

Linear vs. Rotational Motion

Every term in a *linear* equation has a similar term in the analogous *rotational* equation.

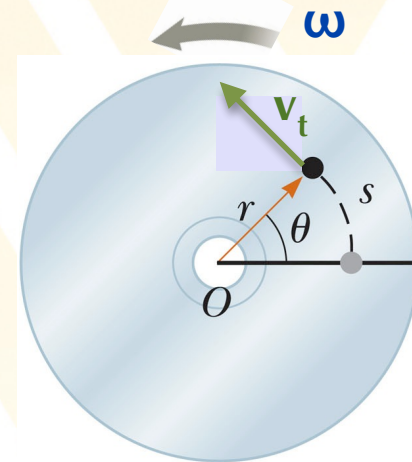
Linear Motion with a Constant
(Variables: x and v)

$$v = v_i + at$$
$$\Delta x = v_i t + \frac{1}{2}at^2$$
$$v^2 = v_i^2 + 2a\Delta x$$

Rotational Motion About a Fixed Axis with α Constant (Variables: θ and ω)

$$\omega = \omega_i + \alpha t$$
$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$$

- Displacements: $s = r \theta$
- Speeds: $v_t = \omega r$
- Accelerations: $a_t = \alpha r$



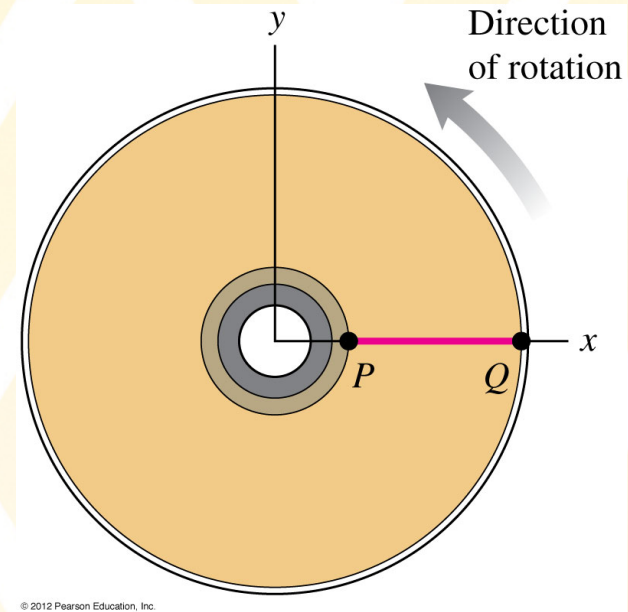
Every point on a rotating object has the same angular, but not the same linear motion!

Example problem: Rotational motion

A DVD is initially at rest so that the line PQ on the disc's surface is along the $+x$ -axis. The disc begins to turn with a constant $\alpha = 5.0 \text{ rad/s}^2$.

At $t = 0.40 \text{ s}$, what is the angle between the line PQ and the $+x$ -axis?

$$\theta(t) = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$



Circular motion at constant speed

Circular motion at constant speed is an accelerated motion, because the direction of the velocity vector is changing!

The magnitude of the velocity remains the same, but its direction changes - this requires an acceleration:

$$\vec{a}_{avg} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

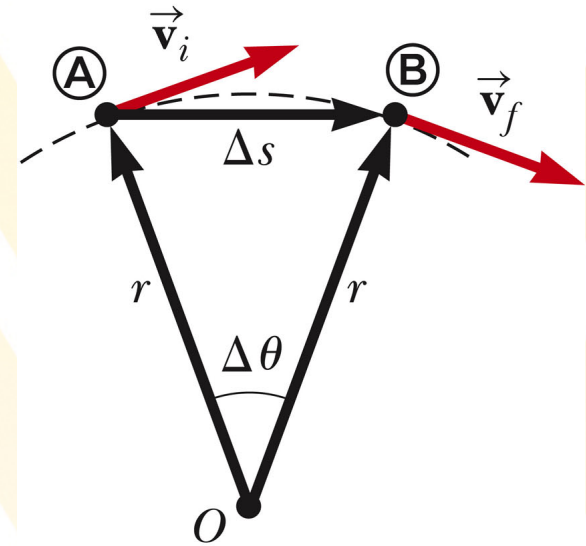
We can determine this acceleration under the assumption that $v_f = v_i = v$ (magnitude remains the same).

Then, we consider both triangles in plots (a) and (b):

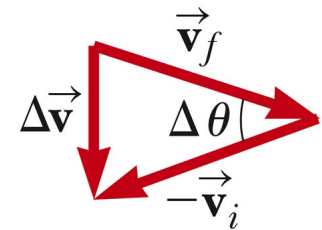
$$\frac{\Delta v}{v} = \frac{\Delta s}{r} \quad \rightarrow \quad \Delta v = \frac{v}{r} \Delta s$$

Dividing by Δt yields:

$$a_{avg} = \frac{v}{r} \frac{\Delta s}{\Delta t} = \frac{v^2}{r}$$



a



b



Centripetal Acceleration and Force

We call this acceleration **centripetal acceleration**, since it is directed towards the center (centripetal = center-seeking):

$$a_c = \frac{v^2}{r} = r\omega^2$$

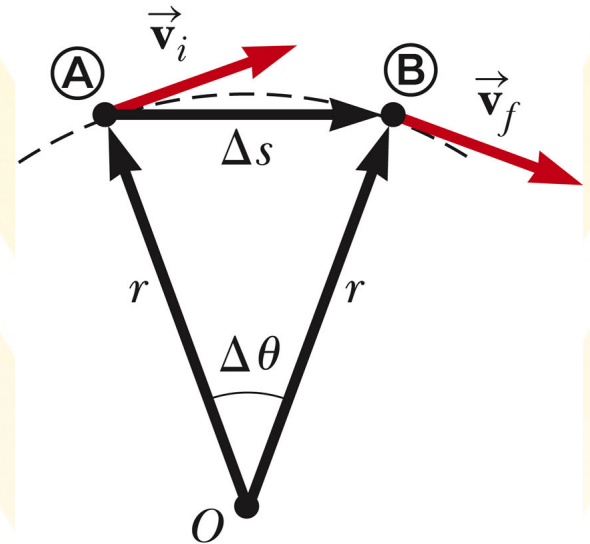
In the last step, we used $v = r\omega$.

As we did for linear motion, first we developed the equations describing the motion (displacement, velocity, acceleration).

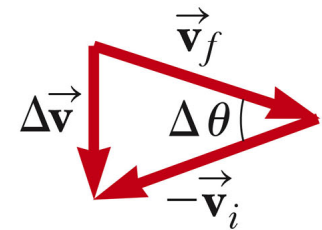
Now we can discuss the forces that cause this motion.

In case of rotational motion at constant speed this is the **centripetal force** (directed towards the center):

$$F_c = ma_c = m \frac{v^2}{r} = mr\omega^2$$



a



b

Total acceleration

If an object rotates, there will always be a centripetal acceleration of magnitude a_c :

$$a_c = \frac{v^2}{r} = r\omega^2$$

Additionally, there can be a tangential acceleration of magnitude a_t , that is linked to the angular acceleration, α .

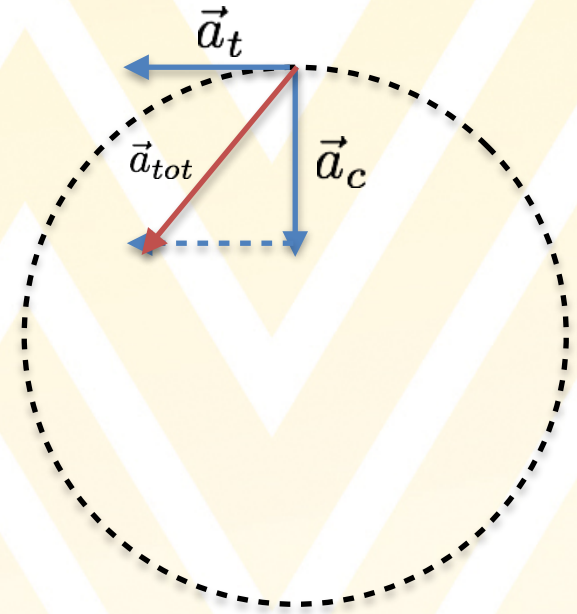
$$a_t = \alpha r$$

Both accelerations are vectors!

In order to find the total acceleration of the object both vectors must be added:

$$\vec{a}_{tot} = \vec{a}_c + \vec{a}_t$$

$$\rightarrow |\vec{a}_{tot}| = \sqrt{a_c^2 + a_t^2}$$



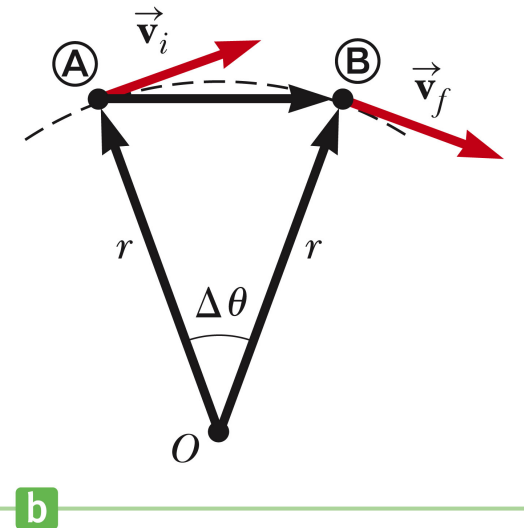
Understanding circular motion

Newton's 1st law: *An object in motion will stay in motion with constant speed and direction unless acted on by an external force.*

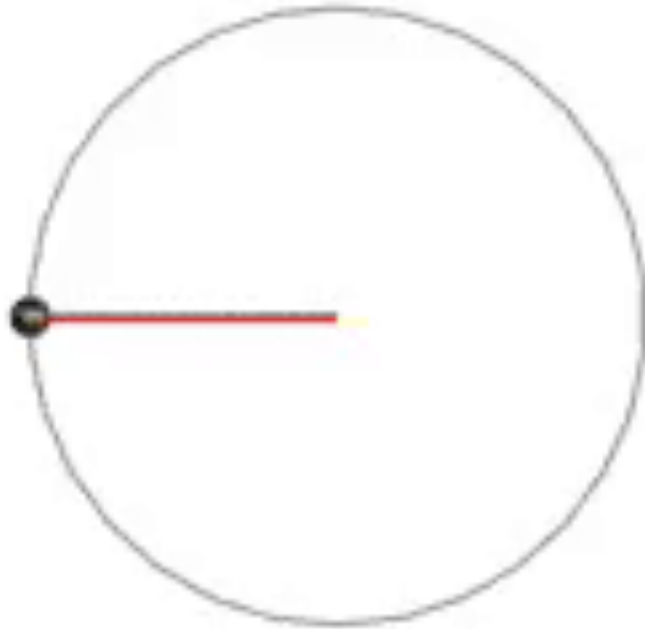
An object moving in a circle permanently “wants” to go straight, but the centripetal force accelerates it towards the center and changes its direction.

If you swing a bucket connected to a rope in a circle, the centripetal force will be provided by the tension in the rope.

The object will never make it to the center, since there is always a tangential velocity that causes it to move away from the center.



Understanding circular motion



Centrifugal force (fictitious)

Imagine yourself on a merry-go-round.

The faster you go, it feels like there is an increasing force pushing you away from the center (centrifugal force).

This force does not exist!

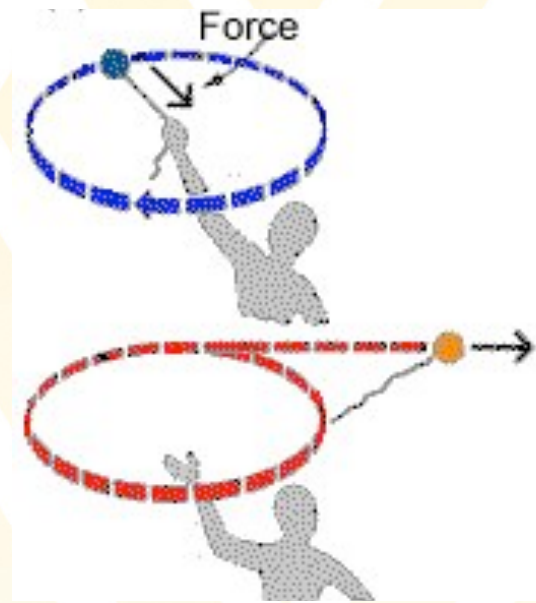
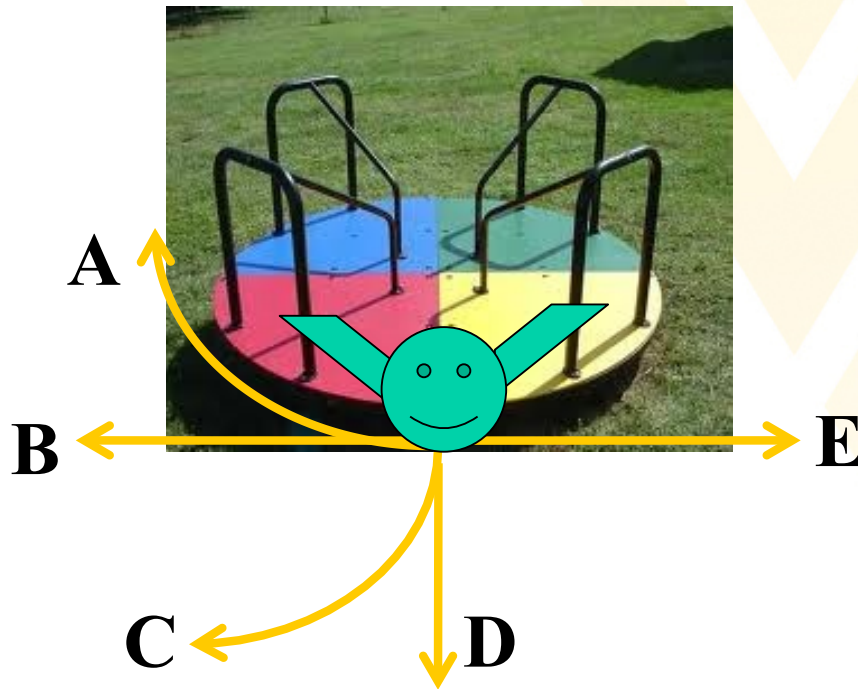
Due to Newton's first law your body wants to continue moving straight.

If you hold onto the merry-go-round you feel a centripetal force pulling you towards the center and causing you to move in a circle.



Clicker question 63

The merry go round is turning **clockwise**. If you are standing on the bottom edge (boundary between red and yellow), what direction would you go if you let go?



Example problem: Racetrack

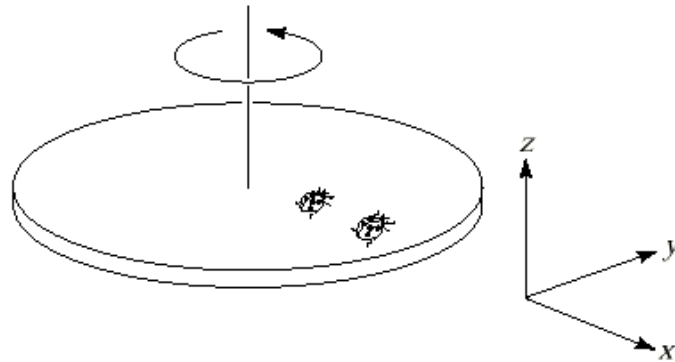
A race car accelerates uniformly from a speed of 40 m/s to a speed of 60 m/s in 5 s while traveling counterclockwise around a circular track of radius 400 m . When the car reaches a speed of 50 m/s , find

- (a) the magnitude of the car's centripetal acceleration.
- (b) the angular speed.
- (c) the magnitude of the tangential acceleration.
- (d) the magnitude of the total acceleration.



Clicker question 64

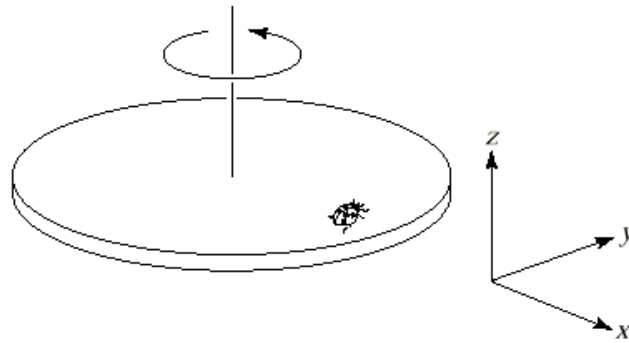
A ladybug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once each second. The gentleman bug's angular speed is



- (a) half the ladybug's.
- (b) the same as the ladybug's.
- (c) twice the ladybug's.
- (d) impossible to determine

Clicker question 65

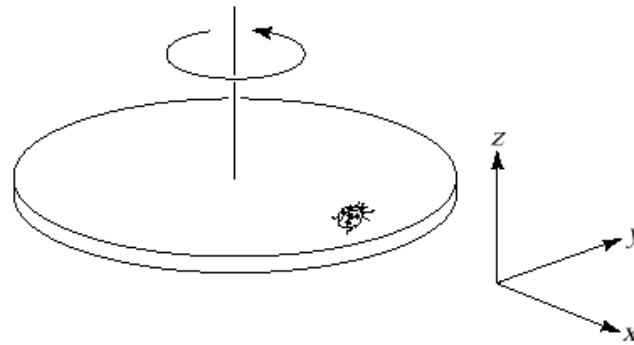
A ladybug sits at the outer edge of a merry-go-round, that is turning and slowing down. At the instant shown in the figure, the radial component of the ladybug's (Cartesian) acceleration is



- (a) in the $+x$ direction.
- (b) in the $-x$ direction.
- (c) in the $+y$ direction.
- (d) in the $-y$ direction.
- (e) in the $+z$ direction.

Clicker question 66

A ladybug sits at the outer edge of a merry-go-round that is turning and slowing down. The tangential component of the ladybug's (Cartesian) acceleration is



- A. in the $+x$ direction.
- B. in the $-x$ direction.
- C. in the $+y$ direction.
- D. in the $-y$ direction.
- E. in the $+z$ direction.

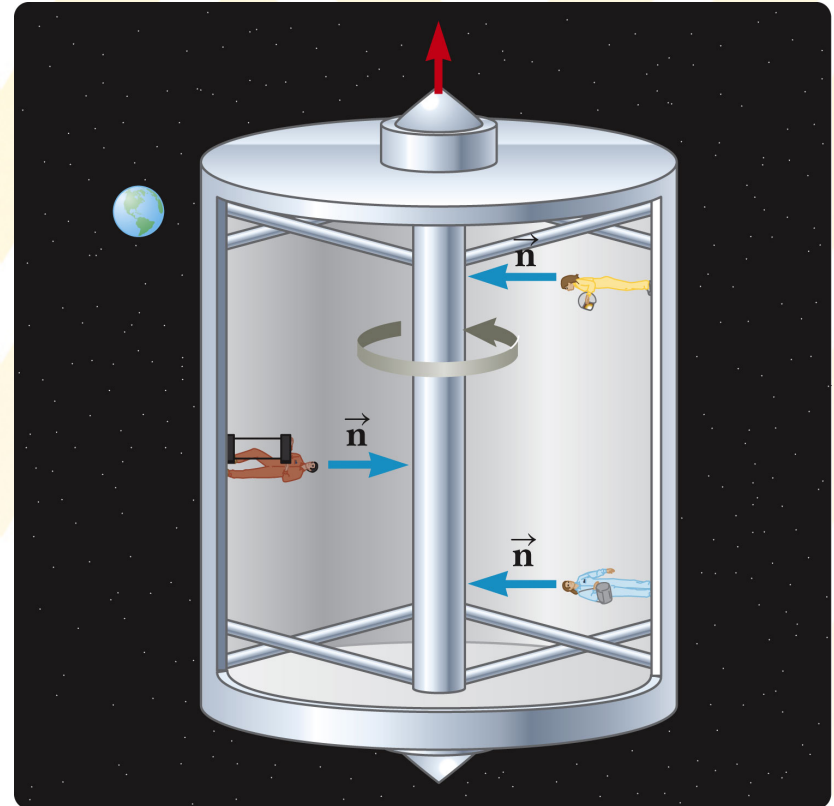
Artificial gravity

In space stations, artificial gravity can be created by rotating the entire station.

Any astronaut in the station will then experience a centripetal force exerted on him/her by the inner walls of the station (normal force).

This is useful for long stays in space to avoid weakening of muscles.

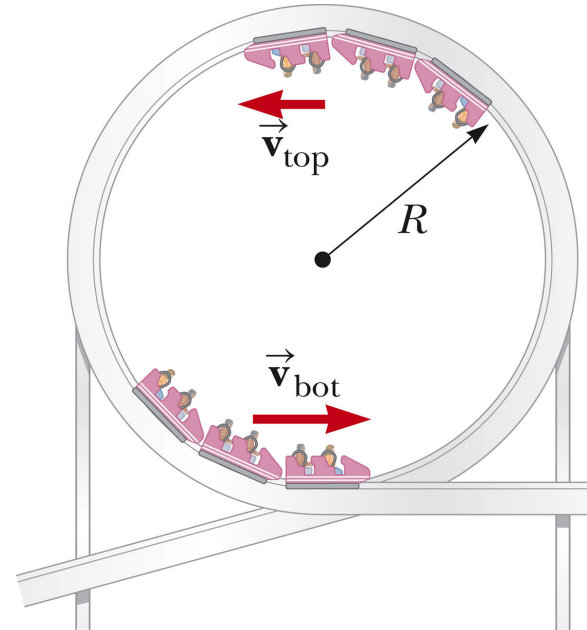
It works similarly to gravitation on earth, where any object would continue moving straight, if gravity did not cause a centripetal force on us to keep us at the surface.



Example problem: Loop the Loop

A roller-coaster car is moving around a circular loop of radius R .

- (a) What speed must the car have so that it will just make it over the loop?
- (b) From what height must the car initially be released (from rest) in order to make it over the loop?



a

Fig. 7-15a, p. 213

Why does the Moon orbit the Earth?

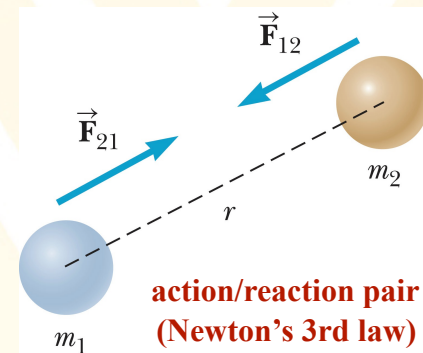
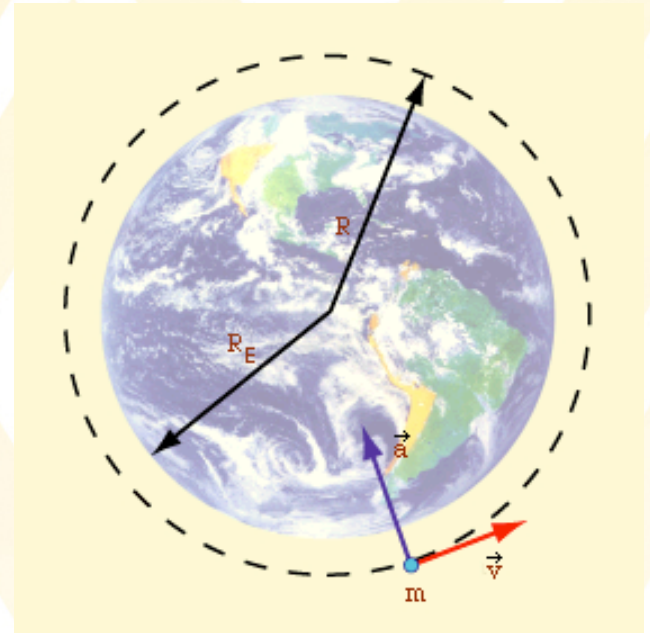
The Moon orbits the earth, because it is attracted by earth due to the **gravitational force**, which acts as the centripetal force on the Moon.

Gravitational Force: If two particles with masses m_1 and m_2 are separated by a distance r , an attractive force will act along a line joining them with magnitude:

$$F = G \frac{m_1 m_2}{r^2}$$

Here, G is “Newton’s constant of gravitation” ($G = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$).

The Moon also attracts earth and causes ocean tides.



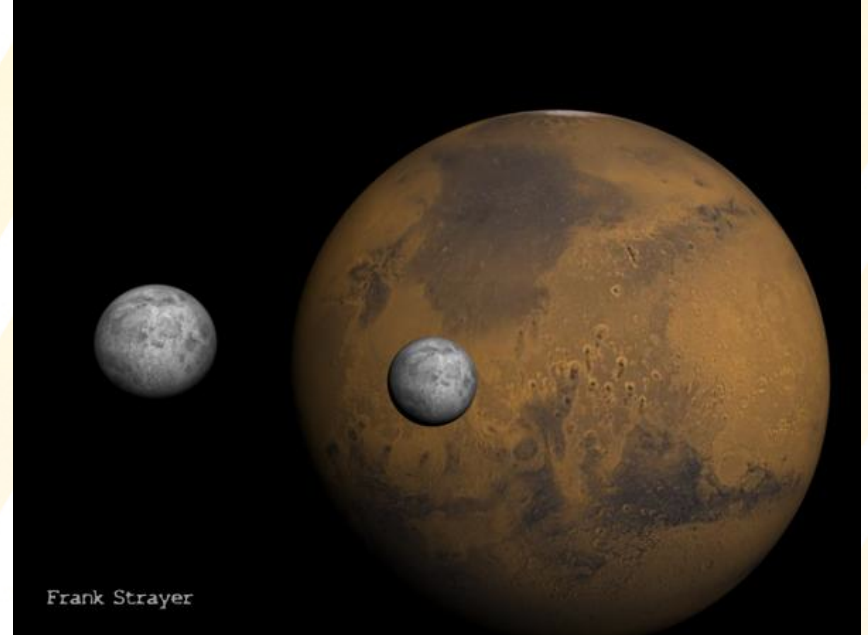
Clicker question 67

A planet has two moons with identical mass.
Moon 1 is in a circular orbit of radius r .
Moon 2 is in a circular orbit of radius $2r$.

The magnitude of the gravitational force exerted by the planet on Moon 2 is

- A) Four times as large
- B) Twice as large
- C) the same
- D) half as large
- E) one-fourth as large

as the gravitational force exerted by the planet on Moon 1.



$$F = G \frac{m_1 m_2}{r^2}$$

Summary

- Circular motion at constant speed is an accelerated motion (the direction of the velocity vector changes).
- The corresponding **centripetal acceleration** is:

$$a_c = \frac{v^2}{r} = r\omega^2$$

- The **centripetal force** is:

$$F_c = ma_c = m\frac{v^2}{r} = mr\omega^2$$

- If the centripetal force stops, the object will continue to move straight based on Newton's 1st law.
- The **centrifugal force is a fictitious force**, i.e. it does not exist.
- Two masses separated by a distance, r , attract each other based on **Newton's law of gravitation**:

$$F = G\frac{m_1 m_2}{r^2}$$

