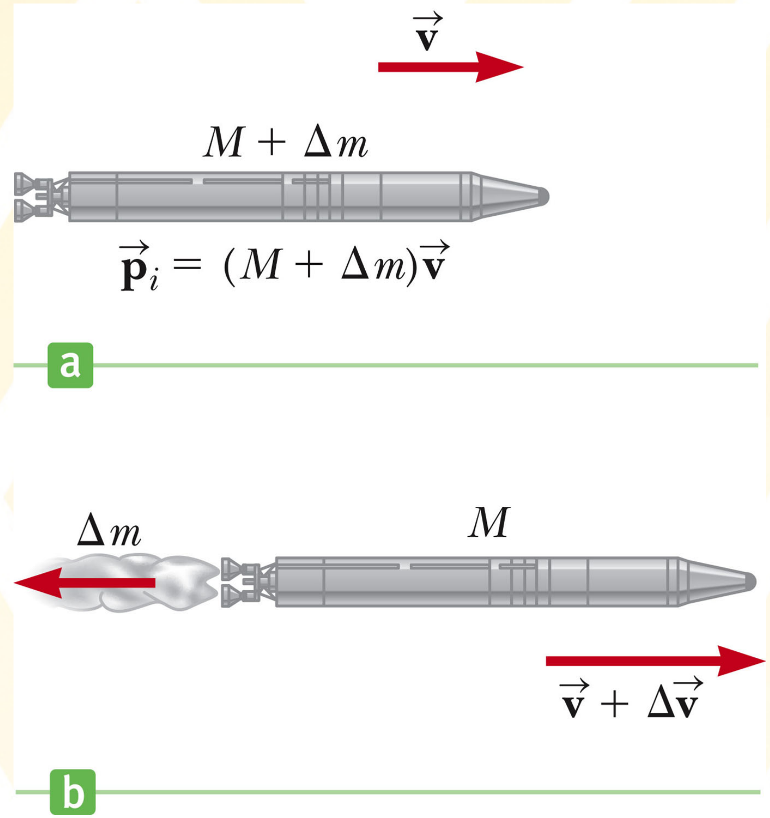
It's all momentum balance:

$$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e)$$

Momentum: Rocket + fuel

Momentum: Rocket without fuel

Momentum: Fuel



Rocket propulsion - the physics

$$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e)$$

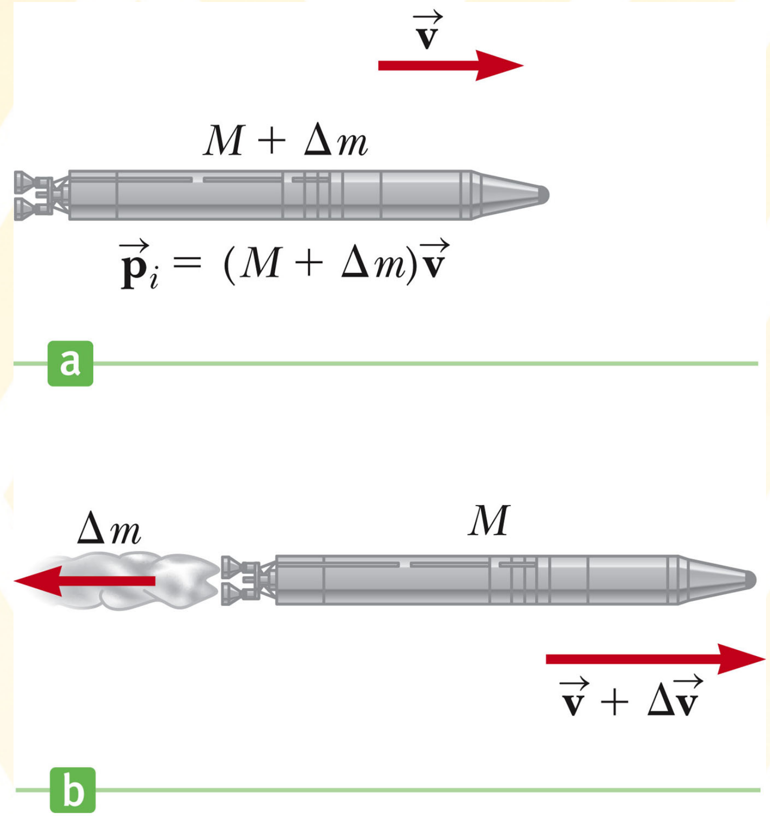
This can be simplified to:

$$M\Delta v = v_e \Delta m$$

The increase Δm in the mass of the exhaust corresponds to a decrease of the total mass of the rocket (Δm = -ΔM):

$$\rightarrow \quad M\Delta v = -v_e \Delta M$$

$$\rightarrow \quad \Delta v = -v_e \frac{\Delta M}{M}$$



Using methods of calculus this can be integrated:

$$\int_{v_i}^{v_f} dv = -v_e \int_{M_i}^{M_f} \frac{dM}{M} \longrightarrow v_f - v_i = v_e ln\left(\frac{M_i}{M_f}\right)$$

Rocket Equation

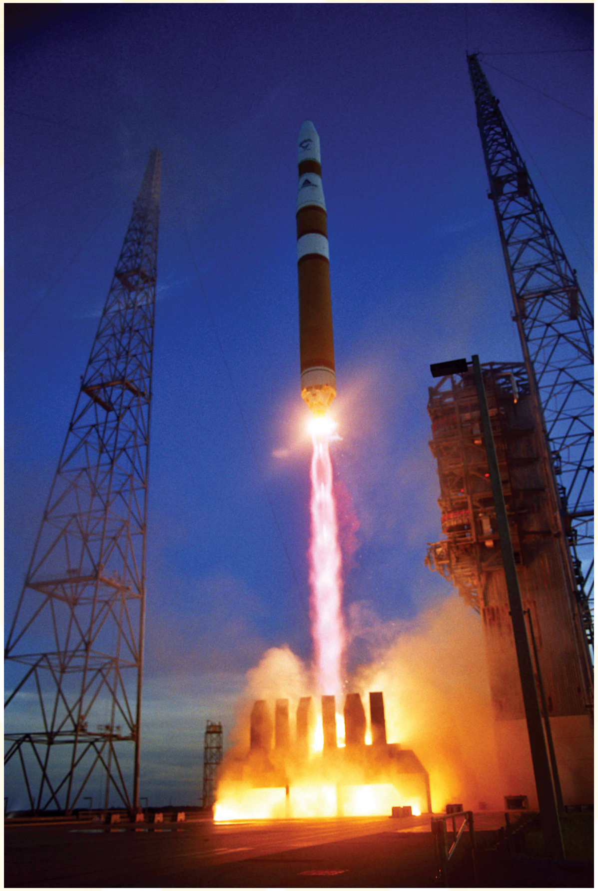
$$v_f - v_i = v_e ln\left(\frac{M_i}{M_f}\right)$$

What does this equation tell us?

The increase of the rocket's velocity is bigger

- the faster the exhaust speed.
- the more mass is ejected.

The ratio of the initial to the final mass, M_i/M_f is limited to a factor of about 10.



Thrust

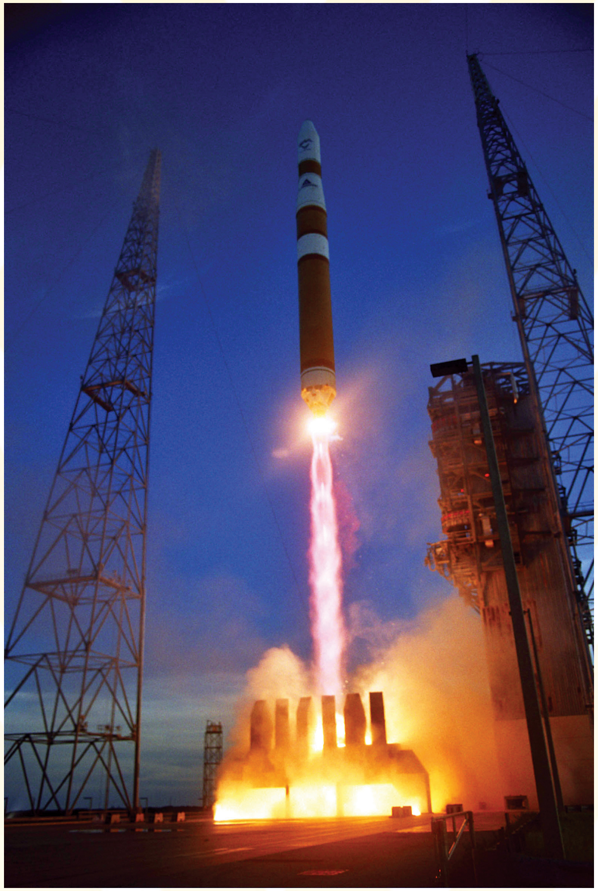
What is the force (= thrust) exerted on the rocket by the ejected gas?

$$M \Delta v = -v_e \Delta M$$

$$\rightarrow \quad \text{Thrust} = Ma = M\frac{\Delta v}{\Delta t} = -v_e \frac{\Delta M}{\Delta t}$$

The absolute value of the thrust increases

- the higher the exhaust velocity is.
- the higher the rate of change of the mass is.

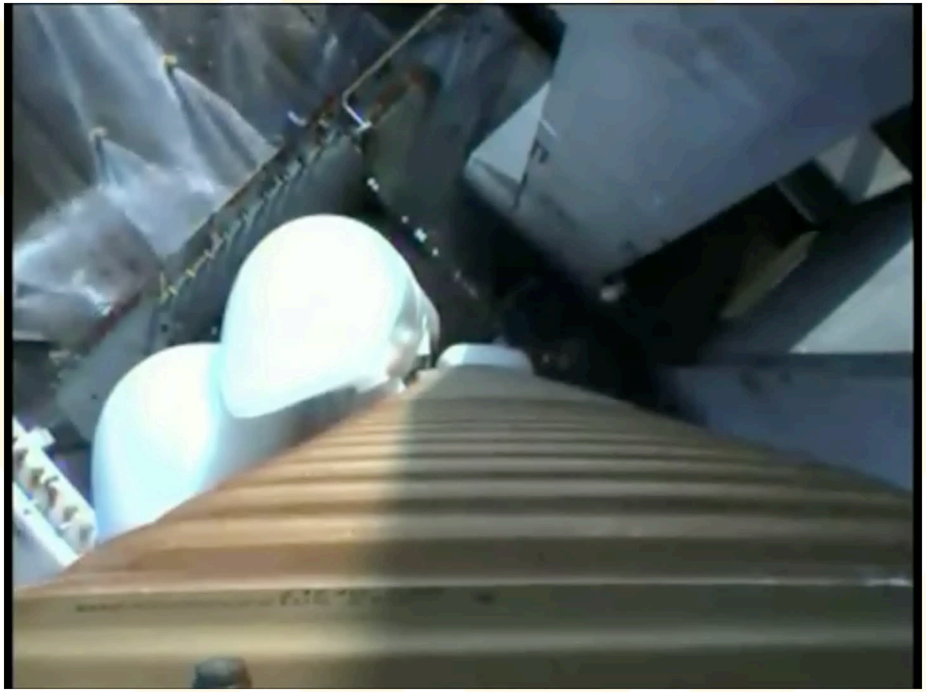


Multistage Rockets

$$v_f - v_i = v_e ln\left(\frac{M_i}{M_f}\right)$$

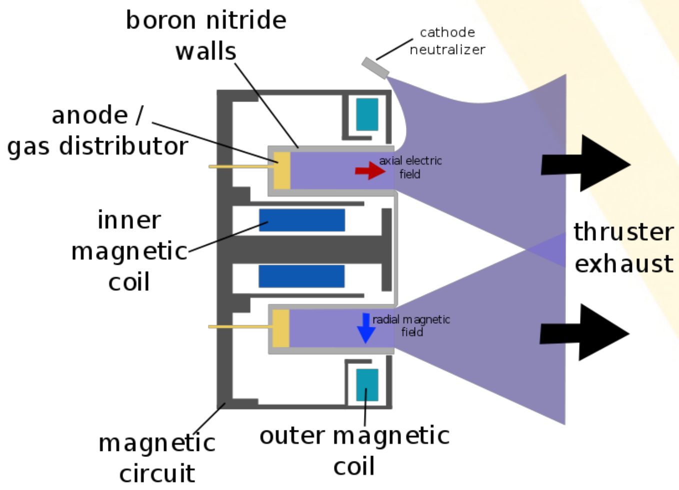
Current maximum exhaust speeds are about v_e = 4500 m/s (liquid hydrogen + oxygen).

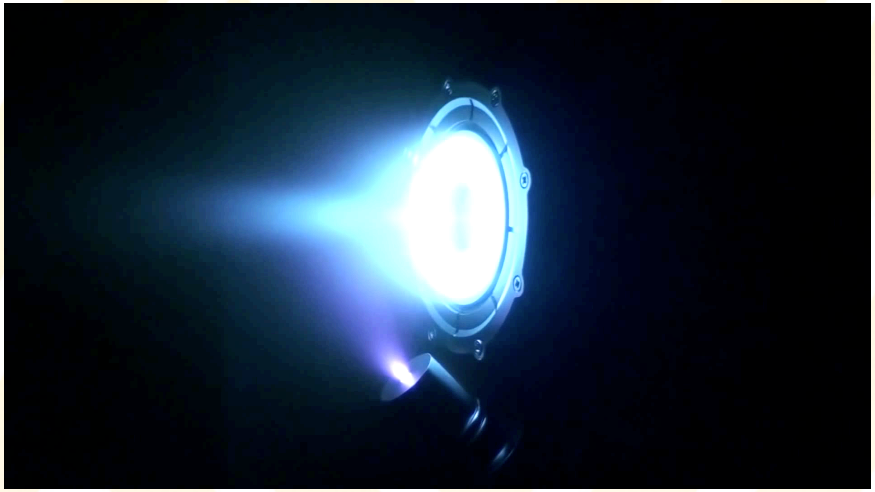
For a given rocket with a mass ratio of M_i/M_f = 10 this results in a maximum rocket speed of 10,000 m/s.



In order to reach higher speeds multistage rockets are used. By dropping stages the rocket becomes lighter and can be accelerated to higher velocities by fuel burned after the engine separation.

Plasma thrusters





Classical rockets produce a lot of thrust, but cannot be used for small course corrections of e.g. satellites. Plasma thrusters are used instead.

A Xenon-Plasma (heavy gas, easy to ionize) is produced and ions are accelerated out of the thruster by an electric field.

This yields very little thrust (about 100 mN), that can be controlled precisely.

Example Problem: Rocket propulsion

A rocket has a total mass of 1×10^5 kg and a burnout mass of 1×10^4 kg, including engines, shell, and payload. The rocket blasts off from earth and exhausts all its fuel in 4 min, burning the fuel at a steady rate with an exhaust velocity of 4.5×10^3 m/s.

(a) If air, friction, and gravity are neglected, what is the speed of the rocket at burnout?

(b) What thrust does the engine develop at liftoff?