Recap

- Classical rocket propulsion works because of momentum conservation. Exhaust gas ejected from a rocket pushes the rocket forwards, i.e. accelerates it.

$$v_f - v_i = v_e ln\left(\frac{M_i}{M_f}\right)$$

- The bigger the exhaust speed, v_e, the higher the gain in velocity of the rocket.
- The higher the ratio of initial to final mass of the rocket, the bigger the gain in velocity.
- Multi-stage rockets are used to minimize the final mass, M_f.
- Plasma thrusters are used for precise thrust control of satellites. They are based on ejecting ions at high velocities.

Today's lecture

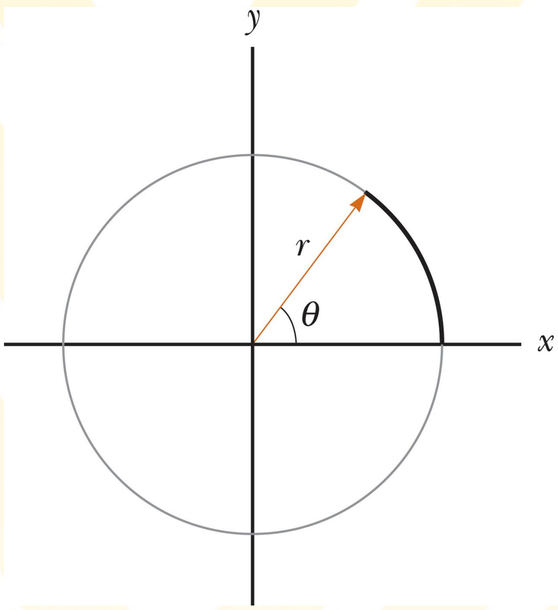
Rotational Motion:

- Angular Displacement, Speed, and Acceleration
- Rotational Motion with constant angular acceleration
- Relations between Angular and Linear Quantities

Chapter 7: Rotational Motion and the Law of Gravity

Today's goal

Describe the rotational motion of a rigid body as a function of time mathematically.



We already know how to do this for a linear motion with constant acceleration using the object's position, x, linear velocity, v, and linear acceleration, a.

For circular motion it works very similarly, but using the angle, θ, the angular velocity, ω, and the angular acceleration, α:

Linear motion (constant a):

$$x(t) = x_i + v_i t + \frac{1}{2}at^2$$

$$v(t) = v_i + at$$

Rotational motion (constant α):

$$\boxed{\begin{aligned}\theta(t) &= \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega(t) &= \omega_i + \alpha t\end{aligned}}$$

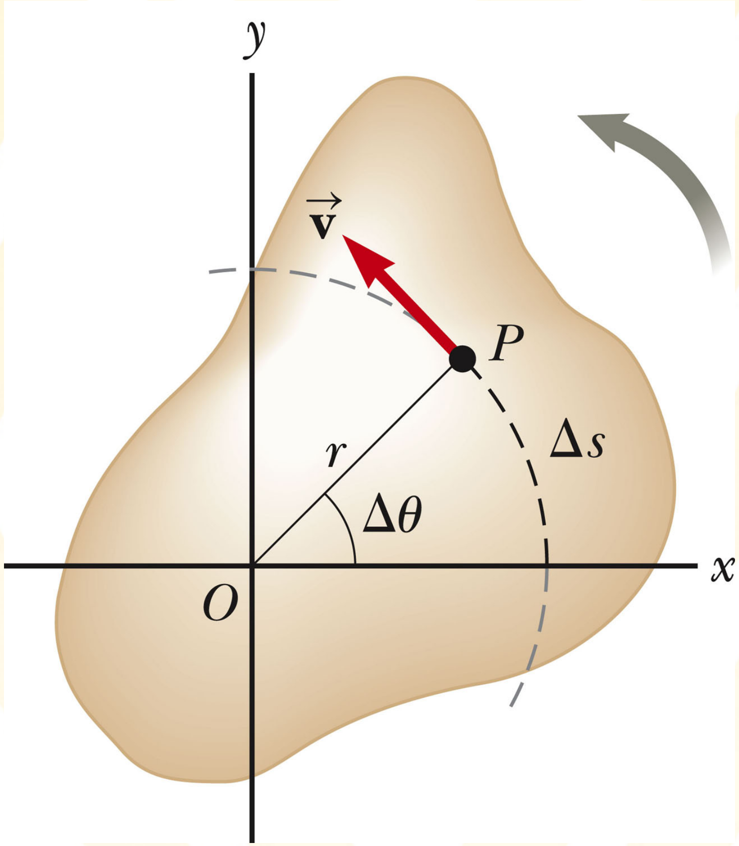
Rigid body

We will discuss rotational motion only for so-called rigid bodies:

This means that each part of the body is fixed in position relative to all other parts of the body.

The shape of the body can be complicated - that does not matter.

Examples for rigid bodies are discs, wheels, etc.



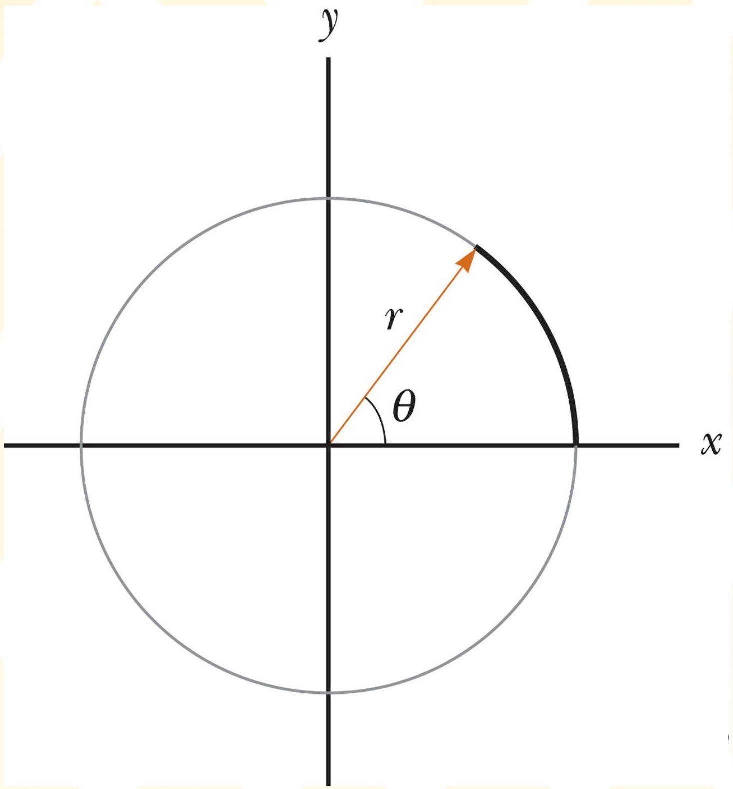
Period and frequency

The duration of one revolution is called the period, T.

Unit: seconds

The inverse of one period is called the frequency, $f = 1/T$.

Unit: $1/s = Hz$



Angular Position

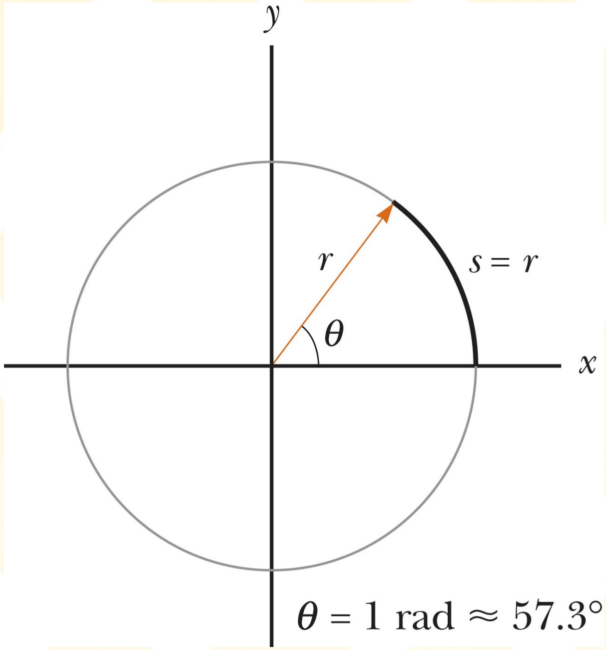
Different points located at different radii cross different distances per unit time.

However, each point on a rigid body crosses the same angle, θ, during a given time interval!

→ θ is a better quantity to describe rotational motion.

Definition of the angular position, θ:

$$\boxed{\theta = \frac{s}{r}}$$



Here, s is the corresponding displacement along the circular arc. r is the radius.

θ is measured in radians and not in degrees!

1 rad = 57.3 degree

Make sure to set your calculator to rad instead of deg!

Clicker question

What is *s* for one rotation (circumference of a circle)?

A. πr

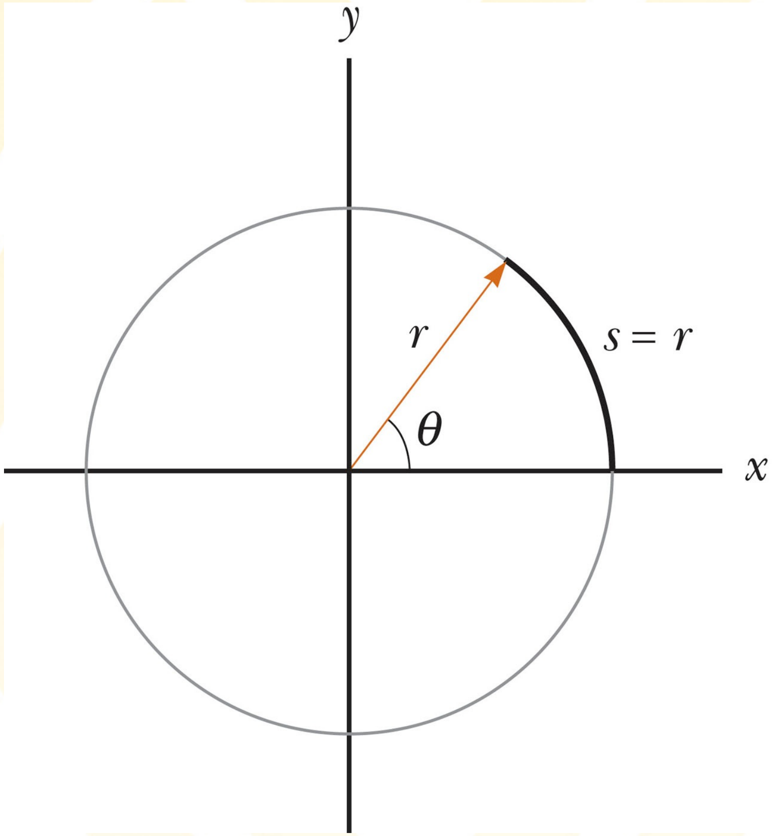
B. $2\pi r$

C. πr^2

D. $2\pi r^2$

$$\boxed{\theta = \frac{s}{r}}$$

1 rad = 57.3 degree



$s = 2\pi r \rightarrow \theta = 2\pi$ for a full circle!

Practical tip for conversions from deg to rad:

θ [rad] = θ [deg] $\cdot\ \pi/180$

Clicker question

$\pi/3$ radians of a pizza. What is the angle?

A. 60 Degree

B. 120 Degree

C. 180 Degree

D. 240 Degree

Practical tip for conversions from deg to rad:

θ [rad] = θ [deg] $\cdot$ $\pi/180$

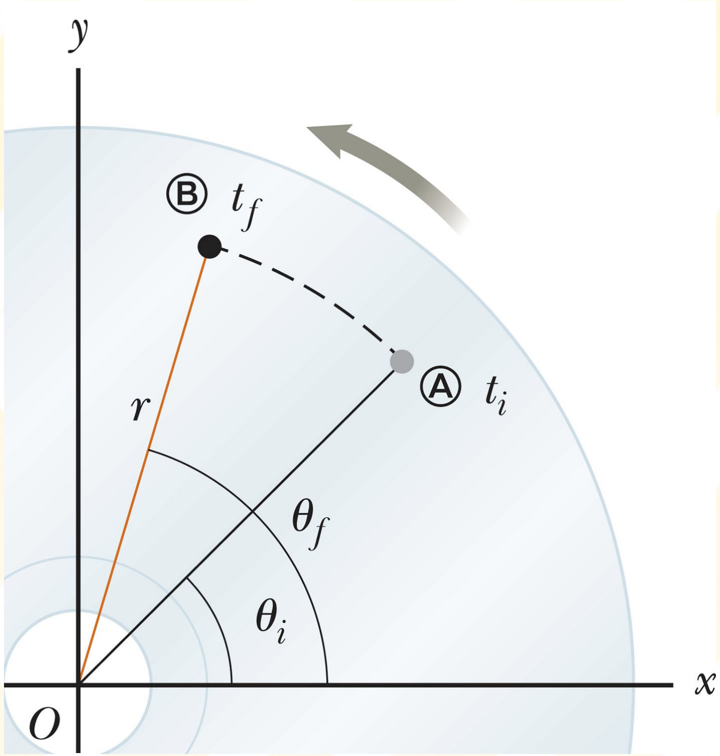
Angular Displacement

- The angular displacement is defined as the angle the object rotates through during some time interval:

$$\boxed{\Delta\theta = \theta_f - \theta_i}$$

Unit: rad

- The unit of angular displacement is the radian
- Each point on a rigid body undergoes the same angular displacement.
- It is analogous to the linear displacement $\Delta x = x_f - x_i$.



Example Problem: Angular Displacement

The tires on a car have a diameter of 2.0 ft and are warranted for 60,000 miles. **Determine the angle** (in radians) through which one of these tires will rotate during the warranty period.

$$\Delta\theta = \frac{\Delta s}{r}$$

How many revolutions of the tire are equivalent to our answer?

Angular Velocity

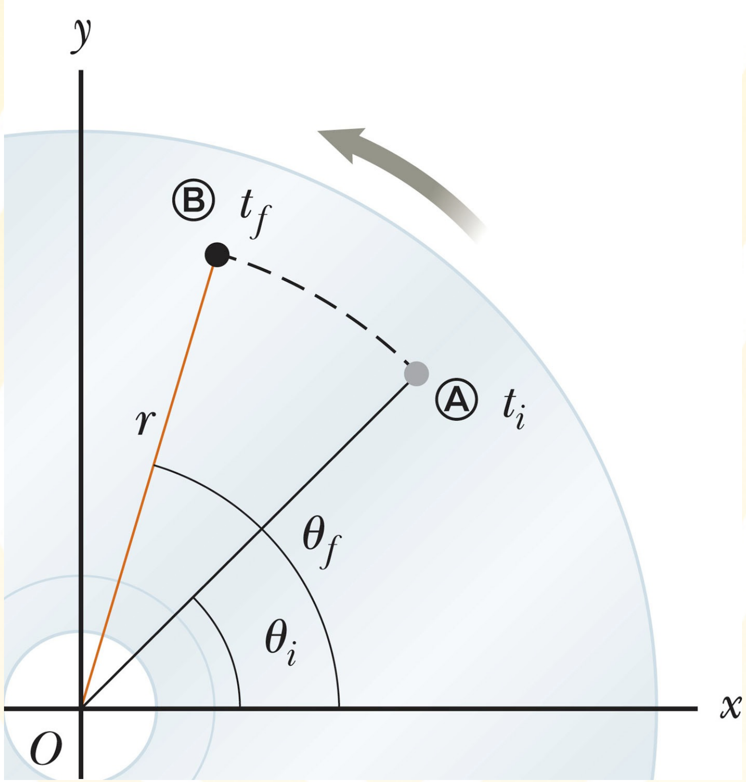
Definition of the average angular velocity:

$$\boxed{\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}}$$

Unit: rad/s

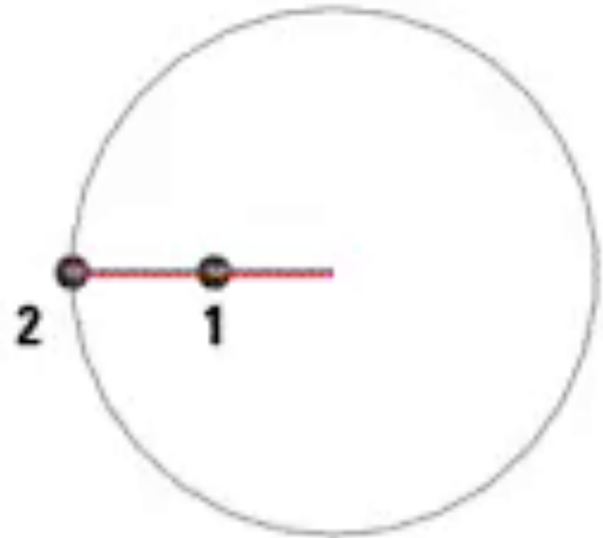
The average angular velocity of a rotating rigid object is the ratio of the angular displacement to the time interval.

Each point on the disc has the same angular velocity, but can have different tangential velocities depending on the radius.



$$\boxed{v_t = \omega r \qquad \omega = 2\pi f}$$

Angular Displacement + Velocity



Clicker question

BIG BEN in London and a little alarm clock both keep perfect time. Which minute hand has the bigger angular velocity?

A) Big Ben

B) little alarm clock

C) Both have the same

Clicker question

My daughter's tricycle:

I put stickers on the bottom of the front and back wheels of different sizes. As I roll the bike (without slipping), the stickers complete a circle (360 degrees) at:

A. The same time

B. Different times

C. Depends on the speed of the bike

$$\theta = \frac{s}{r}$$

Angular Acceleration

An object's average angular acceleration α_{av} during time interval Δt is the change in its angular speed $\Delta\omega$ divided by Δt:

$$\boxed{\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}}$$

Unit: rad/s^2

Positive angular accelerations are *counterclockwise* and negative accelerations are *clockwise.*

When a rigid object rotates about a fixed axis, every point on the object has the ***same*** angular speed and the **same** angular acceleration (but <u>not</u> the same tangential linear speed!).

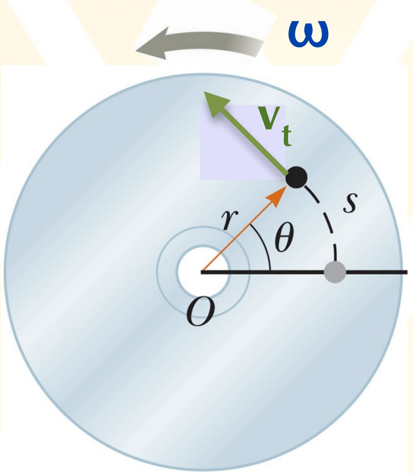
$$a_t = \alpha r$$

Linear vs. Rotational Motion

Every term in a *linear* equation has a similar term in the analogous *rotational* equation.

Linear Motion with *a* Constant (Variables: *x* and *v*)	**Rotational Motion About a Fixed Axis with *α* Constant (Variables: *θ* and *ω*)**
$v = v_i + at$	$\omega = \omega_i + \alpha t$
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$

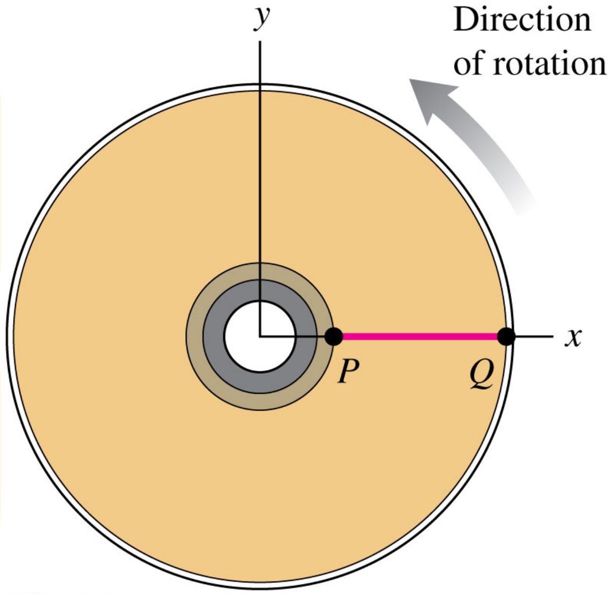
- Displacements: $\mathbf{s = r\,\theta}$
- Speeds: $\mathbf{v_t = \omega\, r}$
- Accelerations: $\mathbf{a_t = \alpha\, r}$



Every point on a rotating object has the same angular, but not the same linear motion!

Example problem: Rotational motion

A DVD is initially at rest so that the line *PQ* on the disc's surface is along the +*x*-axis. The disc begins to turn with a constant α = 5.0 rad/s^2.

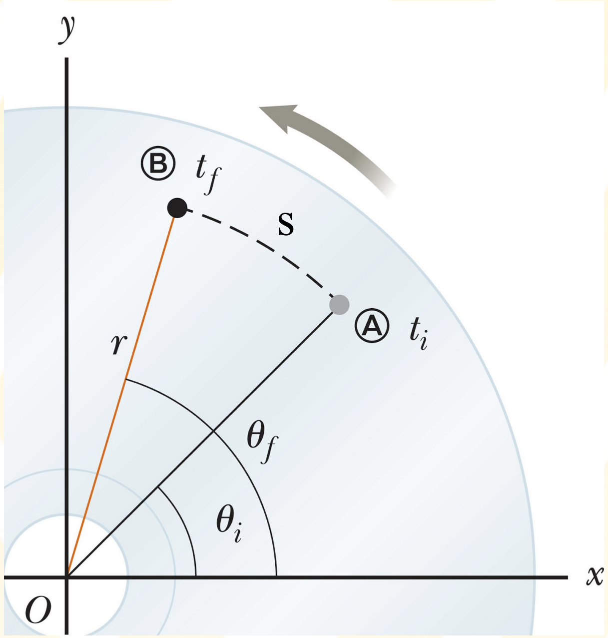


At t = 0.40 s, what is the angle between the line *PQ* and the +*x*-axis?

$$\theta(t) = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

Summary

- Angular position: $\theta = \dfrac{s}{r}$
- Angular displacement: $\Delta\theta = \theta_f - \theta_i$
- Angular velocity: $\omega_{av} = \dfrac{\theta_f - \theta_i}{t_f - t_i} = \dfrac{\Delta\theta}{\Delta t}$
- Angular Acceleration: $\alpha_{av} = \dfrac{\omega_f - \omega_i}{t_f - t_i} = \dfrac{\Delta\omega}{\Delta t}$
- Every point on a rotating rigid object has the same angular, but not the same linear motion!



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$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$