Trigonometry I

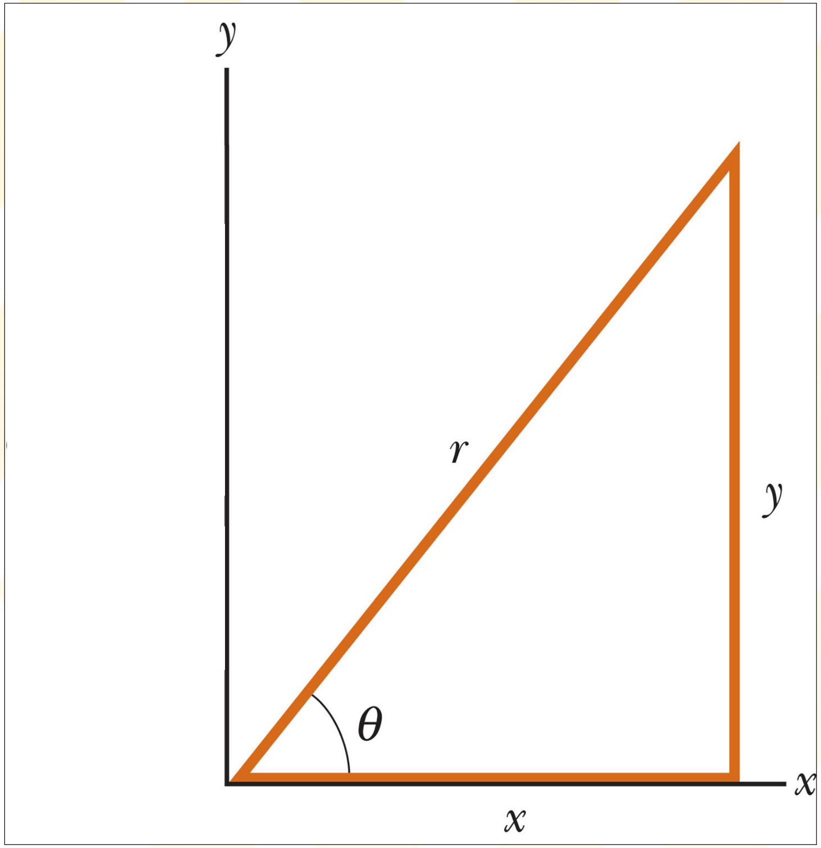
$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

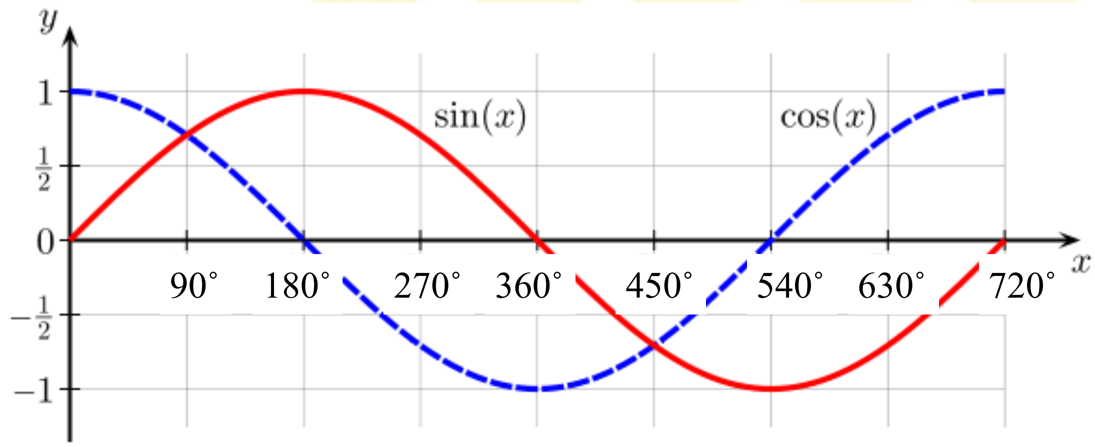
$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

Pythagorean theorem:

$$x^2 + y^2 = r^2$$



Trigonometry II



sin(x) and cos(x) are mathematical functions that describe oscillations.

This will be important later, when we talk about oscillations and waves.

Trigonometry III

- To find an angle, you need the inverse trig function.
  - For example, $\theta = \sin^{-1} 0.707 = 45°$
- Be sure your calculator is set for the appropriate angular units for the problem!
  - For example:

$$\tan^{-1} 0.5774 = 30.0°$$

$$\tan^{-1} 0.5774 = 0.5236 \text{ rad}$$

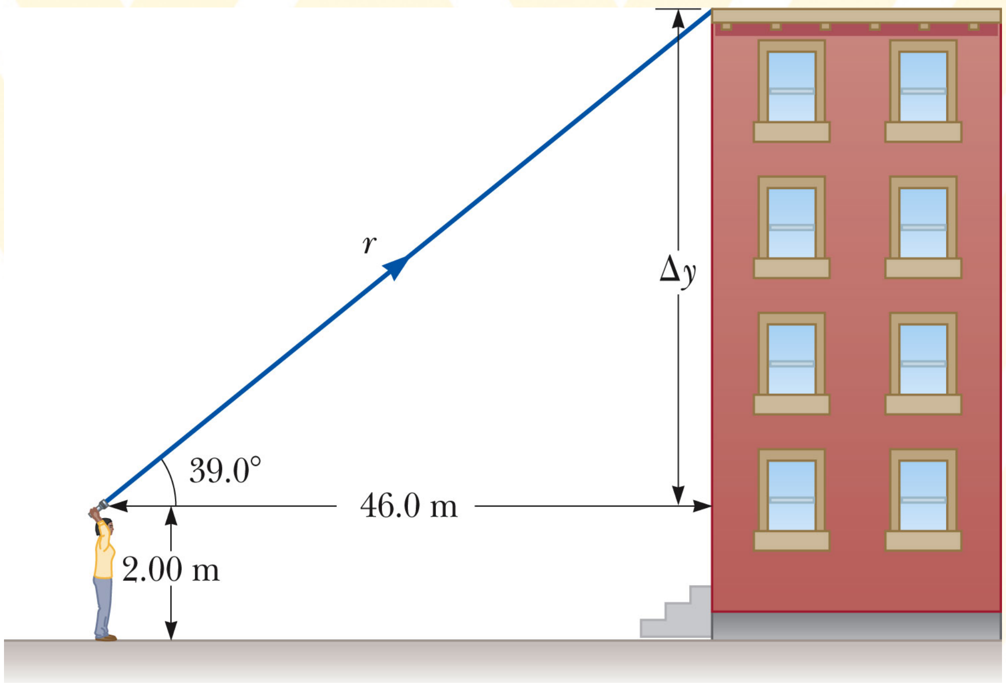
A detailed recap of mathematics required for this class can be found in Appendix A of our textbook.

Typical trigonometry problem

A person measures the height of a building by walking out a distance of 46.0 m from its base and shining a flashlight beam to its top. When the beam is elevated at an angle of 39.0° with respect to the horizontal, the beam just strikes the top of the building.

(a) If the flashlight is held at a height of 2.0 m, find the height of the building.

(b) Calculate the length of the light beam.

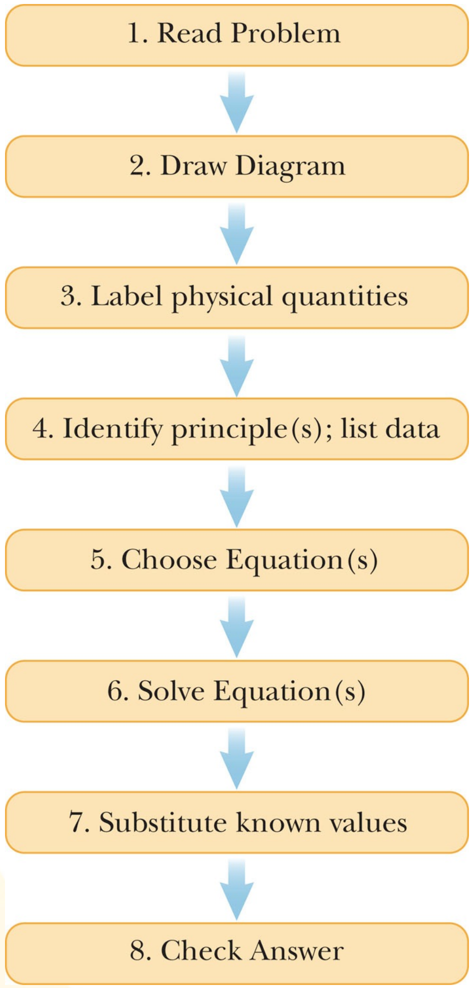


Problem Solving Strategy

1. Read Problem
2. Draw Diagram
3. Label physical quantities
4. Identify principle(s); list data
5. Choose Equation(s)
6. Solve Equation(s)
7. Substitute known values
8. Check Answer

- read the problem twice! maybe three times!
  - can you estimate the answer's order of magnitude?
- identify what *type* of problem it is
- label your diagram with the given information: variables, values, coordinates...
- do algebraic steps carefully
- check your answer for *sense* and *units*
  - should it be *positive* or *negative*?
- is it consistent with your initial estimate?

Problem Solving Strategy

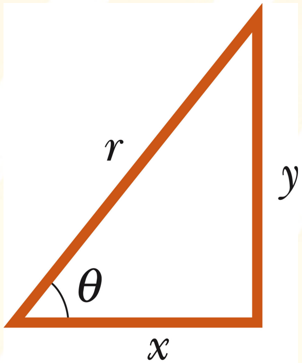


- Equations are the tools of physics
  - Understand what the equations mean and how to use them
- Carry through the algebra as far as possible
  - Substitute numbers at the end
- *Be organized*!

Summary

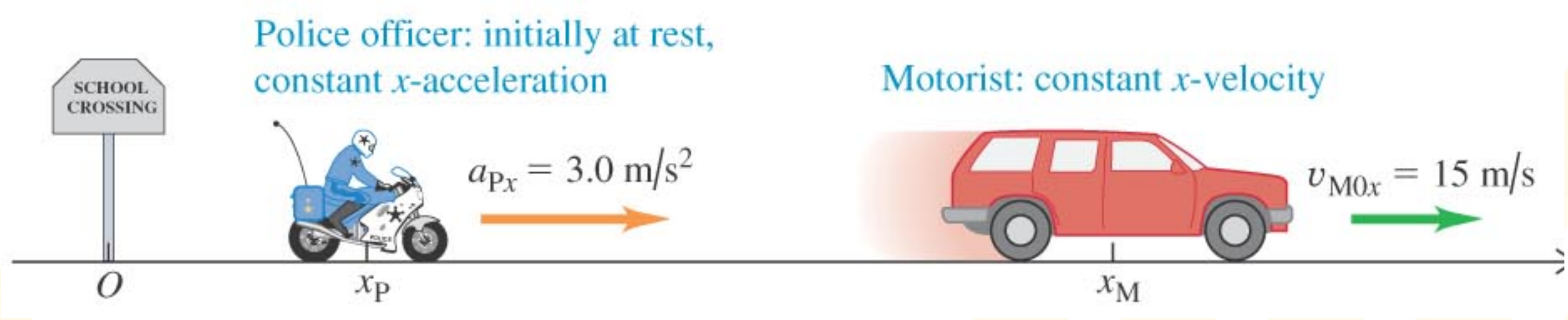
- All physical quantities in mechanics can be expressed by meters (m), kilograms (kg), and seconds (s).
- Every physical quantity must have a unit. Units must be consistent. We use SI-units.
- Dimensional analysis can be used to check equations.
- No physical quantity can be determined without an uncertainty, that determines its significant figures.
- An order of magnitude estimation can be useful to check the result of a calculation.
- The cartesian coordinate system consists of two perpendicular and labeled axes (x- and y-axis). Points are located by specifying their x- and y-values.
- Trigonometry: $\sin(\theta) = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$ $\tan(\theta) = \frac{y}{x}$

$$x^2 + y^2 = r^2$$ Pythagorean theorem

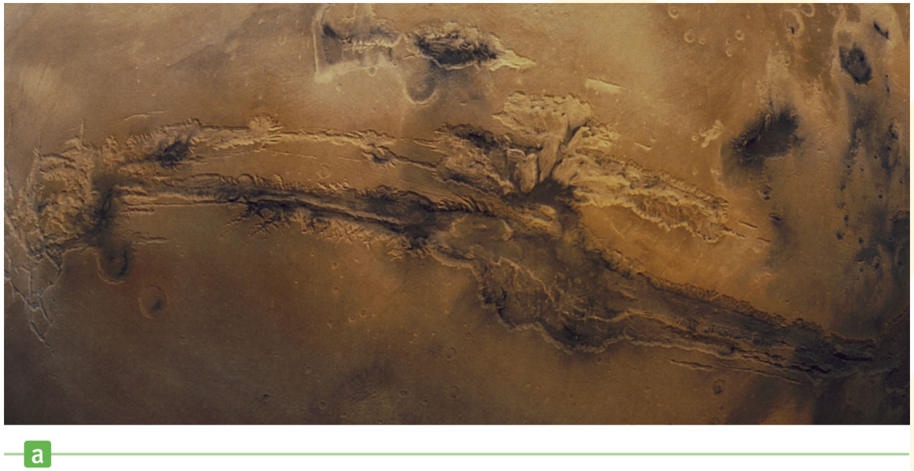


Chapter 2: Motion in one Dimension

- Velocity and motion diagrams
- 1d motion with constant acceleration
- Free fall



Frame of Reference I

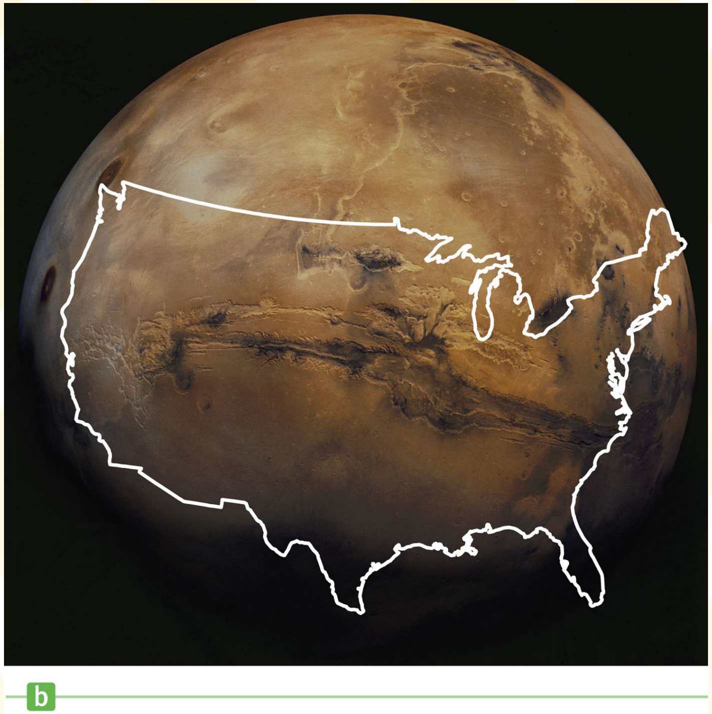


This picture shows a canyon on Mars.

If you just look at the picture, you will not have any idea how long it is.

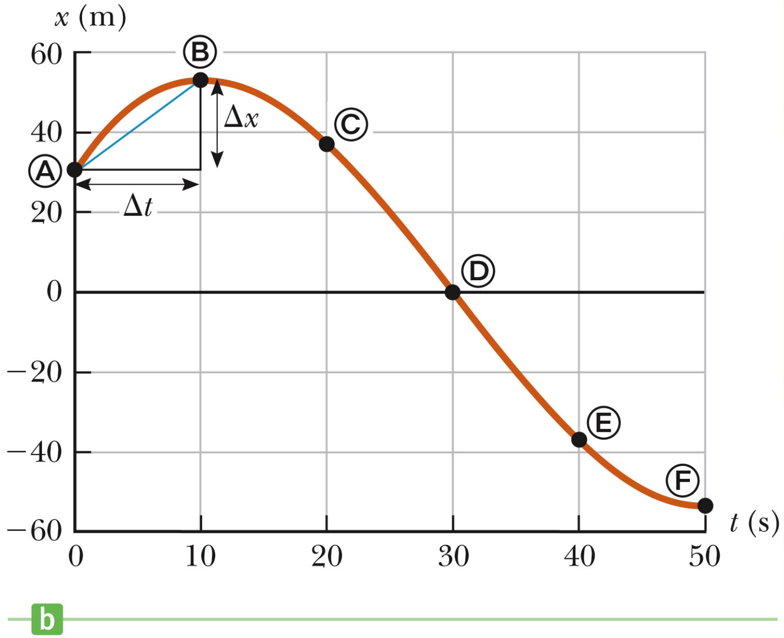
It could be 100 m or 10000 km.

We need a Reference Frame!



The reference frame shows that the canyon is about 2000 km long.

Frame of Reference II



In science, a frame of reference is a coordinate system, whose origin is placed at a particular position.

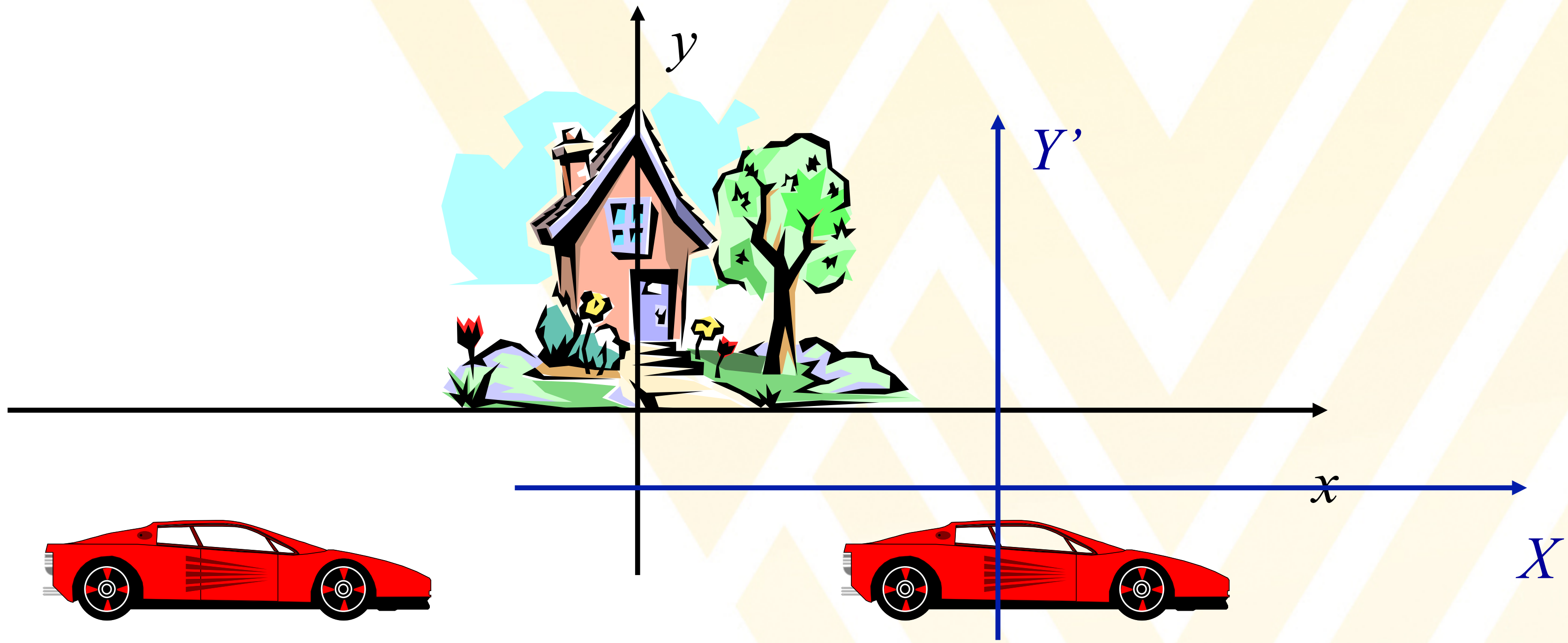
In this reference system the displacement of an object is defined as:

$$\Delta x \equiv x_f - x_i$$

(*i* for initial, *f* for final, Δ for difference)

Displacement - Example

Change of the initial position of an object



x_f = -5.0 m

x'_f = -8.0 m

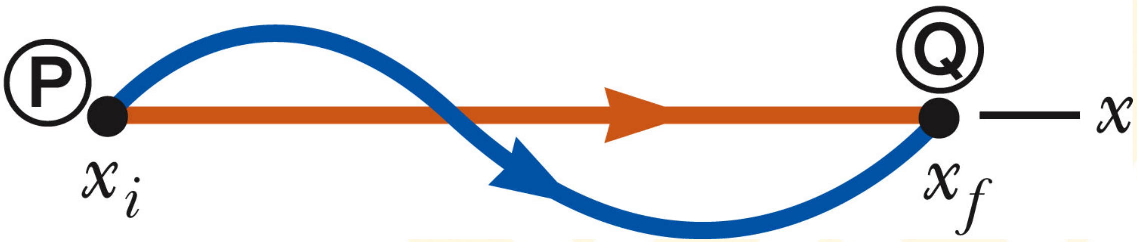
x_i = +3.0 m

x'_i = 0.0 m

Displacement: $\Delta x = x_f - x_i$ $\quad \Delta x$ = -5.0 m - (+3.0 m) = -8.0 m

Other reference frame: $\Delta x = x_f - x_i$ $\quad \Delta x$ = -8.0 m – (0.0 m) = -8.0 m

Displacement vs. Path Length



Displacement is not the same as Path Length!

Displacement, Δx, depends only on the endpoints of the path, while the path length, l, depends on the actual route taken.

Thus: $l \gtrsim \Delta x$

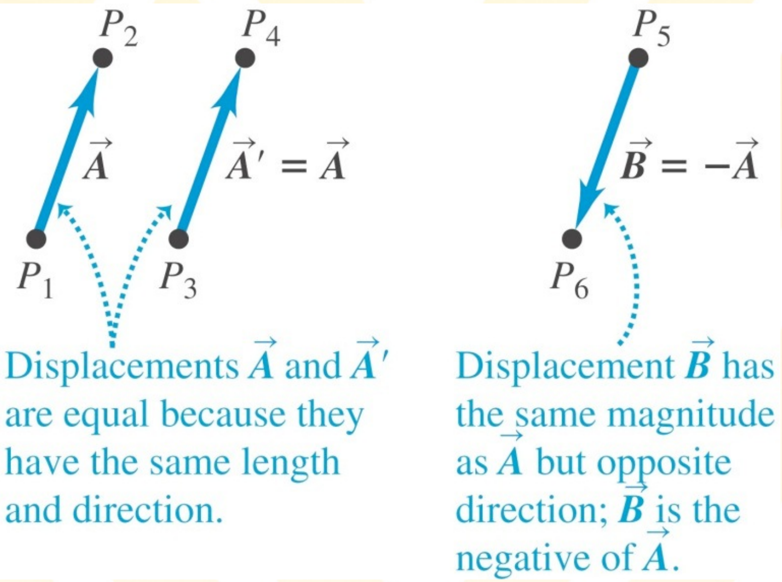
Vectors vs. Scalars

There are two fundamentally different types of physical quantities:

1) Vectors
2) Scalars

A scalar quantity is fully characterized by its magnitude.

A vector must be characterized by magnitude and direction. It is represented by an arrow.



Do you know any examples for scalar and vector quantities?

Scalars: Mass, time, volume, distance

Vectors: Displacement, velocity, acceleration, flux

More on vectors on Jan 28th!

In 1d we do not need vectors, since there is only 1 direction.

Velocity and Speed

The average velocity during a time interval, Δt, is defined as the displacement, Δx, during this time interval divided by Δt:

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Velocity is a vector quantity, since it has a magnitude and a direction!

In contrast to velocity, the average speed is a scalar and defined as:

$$\text{Average speed} = \frac{\text{path length}}{\text{elapsed time}}$$

Speed is always a positive number. Velocity can be negative.

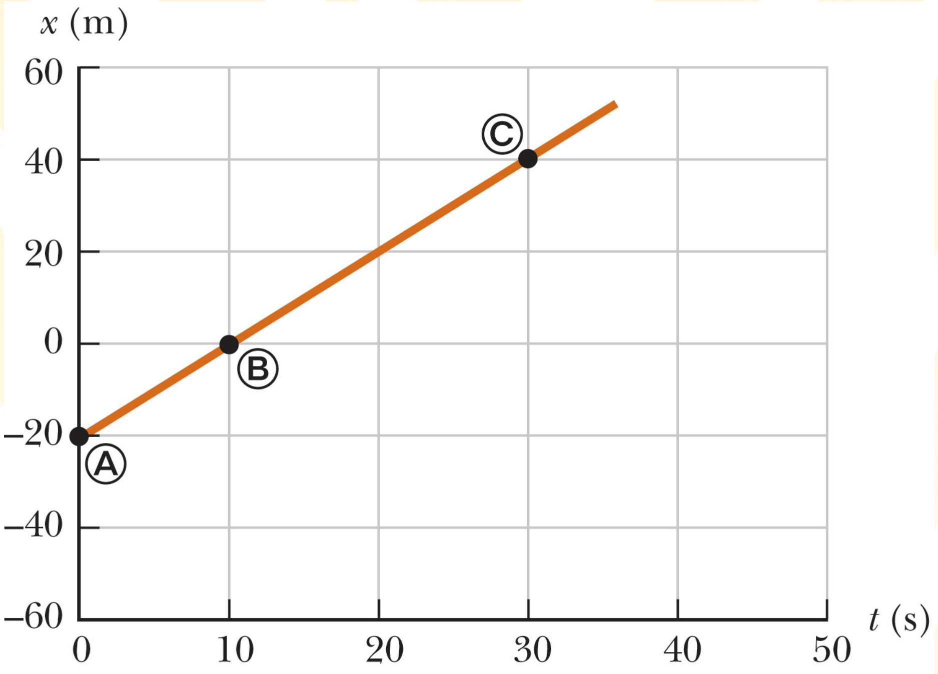
The unit of velocity and speed is m/s.

Graphical interpretation of velocity

- The average velocity between times t_1 and t_2 can be determined from a position-time graph.
- It equals the slope of the line joining the initial and final positions.

$$slope = \frac{change\ in\ vertical\ axis}{change\ in\ horizontal\ axis}$$

- Objects moving at constant velocity will result in straight line in such a plot.
- Slopes have units!



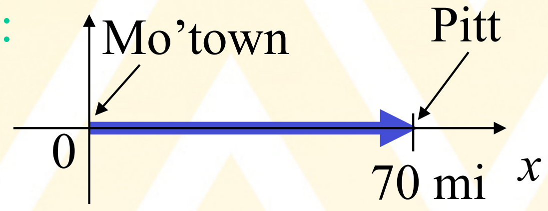
Average velocity - Example

Go from Morgantown to Pittsburgh in 2h and return back to Morgantown 3h after leaving.

Average velocity going to Pitt:

$x_i = 0 \quad t_i = 0$

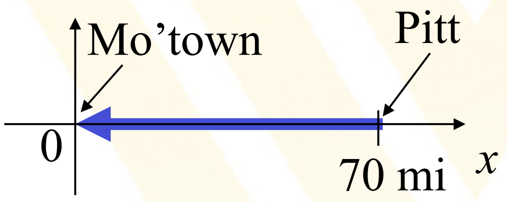
$x_f = +70\text{ mi} \quad t_f = 2\text{ hrs}$



Average velocity coming back from Pitt:

$x_i = +70\text{ mi} \quad t_i = 2\text{ hrs}$

$x_f = 0\text{ mi} \quad t_f = 3\text{ hrs}$



Instantaneous velocity

The instantaneous velocity is the velocity at a particular moment in time, while the average velocity is the velocity averaged over a time interval.

There can be a huge difference between average and instantaneous velocity.

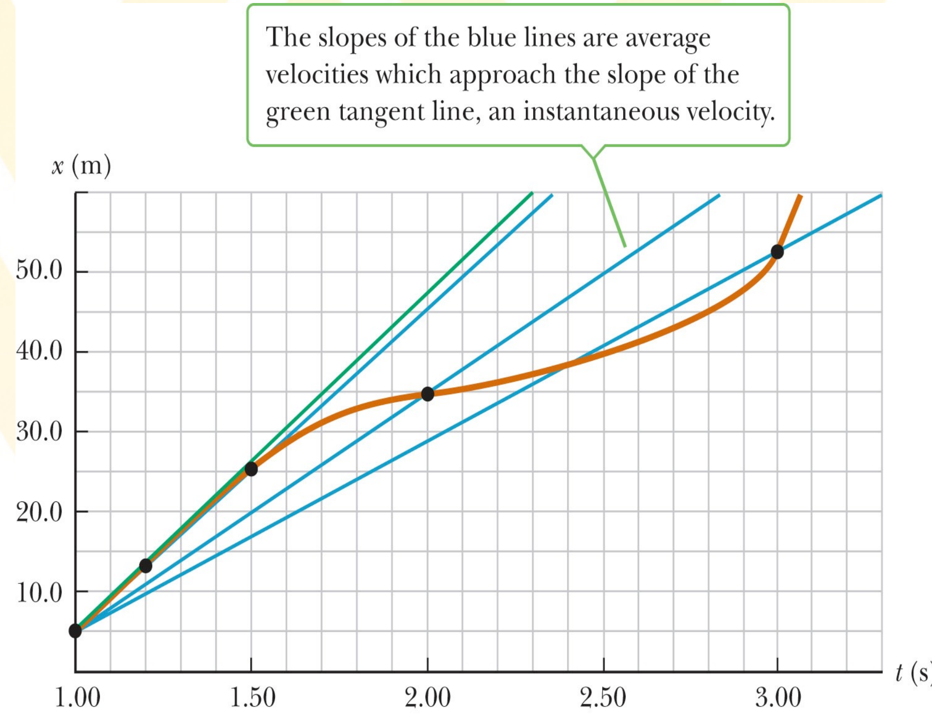
You can go from Morgantown to Pittsburgh by car in 2 h in many different ways:

1. You travel at 35 mi/h for 2 hours. In this case your average and instantaneous velocities are both 35 mi/h

2. You travel at 10 mi/h for 1.5 hours and at 110 mi/h during the remaining 0.5 hours. Your average velocity will also be 35 mi/h, but your instantaneous velocity will be 10 mi/h at every moment during the first 1.5 hours and 110 mi/h during the last 0.5 h.

Mathematical and graphical determination

- This graph illustrates the motion of an object whose velocity is changing in time (not a straight line).
- The slopes of the blue lines correspond to the average velocities during the respective time intervals.
- By reducing the time interval as much as possible around t = 1s, the instantaneous velocity at this time is determined.
- The instantaneous velocity corresponds to the slope of the tangent at t = 1 s.



$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Acceleration

Acceleration is defined as the change of velocity during a time interval, Δt.

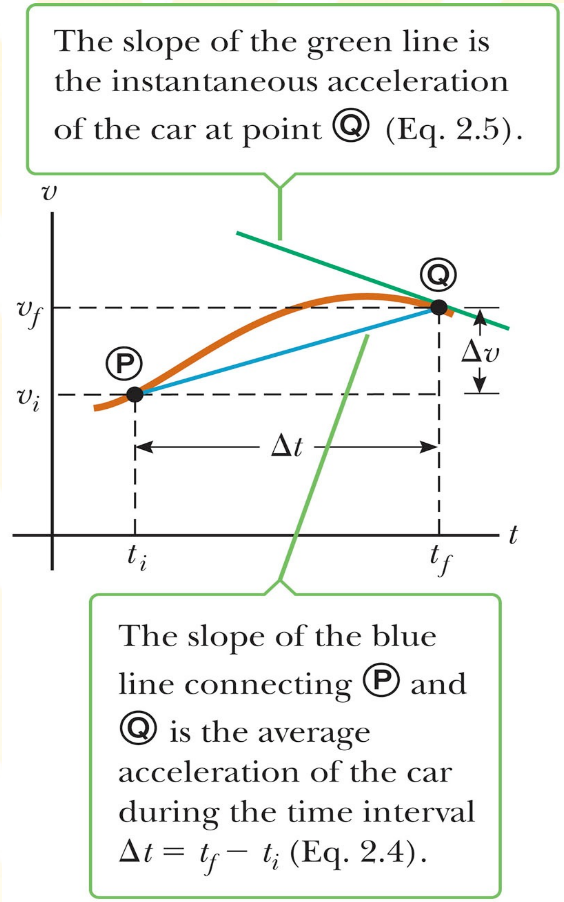
Remember: Velocity is the change of position during Δt.

Again, there is an average and an instantaneous acceleration:

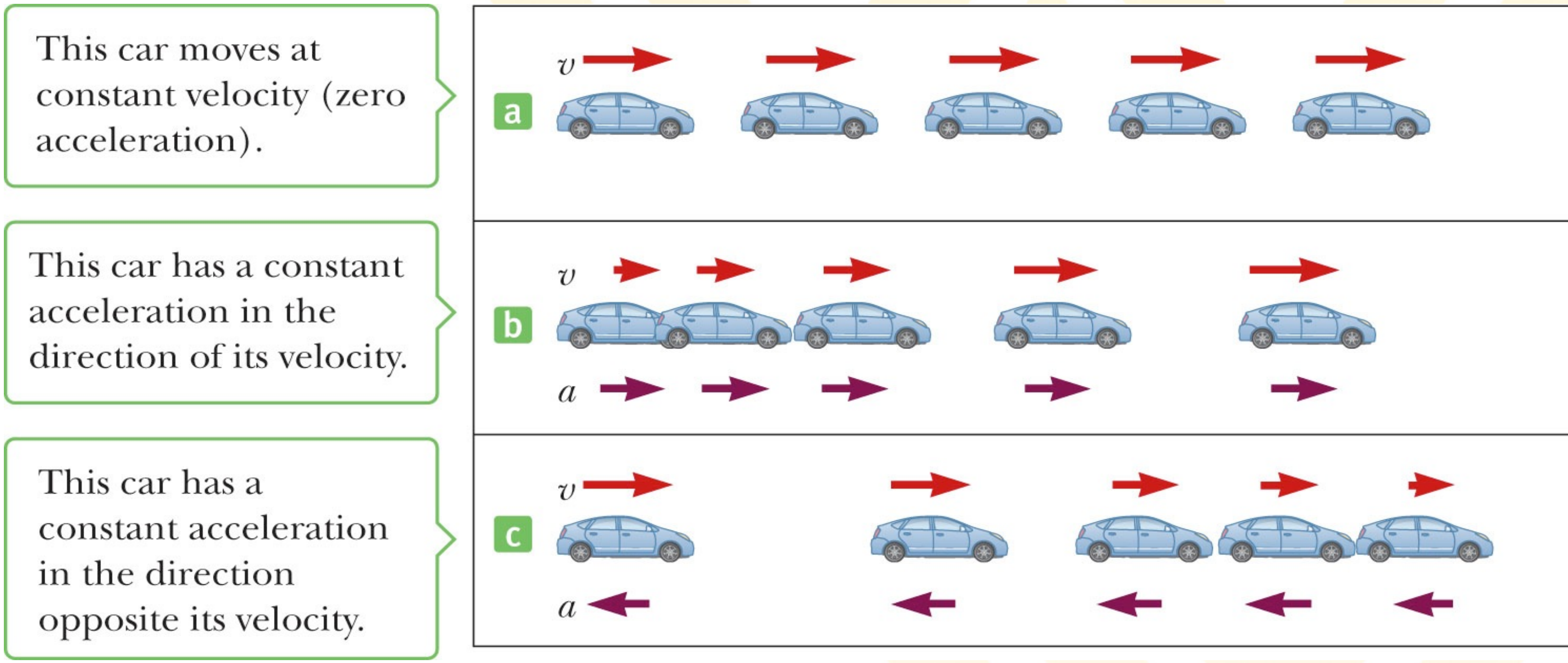
$$\bar{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Acceleration is a vector quantity, since it has a magnitude and direction. Its unit is m/s^2.

The instantaneous acceleration of an object at time, t_f, is the slope of the tangent of the velocity-time graph at this time.



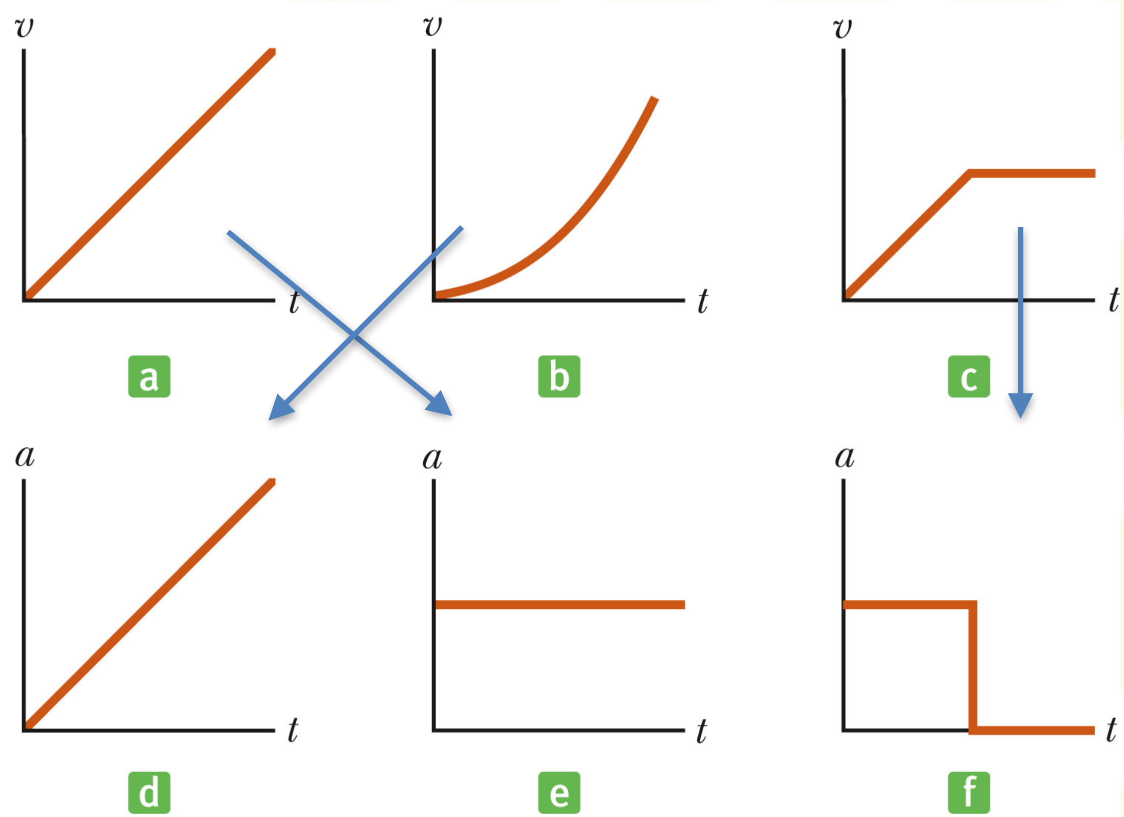
Relating velocity and acceleration I



If velocity and acceleration point into the same direction, the object will speed up.
If they point into opposite directions, it will slow down.
An object’s velocity can be zero instantaneously, while its acceleration is not zero!

Relating velocity and acceleration II

Which velocity-time and acceleration-time graphs match?



Hands on graphing work will be done in the lab.

What you learn from graphs

Type of graph	Slope gives:	Change of direction
Position vs Time	Velocity	At maximum or minimum
Velocity vs Time	Acceleration	When curve crosses axis
Acceleration vs Time	---	Can't determine

Mathematical description of 1d motion with constant acceleration

An objects moves in one direction with constant acceleration [a(t) = const.].

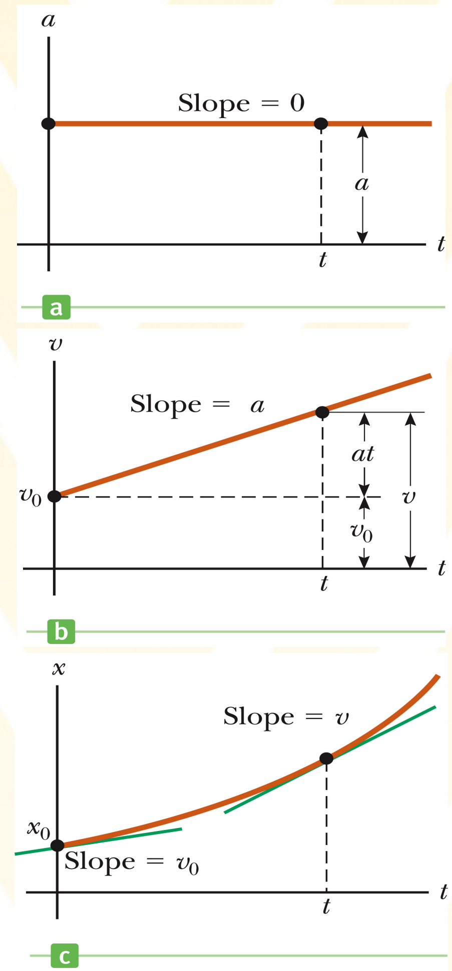
a(t) = const. results in a horizontal line in the a-t-graph.

How can we calculate v(t) and x(t)?

$$v(t) = v_0 + at \qquad x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

Initial velocity at t = 0s   Initial position at t = 0s

- v(t) is a linear function and results in a straight line in the v-t-graph.
- x(t) is a quadratic function and results in a parabola in the x-t-graph.



Mathematical description of 1d motion with constant acceleration

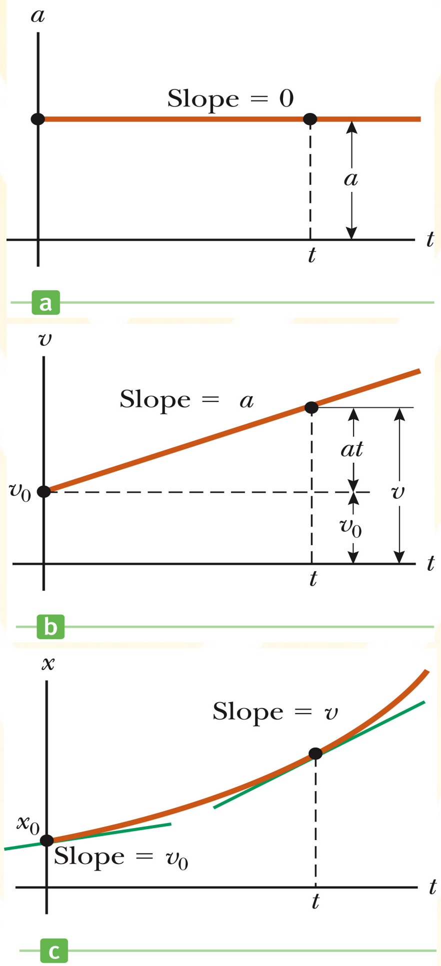
Other useful formula:

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t \qquad \Delta x(t) = x(t) - x_0$$

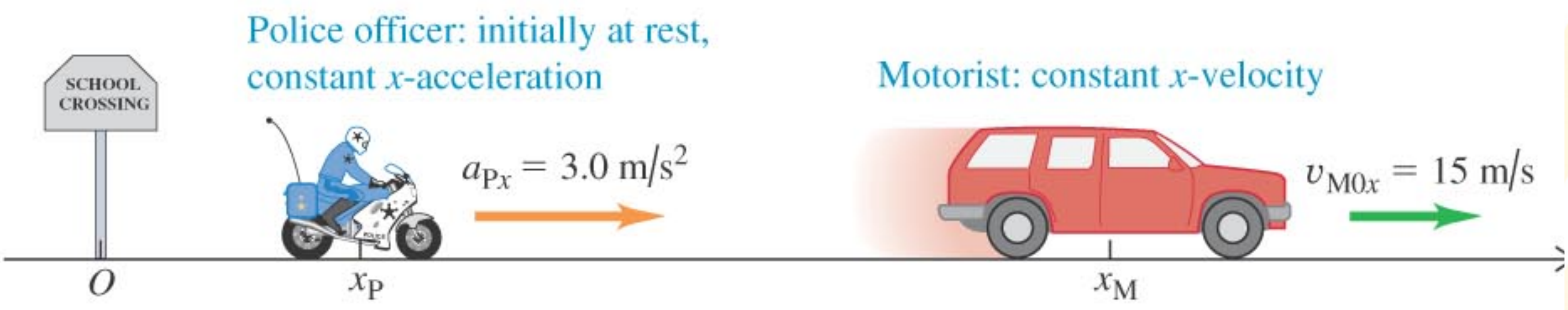
$$v^2(t) = v_0^2 + 2a\Delta x^2(t)$$

These equations (this and previous slide) are only valid for 1d motion with **constant** acceleration.

These formula are extremely important for solving many problems relevant for real life, homework, and exams.



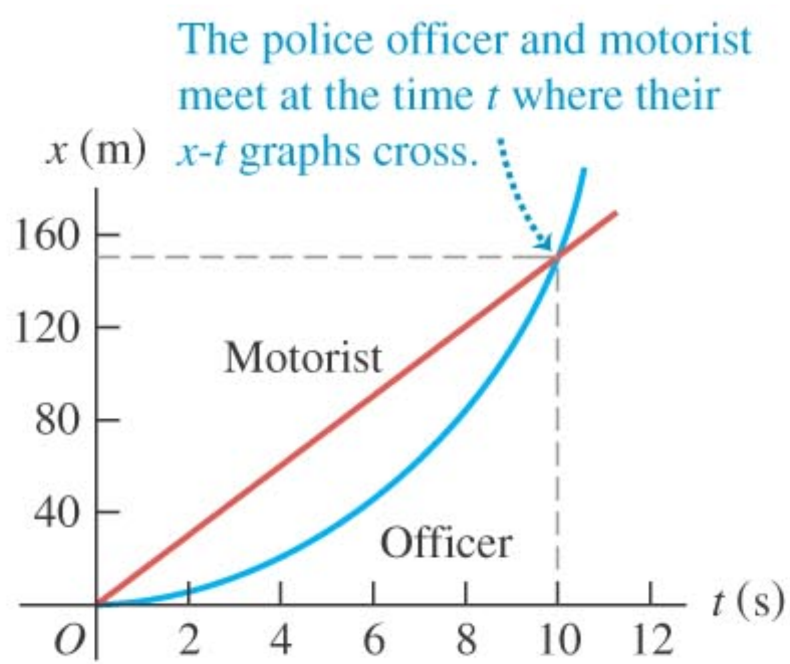
Example problem



A car is traveling at 15 m/s, when it passes a trooper, who does not move. The trooper sets off in chase immediately with a constant acceleration of 3.0 m/s^2.

(a) How long does it take the trooper to overtake the car?

(b) How fast is the trooper going at that time?



Definition of free Fall

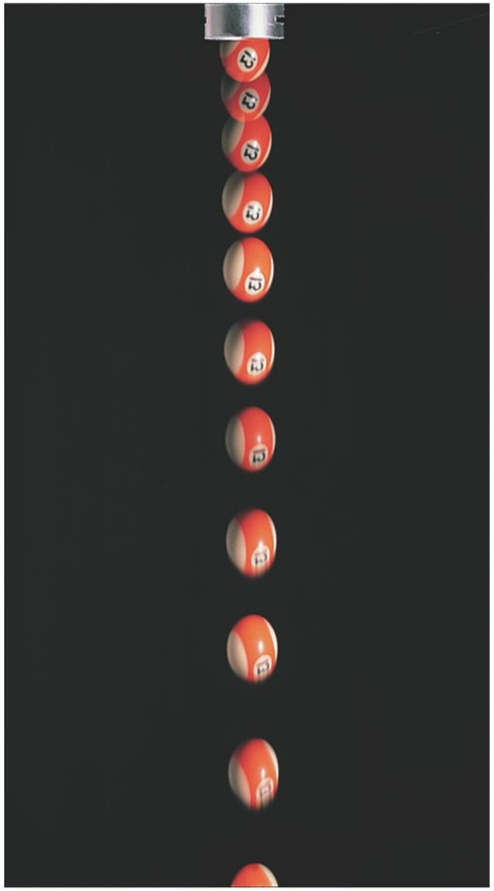
- Free Fall is the motion of an object under the influence of gravity alone (no other forces/accelerations).

- Such objects do not have to start from rest, but can have an initial velocity pointing upwards.

  Example: A ball is thrown upwards. After the ball is thrown (no more acceleration in upward direction), it moves upwards. This is an example of free fall, since only gravity influences the ball's motion.

- The velocity change in each time interval is constant:

$$a = \frac{\Delta v}{\Delta t} = -g = -9.81 m/s^2$$

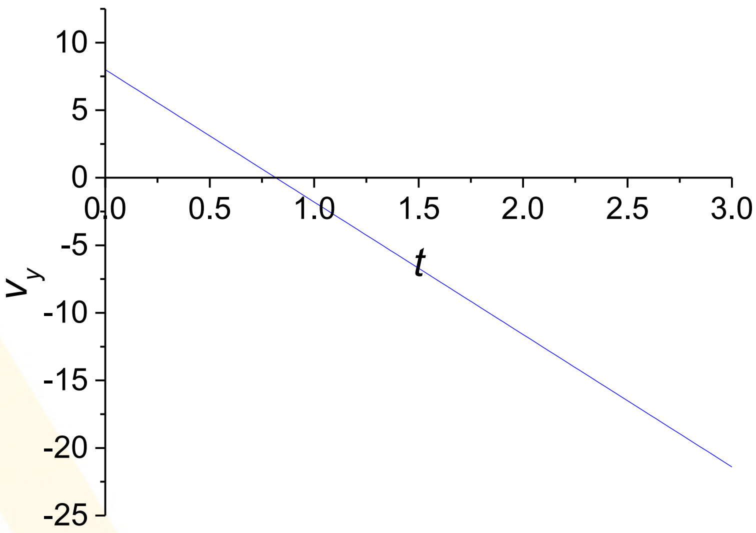


Graphing freely falling bodies (a = -g = const.)

$$v_y = v_0 - gt$$

This is a linear function of t. The slope is negative (-g).

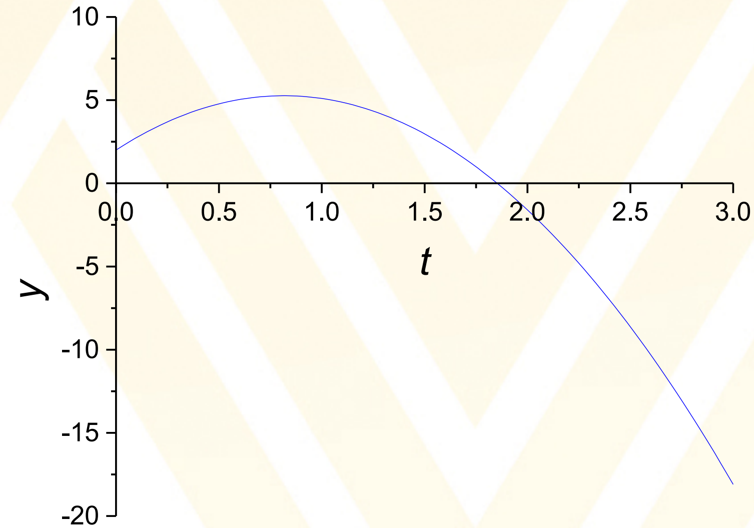
→ Straight line in the v_y - t - diagram.



$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

This is the sum of a linear function and a quadratic function of t.

→ For low values of t, it is a straight line.
→ For high values of t, it is a parabola.



Example problem: Free Fall

A ball is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The ball just misses the edge of the roof on its way down. Determine

A. the time needed for the ball to reach its maximum height.

B. the maximum height itself.

C. the time needed for the ball to return to the height from which it was thrown and the velocity of the ball at that instant.

D. the time needed for the ball to reach the ground.

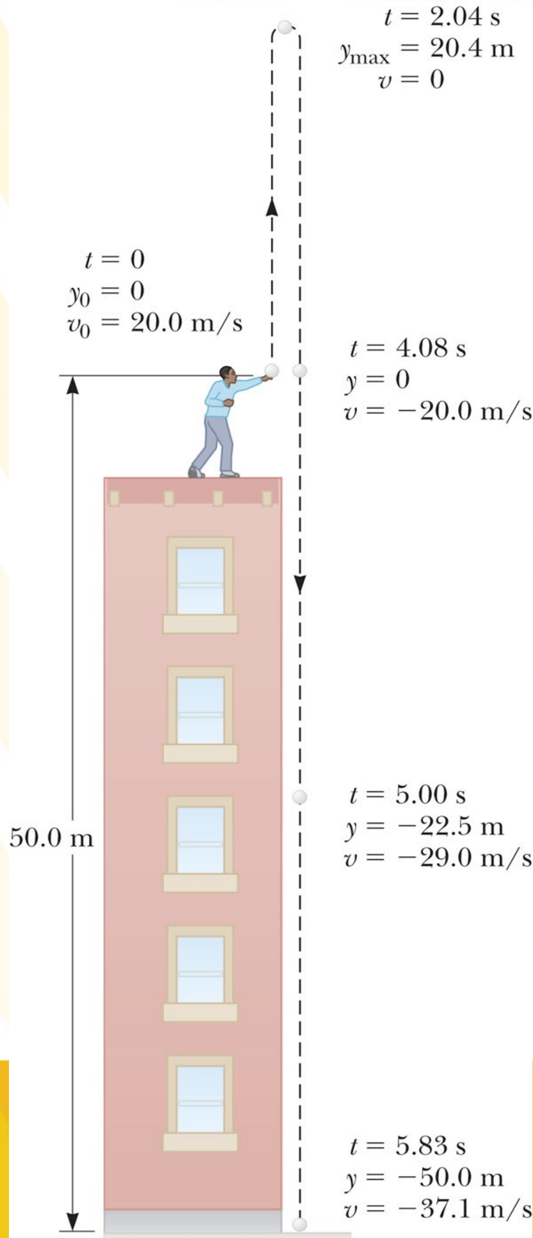
E. the velocity and position of the ball at t = 5s.

Neglect air drag.

Some helpful equations:

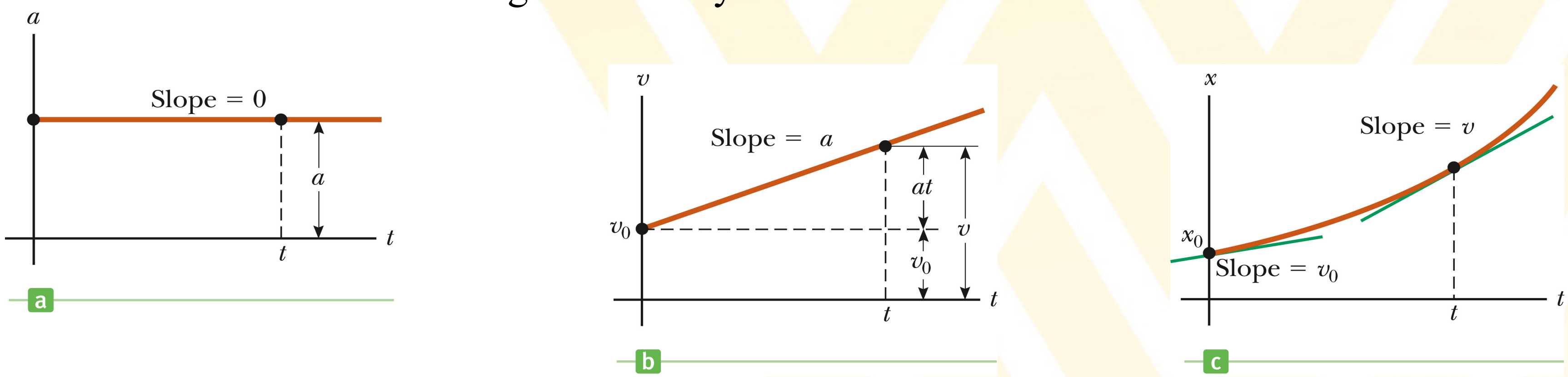
$$v(t) = v_0 - gt$$

$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$



Summary

- Velocity is the rate of change of position: v = Δx/Δt.
- Acceleration the rate of change of velocity: a = Δv/Δt.



- A constant acceleration results in:
  - a horizontal line in the a-t-graph.
  - a straight line in the v-t-graph.
  - a parabola in the x-t graph.
- Free Fall is the motion of an object under the influence of gravity alone (no other forces/accelerations).
- Same for all objects without air drag, different with drag.
- Equations relevant for 1d problems with constant acceleration:

$$v(t) = v_0 + at \qquad x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \qquad \Delta x(t) = \frac{1}{2}[v_0 + v(t)]t \qquad v^2(t) = v_0^2 + 2a\Delta x^2(t)$$