Announcements

- The final exam takes place in White B51 on **Thursday, May 2, 8 PM - 10 PM**.
- The exams covers **all** sections that we have covered in class through the end of class today (i.e. Chapters 1-13 will be included, Chapter 14 will **not**).
- The formula sheet, some old exam and extra problems, and my own worked solutions to them, will be available on the course webpage on/after Wednesday (today).
- Best way to prepare: first, go through all worked examples and clickers from lecture slides, then, if you have time, go through homework and WebAssign "practice tests" (i.e. extra problems). Next, try the old exam without reference materials, and re-review the worked examples from step 1 for any problem areas. Finally (if you still have more time after all that) try additional problems from the text.

Today's lecture

Review of chapters 1 - 13

Velocity and acceleration (1d)

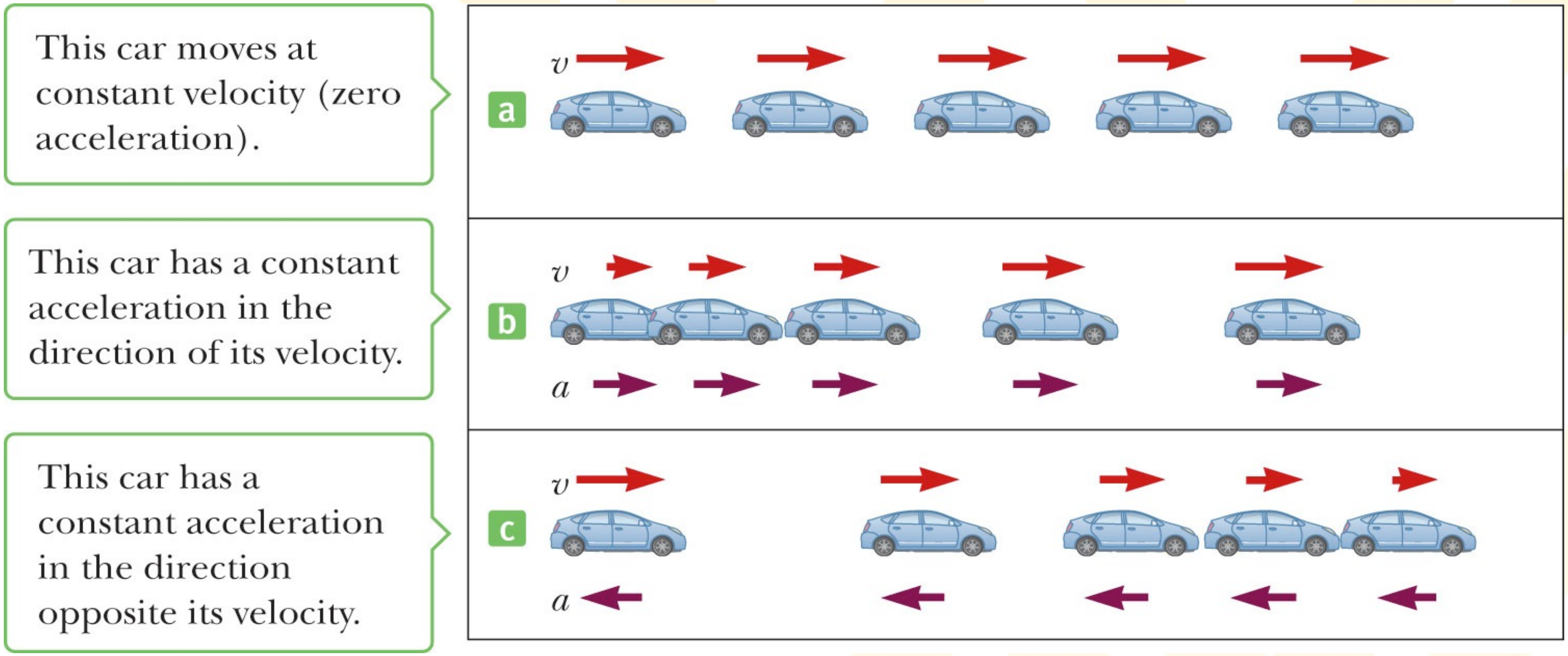
- The average and instantaneous velocities are defined as:

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- The average and instantaneous accelerations are defined as:

- $$\bar{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Motion diagrams



Velocity is the rate of change of position.

$$v_{avg} = \frac{\Delta x}{\Delta t} \qquad a_{avg} = \frac{\Delta v}{\Delta t}$$

Acceleration is the rate of change of velocity.

Typical problem: What are the signs of the velocity and acceleration in plot c?

1d motion with constant acceleration - Free Fall

- Free Fall is the motion of an object under the influence of gravity alone (no other forces/ accelerations).

$$a = \frac{\Delta v}{\Delta t} = -g = -9.81 m/s^2$$

- For most problems (1d motion with constant acceleration) the following equations are sufficient:

$$v(t) = v_0 - gt \qquad y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$

- Two objects of different masses will hit the ground at the same time, if dropped from the same height (no friction).

Projectile motion

- In 2d problems, displacement, velocity, and acceleration are vectors:

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i \qquad \vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad \vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

An acceleration will happen, if the magnitude and/or direction of the velocity vector is changed as a function of time.

- A projectile is an object, given an initial velocity, that moves under the influence of gravitation.
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- For projectile motion, the motions in horizontal and vertical direction are completely independent. Without air resistance, the velocity in horizontal direction is constant (no horizontal acceleration). In vertical direction a = -g = const.

- We can use the equations for 1d motion for each direction separately:

$$x(t) = x_0 + v_{x0}t + \frac{1}{2}at^2 \rightarrow x(t) = v_{x0}t \qquad y(t) = y_0 + v_{y0}t + \frac{1}{2}at^2 \rightarrow y(t) = v_{y0}t - \frac{1}{2}gt^2$$

$$v_x(t) = v_{x0} + at \rightarrow v_x(t) = v_{x0} \qquad v_y(t) = v_{y0} + at \rightarrow v_y(t) = v_{y0} - gt$$

Forces

- Newton's first law:

An object moves with a velocity that is constant in magnitude and direction unless a non-zero net force acts on it.

- Newton's second law:

$$\boxed{\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}}$$

- Newton's 3rd law:

$$\boxed{\text{Action} = \text{- Reaction}}$$

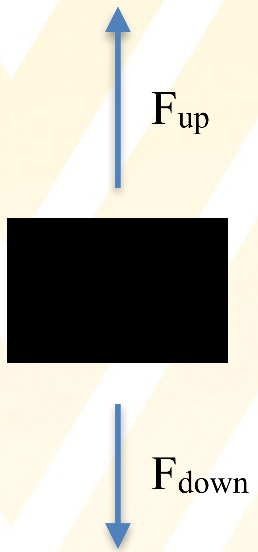
- Friction forces are directed into the opposite direction of an externally applied force.

If an object moves at constant velocity, its acceleration is zero. This means that there is either no force acting on the object or multiple forces compensate each other, e.g. gravity and friction.

Example problem: Newton's 2nd law

An upward and a downward force act on an object of mass $m = 20$ kg: $F_{up} = 20$ N, $F_{down} = 10$ N.

What is the acceleration of the box?



Different types of energies

1. Translational kinetic energy: $$KE_T = \frac{1}{2}mv^2$$

2. Rotational kinetic energy: $$KE_R = \frac{1}{2}I\omega^2$$

3. Gravitational potential energy: $$PE_g = mgy$$

4. Spring potential energy: $$PE_s = \frac{1}{2}kx^2$$

Clicker question

If I double the velocity of a given object, what happens to its translational kinetic energy?

A. It decreases by a factor of 4.

B. It decreases by a factor of 2.

C. It remains the same.

D. It increases by a factor of 2.

E. It increases by a factor of 4.

$$KE_T = \frac{1}{2}mv^2$$

Conservation laws

We know three fundamental conservation laws in physics:

1. Conservation of mechanical energy, if there is no work done by non-conservative forces:

$$(KE_t + KE_r + PE_g + PE_s)_i = (KE_t + KE_r + PE_g + PE_s)_f$$

2. Conservation of linear momentum, if there is no net external force on the system (two objects):

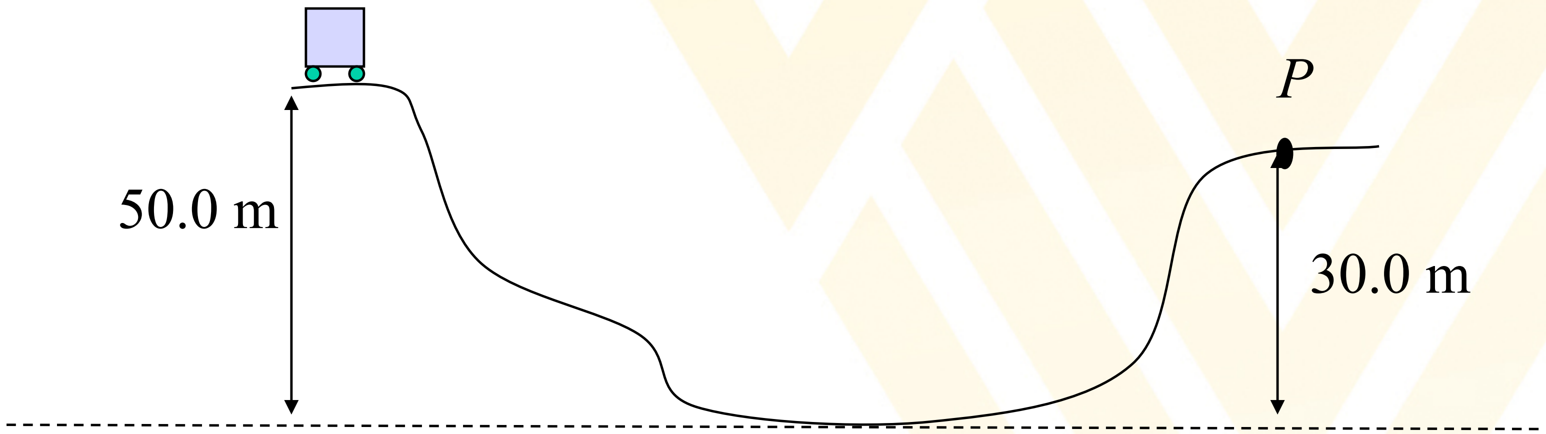
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

3. Conservation of angular momentum, if there is no net torque on the system:

$$I\omega_i = I\omega_f$$

Example problem: Conservation of energy

A rollercoaster car is at the top of a hill. If its speed at the top of the hill is 2.0 m/s, calculate the speed ignoring friction at the point *P* shown below:



Power

Power is the rate at which energy is transformed from one type to another:

Average power: $$\bar{P} = \frac{W}{\Delta t}$$ Power is a scalar quantity.

Unit: $$1W = 1J/s = 1kg \cdot m/s^2 \cdot m \cdot s^{-1} = 1kg \cdot m^2/s^3$$

Alternative expression for power:

$$W = F \cdot \Delta x$$ if F is parallel to Δx.

$$\rightarrow \quad \bar{P} = F \cdot \frac{\Delta x}{\Delta t} = F\bar{v}$$

Momentum and Impulse

- Definition of linear momentum: $\vec{p} = m\vec{v}$ Unit: kg · m/s

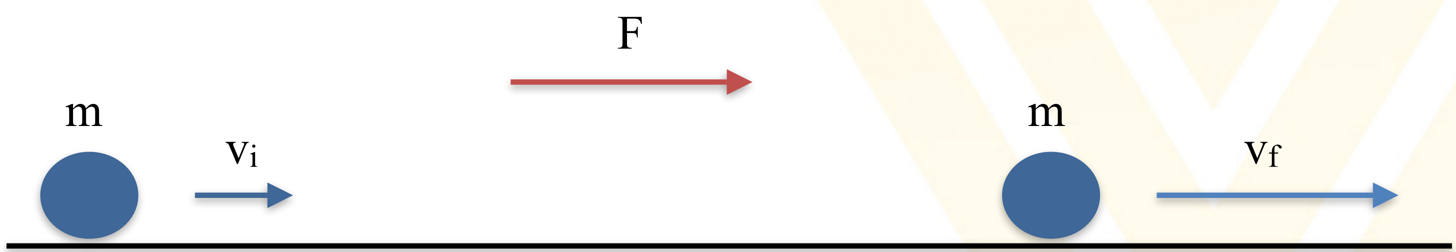
  Momentum is a vector quantity. It points into the same direction as the object's velocity.

- Relation between momentum and kinetic energy: $KE = \dfrac{p^2}{2m}$

- Relation between force and impulse: $\vec{F}_{net} = \dfrac{\Delta\vec{p}}{\Delta t}$

- Definition of impulse: $\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = \vec{F}_{net} \cdot \Delta t$

  Impulse corresponds to a change of momentum during a time interval Δt.

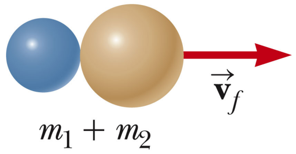


Collisions

- There are three types of collision processes:
  Inelastic, elastic, and superelastic collisions

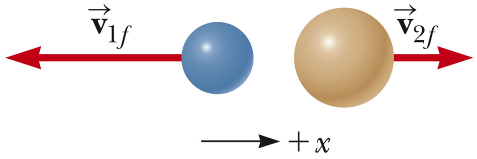
- Inelastic collisions:
  Momentum is conserved, energy is not conserved (lost).
  If the colliding objects stick to each other, the collision is *perfectly inelastic*:

$$\boxed{v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}}$$

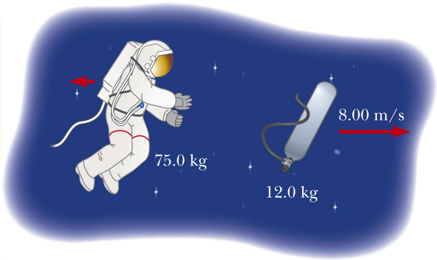


- Elastic collisions:
  Momentum and energy are conserved.

$$\boxed{\begin{gathered} m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \end{gathered}}$$



- Superelastic collisions:
  Momentum is conserved, energy is not conserved (gained)

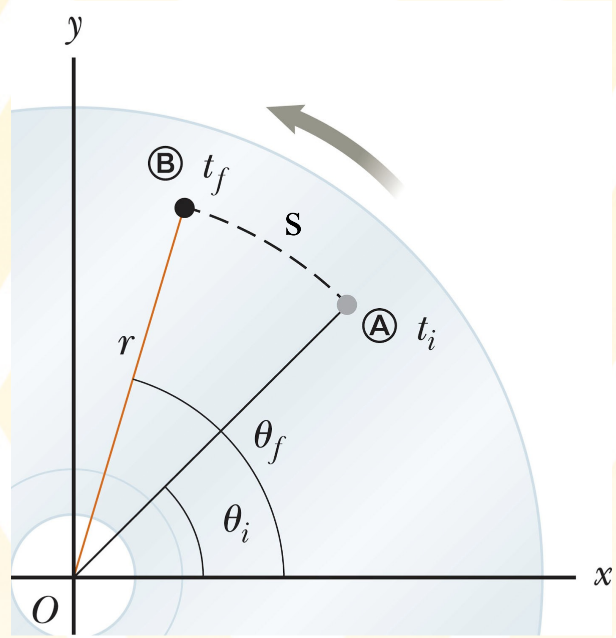


Example problem: Inelastic collision

A 1200 kg car traveling at 20 m/s runs into the rear of a stopped car that has a mass of 2000 kg and they stick together. What is the speed of the combined cars just after the collision?

Rotational motion

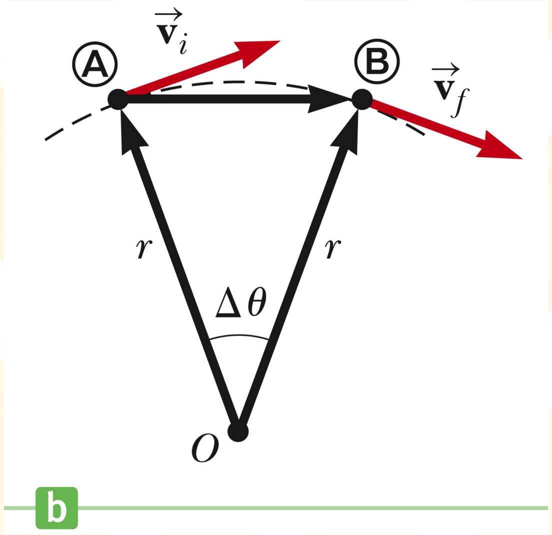
- Angular position: $\theta = \dfrac{s}{r}$
- Angular displacement: $\Delta\theta = \theta_f - \theta_i$
- Angular velocity: $\omega_{av} = \dfrac{\theta_f - \theta_i}{t_f - t_i} = \dfrac{\Delta\theta}{\Delta t}$
- Angular Acceleration: $\alpha_{av} = \dfrac{\omega_f - \omega_i}{t_f - t_i} = \dfrac{\Delta\omega}{\Delta t}$
- Every point on a rotating rigid object has the same angular, but not the same linear motion!



Linear Motion with *a* Constant (Variables: *x* and *v*)	Rotational Motion About a Fixed Axis with *α* Constant (Variables: *θ* and *ω*)
$v = v_i + at$	$\omega = \omega_i + \alpha t$
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$

Centripetal and gravitational forces

- Circular motion at constant speed is an accelerated motion (the direction of the velocity vector changes).



- The corresponding centripetal acceleration is:

$$\boxed{a_c = \frac{v^2}{r} = r\omega^2}$$

- The centripetal force is:

$$\boxed{F_c = ma_c = m\frac{v^2}{r} = mr\omega^2}$$

- If the centripetal force stops, the object will continue to move straight based on Newton's 1st law.
- The centripetal force is always directed towards the center.
- Two masses separated by a distance, r, attract each other based on Newton's law of gravitation:

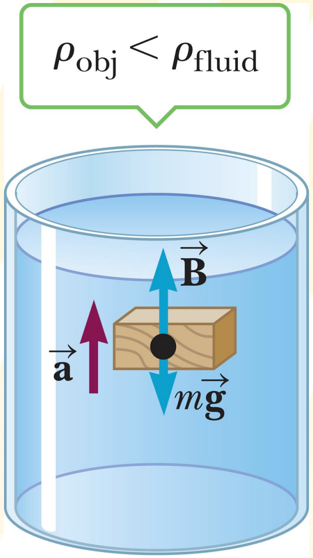
$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

Archimedes principle, continuity, Bernoulli

- The Buoyant Force acts on every object immersed in a liquid (or gas): $\boxed{B = \rho_{fluid} \cdot V_{fluid} \cdot g}$

  It corresponds to the weight of the fluid displaced by the object (Archimedes principle).

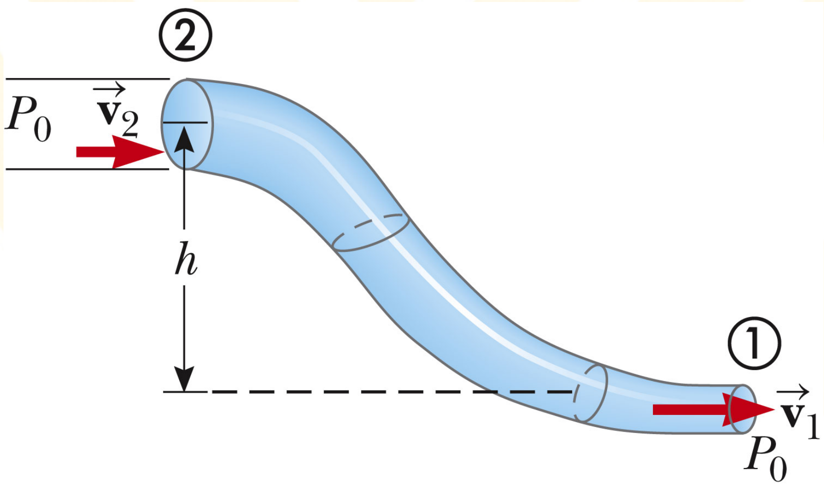
  Such an object also feels the force of gravity due to its own mass. The balance of the Buoyant force and gravity determines if the object floats or sinks (ρ_{obj} vs. ρ_{fluid}).



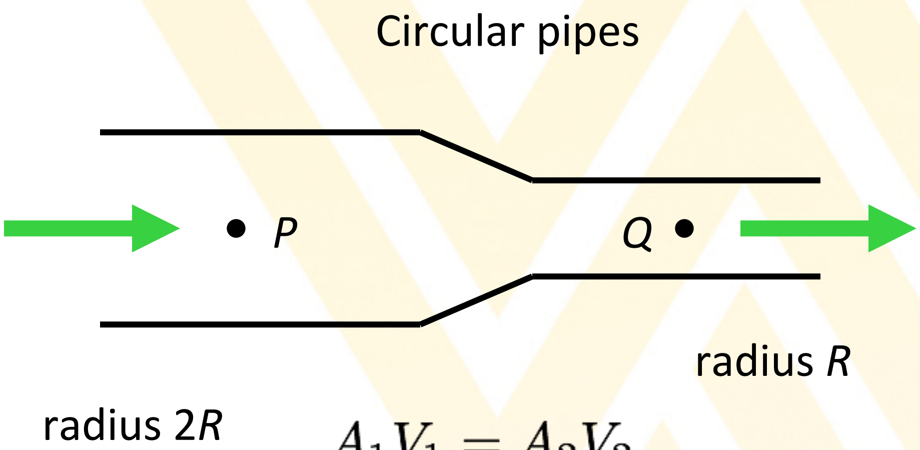
- Equation of continuity: $\boxed{A_1 V_1 = A_2 V_2}$

- Bernoulli equation (energy consevation):

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = const.}$$



Clicker question



$$A_1 V_1 = A_2 V_2$$

A fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point *P*, the fluid at point *Q* has

A. 4 times the fluid speed.

B. 2 times the fluid speed.

C. the same fluid speed.

D. 1/2 the fluid speed.

E. 1/4 the fluid speed.

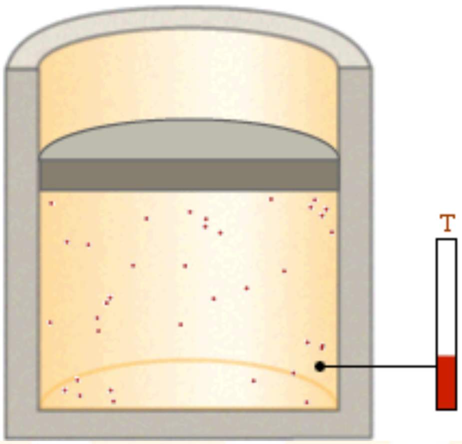
Example problem: Buoyant force

A 10-kg piece of aluminum is suspended in water by a support string. A 10-kg piece of lead is also suspended in water by a support string. The density of lead is greater than the density of aluminum.

Which has the greater buoyant force acting on it?

Thermal Physics I

- Temperature, T, is related to the average kinetic energy of each atom/molecule the given material consists of:



$$\boxed{\frac{1}{2}mv_{avg}^2 = \frac{3}{2}k_BT}$$

- The ideal gas law relates pressure to density and temperature:

$$\boxed{P = \frac{N}{V}k_BT}$$

- There are 3 different temperature scales: Celsius, Kelvin, and Fahrenheit

$$\boxed{T[^\circ C] = T[K] - 273.15}$$

$$\boxed{T[^\circ C] = \frac{5}{9}(T[^\circ F] - 32)}$$

- There are 3 types of thermal expansion:

(i) Length expansion

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

(ii) Area expansion

$$\boxed{\Delta A = \gamma A_0 \Delta T}$$

(iii) Volume expansion

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

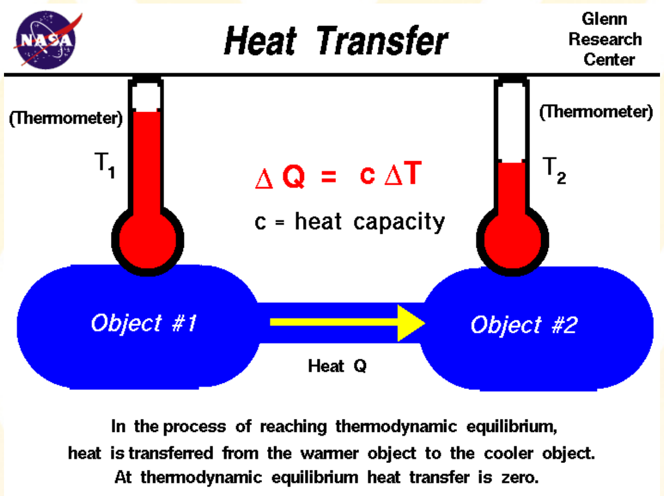
Thermal Physics II

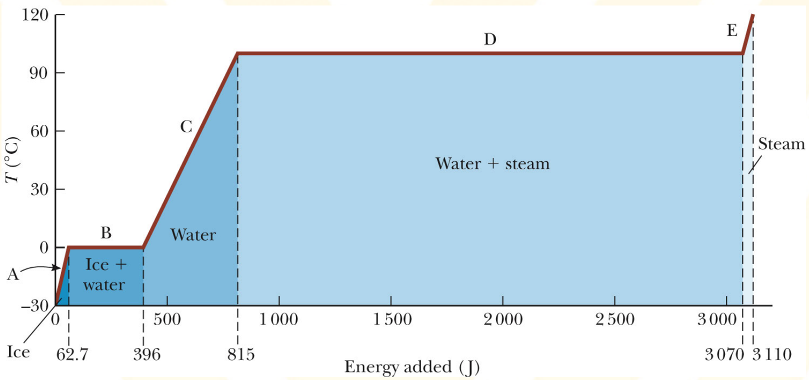
- Internal Energy, U, is the energy associated with the atoms and molecules of the system.
- Heat, Q: Energy *transferred* between a system and its environment due to a *temperature difference:*

$$\boxed{Q = mc\Delta T}$$ Unit: J/(kg °C)

- c is the material dependent specific heat, i.e. the energy required to raise the temperature of 1 kg of the material by 1 °C.
- Latent heat, L, is the energy required to chance the phase a substance and does not cause its temperature to increase.

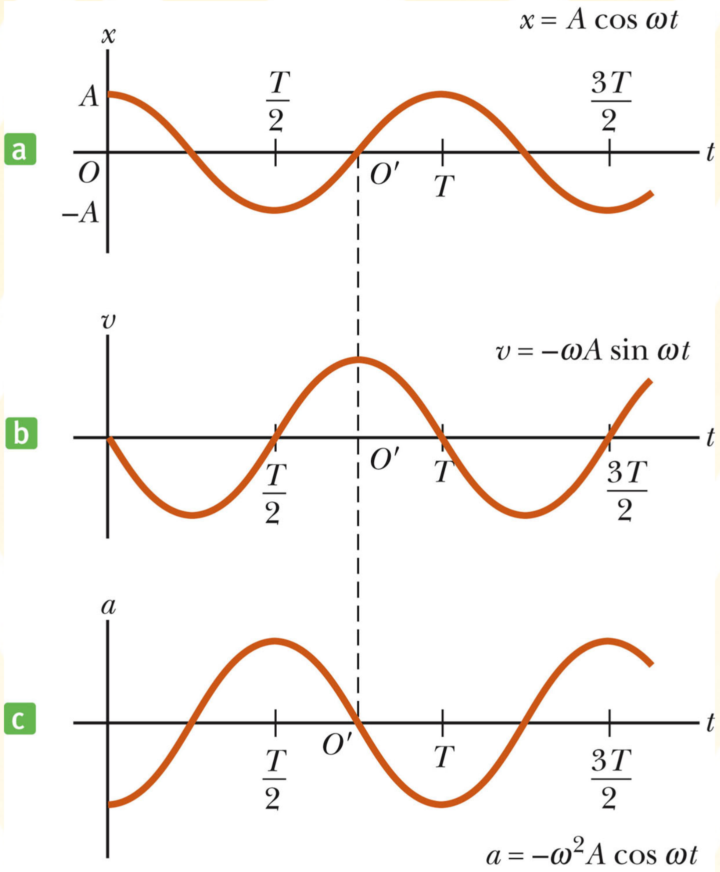
$$\boxed{Q = mL}$$ Unit of L: J/kg





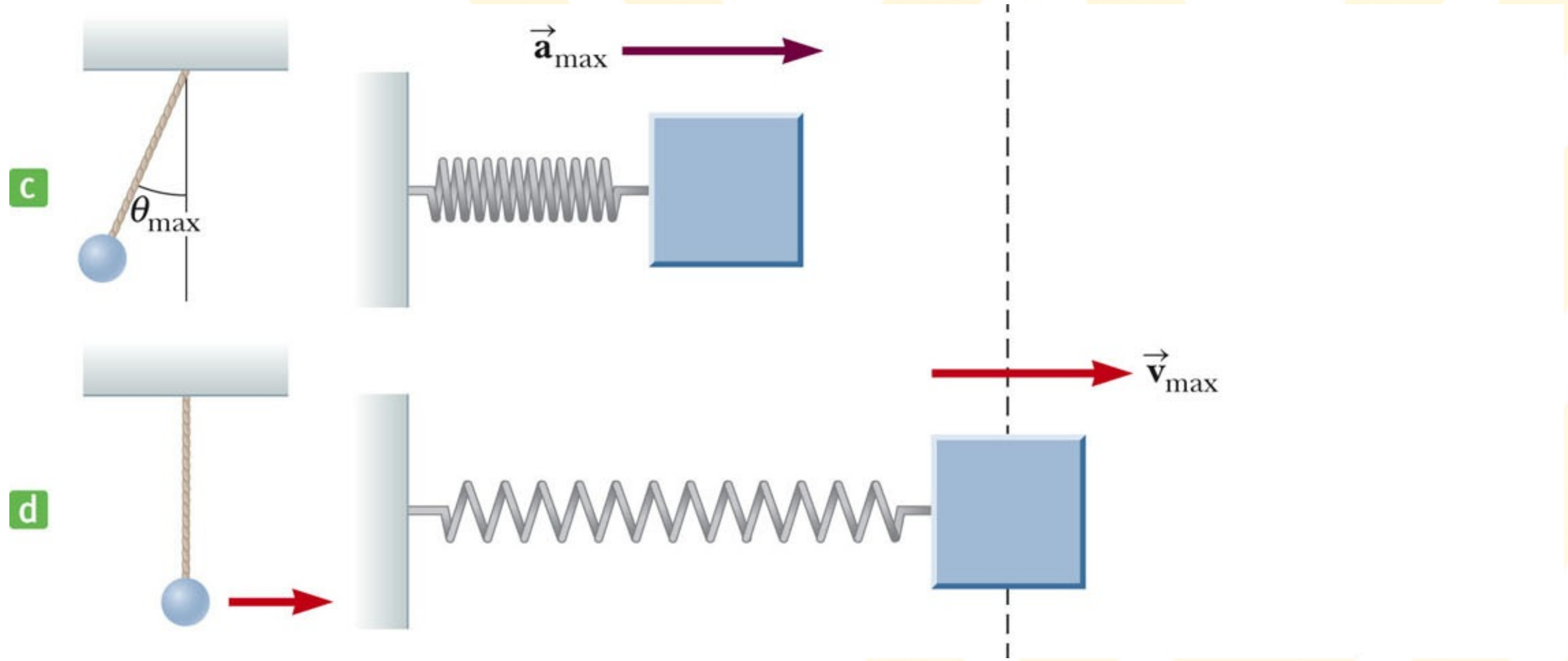
Simple Harmonic Motion (SHM)

- Definition of SHM: The restoring force is proportional to the displacement from equilibrium.
- Hooke's Law: $F_s = -kx$
- In SHM the acceleration is not constant: $a = -\frac{k}{m}x$
- Spring potential energy: $PE_s = \frac{1}{2}kx^2$
- Period of SHM: $T = 2\pi\sqrt{\frac{m}{k}}$
- Position, velocity, acceleration as a function of time:



$x = A\cos(2\pi f t)$ $\quad v = -A\omega\sin(2\pi f t)$ $\quad a = -A\omega^2\cos(2\pi f t)$

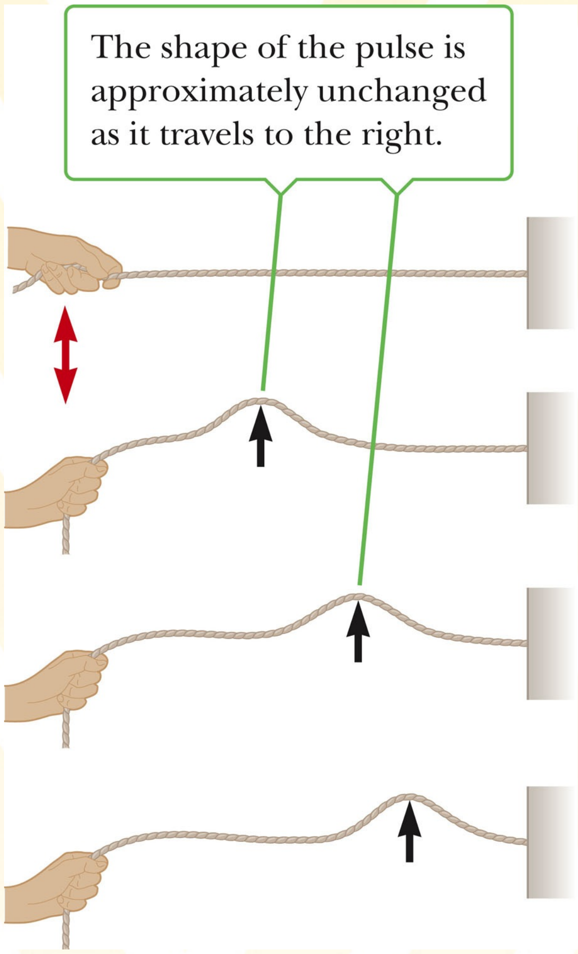
Simple Harmonic Motion (SHM)



- The acceleration and restoring force are maximum at the turning points.
- The velocity and kinetic energy are maximum at the equilibrium point.

Waves

- Waves are moving oscillations, i.e. the equilibrium point moves and is no longer static.
- The medium itself does not move - the disturbance moves.
- There are transverse and longitudinal waves.
- The distance between two successive points that behave identically is called the wavelength, λ.
- The wave speed is: $\boxed{v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f}$
- Sound waves are longitudinal density waves.



- The speed of a wave propagating on a string stretched under some tension, F is:

$$\boxed{v = \sqrt{\frac{F}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}}$$

Example problem

On average, how far apart are the wave peaks for WAJR, which broadcasts on 104.5 FM, and 1440 AM?

FM radio broadcasts in MHz.

AM radio broadcasts in kHz.

Speed of sound in air is $v_s = 346\,\mathrm{m/s}$.