# VCAST: Scalable dissemination of vehicular information with distance-sensitive precision

V. Kulathumani, R. A. Moparthi, Y. P. Fallah

Dept. of Computer Science and Electrical Engineering

West Virginia University

Email: {vkkulathumani, yfallah}@mail.wvu.edu, rmoparthi@mix.wvu.edu

#### Abstract

Real-time information about the state (location, speed, direction) of other vehicles in the system is critical for both safety and navigation applications in future intelligent transportation systems. However, reliably obtaining this information over multiple hops in a wireless network while avoiding network congestion in the shared channel poses a significant challenge. In this paper, we describe an algorithm VCAST that addresses this challenge by exploiting a notion of *distance sensitivity in information propagation*, in which information is forwarded at a rate that decreases linearly with distance from the source. By doing so, traffic information can be obtained with a staleness, which is a measure of error in the traffic information, that is bounded by  $O(d^2)$  where d is the distance from the source of the information. At the same time, the required communication overhead per node can be significantly reduced to  $O(\sqrt{N})$ , thereby reducing channel contention, allowing higher information supply rates and scaling to larger network sizes. VCAST can be used to improve safety against collisions, inform about approaching emergency vehicles and lane merging vehicles (that may even be beyond a single hop communication range) and to enable dynamic routing and navigation techniques by providing aggregate traffic information in an extended neighborhood. VCAST does not require any special hardware or modification to vehicular transmission standards; instead it can simply piggyback on basic Here I am communication for vehicular networks. The performance of VCAST is validated using extensive ns-3 simulations under different network sizes and densities with an IEEE 802.11b transmission model and the advantages of VCAST in comparison to non distance-sensitive approaches are highlighted.

## I. INTRODUCTION

Infrastructure-less, vehicle-to-vehicle (V2V) wireless communication is expected to be the basis for both safety and navigation applications in future intelligent transportation systems [1], [2]. Examples of safety applications in transportation scenarios include collision warnings, guidance on lane change and lane merge, and stopped vehicle alert. Examples of intelligent navigation applications include dynamic travel-time computations and re-routing based on real-time traffic information. A building block for all these applications is a real-time vehicular traffic mapping system on board every vehicle, which portrays information about current position of other vehicles in its vicinity and provides guidance about accidents, approaching emergency vehicles and traffic congestion over an extended neighborhood. In this paper, we design *VCAST*, a scalable, infrastructure-less peer-to-peer wireless network service for computing such a real-time traffic map over a given region surrounding each vehicle  $^1$ 

We assume that vehicles are equipped with a differential GPS that can estimate its location to an accuracy of about 1-2m. As a result, by advertising this information, vehicles can learn about traffic within a one hop communication range. However, estimating traffic maps over a large area using multi-hop wireless communication is a much more challenging task. (1) Firstly, we note that forwarding each vehicle's information over multiple hops at a constant rate is unlikely to be scalable as it will cause the amount of communication required at each vehicle to grow with the number of vehicles in the region over which traffic information is required. As a result, both the allowable broadcast rate and the accuracy of the traffic map obtained at each vehicle will decrease as vehicular density and size of the region increase. *Hence effective forwarding algorithms need to be designed that ensure that the system remains scalable* 

<sup>&</sup>lt;sup>1</sup>This paper is a significant extension of a shorter version that appeared in IEEE VTC 2012 [3]. The specific additions are as follows. (1) Simulations are carried out in ns-3 using an IEEE 802.11b transmission model to quantify the impact of network size, vehicular density, vehicular mobility and time-varying inter-vehicular separations on the achievable staleness. (2) The average communication cost incurred by each vehicle is quantified. (3) The performance is compared analytically and using simulations with schemes that do not incorporate distance-sensitivity, and the significant impact on scalability using *VCAST* is highlighted. (4) Complete proofs are included for all Lemmas and Theorems.

with vehicular density, information supply rate and region size. (2) Secondly, there exist trade-offs in choosing the rate and communication range of each broadcast. While higher broadcast rates and range promise greater tracking accuracy, in reality wireless channel contention can cause an adverse effect in tracking accuracy as these levels exceed a certain limit. Existing broadcast techniques for vehicular safety systems have primarily focused on balancing transmission rate and communication range to maximize the reliability of single hop wireless communication. However, in such an approach information about other vehicles is available only when inter-vehicular distance is too small which may not be enough to avert a collision [4] or to provide a timely re-route in the presence of traffic congestion. On the other hand, increasing the single-hop communication range or moving to multi-hop forwarding techniques to learn about vehicles in a larger area quickly decreases the achievable information supply rate from each vehicle (even those at smaller distances). *Hence, there remains a need for multi-hop broadcasting techniques that are able to supply timely information over large distances without compromising on data supply rates at smaller distances.* 

**Contributions:** To address the above challenges and to ensure scalability when providing traffic maps over large regions in real-time, we exploit a notion of *distance-sensitivity* in propagating individual vehicle information: information about each vehicle is propagated at a rate that decreases linearly with the distance from the vehicle [5], [6]. The rationale behind exploiting distance-sensitivity is that the reaction time available to a vehicle for taking safety actions or for computing new routes towards the destination is lesser with respect to the state of nearby vehicles than that of farther vehicles. *VCAST* maps this distance-dependent reaction time into delivery of information with quality that progressively decays as a function of distance.

We use staleness in vehicular state information as a metric for information quality as it reflects how old the current information about a particular vehicle is, and show that traffic information in VCAST can be obtained with a worst-case staleness that is bounded by  $O(d^2)$  where d is the distance from the source of the information. Thus, VCAST provides traffic information with distance-sensitive precision in which the error does not grow with the number of vehicles in the region and is independent of traffic density and network size. At the same time, the average communication cost (the required transmission rate at each node) also is only bounded by  $O(p\sqrt{N})$ , where p is the broadcast rate at the source. Lower communication overhead per node ensures lower channel contention and higher source broadcast rates, thus benefiting information supply at smaller distances. VCAST can be used to propagate actual vehicle location as well as to propagate aggregate traffic information such as average speed and density of vehicles over individual traffic *cells* inside a region. One possible scenario is to propagate actual vehicle location up to a distance of 500 - 1000m and aggregate traffic information up to several miles. When propagating only aggregate cell information, we note that the required communication rate per node decreases further by a factor  $r_c$ , where  $r_c$  is the radius of each aggregation cell.

The performance of *VCAST* is validated using extensive simulations in ns-3, a discrete event simulator for wireless networks, under different network sizes, network densities, source broadcast rates and communication ranges. The results of our evaluation show that by using distance-sensitive forwarding, *VCAST* is able to scale to larger network sizes as well as support much higher broadcast rates compared to non distance-sensitive techniques. The reason for scalability was shown to be the significantly lower message overhead which reduces the channel contention in the network. We have also characterized the performance of *VCAST* under severe random mobility and studied information staleness when aggregated data is transmitted as opposed to individual vehicle information.

**Impact:** By forwarding traffic information over multiple communication hops, we expect several advantages: (1) vehicle location over a vicinity will be available even if the communication range is extremely low because of high density, thus improving vehicular safety (2) information about approaching emergency vehicles will be available, (3) information about lane changing and merging vehicles will be available even if they are outside a single communication range and (4) information about road blocks, accidents and

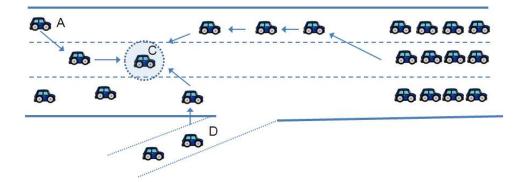


Fig. 1. Utilization of multi-hop vehicular information for safety applications. The arrows indicate multi-hop information flow towards car C. Knowledge of states of A and D will guarantee safe lane change to left and right respectively. Knowledge of congestion (shown as shaded region) will allow safe, timely reaction.

impending congestion will be available from several miles ahead, thus permitting higher level applications that dynamically re-route based on this information. Some of these scenarios, from a vehicular safety perspective, are illustrated in Fig. 1. To achieve scalability, *VCAST* provides traffic information with an accuracy that degrades with distance, but we expect this to be a reasonable condition that is still sufficient for both vehicular safety and intelligent navigation. Finally, we note that *VCAST* does not require any special hardware or modification to vehicular transmission standards; instead it can simply piggyback on the proposed *Here I am* communication [1], [2] for vehicular networks.

**Outline of the paper** In Section 2, we describe related work. In Section 3, we describe our system model, the VCAST algorithm and provide an analysis for the expected accuracy and communication cost. In Section 4, we evaluate the performance of our system in simulations. In Section 5, we present conclusions and state future work.

## II. RELATED WORK

The design of routing protocols for vehicular ad-hoc networks and more generally in mobile ad-hoc networks is a well researched topic and a good survey of these techniques can be found in [7]–[9]. Many of these protocols have focused on delivering aperiodic, low bandwidth data such as emergency vehicle information to either a single vehicle (unicast) or a group of vehicles in a geographic region (geocast) with low latency [10]–[17]. On the other hand, the focus in this paper is on broadcasting information

from each vehicle at high rates to support the design of safety and navigation applications.

The design of wireless broadcast protocols has also received a lot of research attention lately [1], [18]–[22] in the context of vehicular safety applications. There have been several recent papers that have focused on the problem of balancing broadcast range and reliability so as to maximize the number of successful receptions in close proximity of the sender [18]–[21]. A common foundation in these papers to handle the trade-off is to reduce the communication range in regions of high density so as to improve the reliability of reception. However, these papers mainly focus on reliable one-hop message reception and not on multi-hop propagation. As a result information about vehicles outside the communication range are not available even when the range has to be very low because of high density. The trade-off in single-hop broadcast schemes is that a higher communication range causes larger staleness even for nearby vehicles, while a lower communication range prevents information availability from beyond that range. On the other hand, the algorithm developed in this paper can be used to propagate both individual vehicle information and aggregate traffic information over several communications hops and yet retain high data supply rates at smaller distances.

We also note that rate and power control algorithms [1], [18]–[21] developed for a single hop vehicular broadcast are complementary and can be used in conjunction with the distance-sensitive multi-hop for-warding algorithm designed in this paper. Thus decisions on source broadcast rate and range can be made commensurate with traffic density using the techniques proposed in [1], [18]–[21], and *VCAST* can be used to aggregate and forward this information over multiple hops.

Multi-hop broadcast algorithms [23]–[29] for vehicular networks have mainly focused on the choice of optimal forwarding vehicles and on the reduction of redundant forwarding vehicles using one of several heuristics proposed in [30]. A good survey of multi-hop broadcast techniques in mobile ad-hoc networks is presented in [31]. The idea of distance-sensitive broadcasting rates developed in this paper for ensuring scalable all-all information broadcast has thus far not been explored in the context of mobile ad-hoc and

vehicular networks. The improvement in communication overhead gained by exploiting distance-sensitivity is characterized in Section III-C.

The concept of distance-sensitivity has found applications before in several other networking fields in different forms. For example, route aggregation in the Internet utilizes this concept for efficiently distributing routing information [32]. Fisheye routing [33] uses this idea to propagate routing tables in mobile ad-hoc networks. Fractionally cascaded information [34] is a form of distance sensitive key sharing that is widely used for speeding up traversal of data structures. Distance-sensitivity has also been used in wireless sensor network based querying and tracking applications to model communication latency and information quality as functions of distance [35]–[38]. In the context of network-wide continuous broadcast of system states, an algorithm has been designed in [39] for supplying global state information to all nodes in a static sensor network with distance-sensitive latency and error. In this paper, we show that distance-sensitivity is a valuable tool for efficient and scalable propagation of state information even for *networks of mobile nodes (vehicles)*. Unlike the algorithm in [39], we design *VCAST* without assuming an underlying clustering or routing structure, thus avoiding the need for maintenance related communication.

# **III.** SYSTEM DESCRIPTION

In this section, we first state the system model and objective. We then describe *VCAST* and analytically characterize its accuracy and required communication rate.

## A. System model and problem statement

We model the vehicular network as a large geographic area with multiple traffic flows, each with potentially different traffic densities at different places. Let  $\rho$  denote the maximum density, i.e., the maximum number of vehicles per unit area at any time in the whole region. Note that the geographic area will consist of regions with no traffic flows (i.e., no roads), as well as regions with high density traffic flows. When analyzing the impact of aggregation, we assume that the region is partitioned into geographic *cells* which

allow representation of aggregated traffic information for that cell. The cells need not be of the same size, but for ease of explanation we assume that the area of each cell is constant and equal to  $A_C$  with a radius of  $r_c$ . In a realistic environment, these cells could be used to model critical traffic links such as the traffic in a region between two highway exits or traffic between two urban streets. Let  $r_h$  denote the single hop communication range for a vehicle. The objective of our system is to provide each vehicle with information about all vehicles and all cells within a radius R around itself where  $r_h << R$ . We call this area of radius R around each vehicle as the *tracking zone*.<sup>2</sup>

Let N denote the maximum number of vehicles within the tracking zone of each vehicle. Thus  $N = \rho \pi R^2$ . Let L denote the maximum number of cells in a radius of R around each vehicle. Thus  $L = O(\frac{R^2}{A_c})$ . Note that by using a two dimensional model for the vehicular network, the bounds on communication cost in our analysis are expected to hold for dense urban traffic scenarios as well as highway traffic scenarios in which the number of vehicles are expected to be lower.

Let p denote the frequency at which each vehicle broadcasts its own information. Let d(i, j) denote the geographic distance between vehicles i and j. Let  $d_c(i, j)$  denote the distance between vehicle i and cell j in terms of smallest number of cells traversed to reach cell j from i. In the following subsections, we first describe VCAST assuming that only vehicle location is propagated up to a distance R. Then we describe the required changes when aggregate *cell* information is propagated. The communication cost and accuracy for both these cases are characterized in Section III-C.

# B. Distance-sensitive broadcast algorithm

A naive technique would be to require each vehicle to obtain information about all vehicles in its tracking zone at a constant interval of 1/p seconds. However, in such a scenario each vehicle would have to broadcast information about at most N vehicles at p Hz, making the required communication rate at each node per unit time to be O(Np). Note that the required communication rate grows with the number

 $<sup>^{2}</sup>$ We note that our system can also be specialized to propagate individual vehicle information within a smaller radius than aggregate cell information.

of vehicles in the region and is therefore directly proportional to the vehicle density and the area of the region. Also if C denotes the allowable wireless channel transmission capacity at a node in bits per second, we observe that the allowable broadcast rate p is limited by  $p < \frac{C}{N}$ , i.e., inversely proportional to the number of vehicles. This in turn has an adverse effect on the latency with which traffic information is obtained and as a result the accuracy of the obtained traffic map information decreases with increasing vehicular density, which is not desirable.

To address these drawbacks, we propose to forward information about a vehicle at a rate that is proportional to the distance from that vehicle. Also, in *VCAST*, a vehicle suppresses forwarding of information about a vehicle in an interval, if some other vehicle has already forwarded information about the respective vehicle in that interval. By doing so, we show that the communication cost can be significantly reduced, leading to scalability. A more formal description of *VCAST* is presented below. A pseudo-code in guarded command notation [40] is provided in Fig. 2, which shows the program at each vehicle in the form of  $<event \rightarrow action >$  pairs.

In VCAST, each vehicle j maintains a list j.V of vehicles that are within a distance R from itself. Associated with each vehicle  $i \in j.V$  is the location  $j.X_i$  of i as most recently heard by j, a timestamp  $j.T_i$  associated with the location and  $j.\nu_i$  which is the number of times information about i has been heard since the last broadcast interval. A timer is fired at each vehicle every  $\frac{1}{p}$  seconds for broadcasting information and a randomness is introduced in this interval because of CSMA based transmission. Let  $\lambda = 1, 2, ...$  denote the timer sequence at vehicle j. Let  $j.V_{\lambda}$  denote the set of vehicles whose information is forwarded in the interval number  $\lambda$  at vehicle j.  $j.V_{\lambda}$  is initially set to be equal to  $\{j\}$ . Thus information about itself is broadcast by a vehicle in every interval along with the current time which serves as the timestamp for this record. Node i is added into the set  $j.V_{\lambda}$  only if  $\lambda \mod d_h(i, j) = 0$  and if information about i has not been broadcast by any other vehicle within j's range in the last  $d_h(i, j)$  intervals. This ensures that information about nodes at a distance of k communication hops is broadcast at most once Protocol: VCAST Vehicle: j Var: j.V: List of vehicles within radius R $j.X_i \forall i \in V$ : Location of i $j.T_i \forall i \in V$ : Timestamp of *i*'s record  $j.\nu_i \forall i \in V$ : Counter for *i*'s information  $j.\lambda$ : Sequence number of current interval  $j.V_{\lambda}$ : Forwarding list for  $j.\lambda$ Actions:  $\langle A_1 \rangle$  :: Initialization:  $\longrightarrow$  $j.V = j; j.\nu_i = 0; j.\lambda = 0;$ Timer.start  $\left(\frac{1}{n}\right)$  $\langle A_2 \rangle ::$ **Timer fired:**  $\longrightarrow$  $j.\lambda = j.\lambda + 1;$  $j.V_{\lambda} = j;$  $\forall i \in j.V$ if  $(\lambda \mod d_h(i, j) == 0)$ **if**  $j.\nu_i < 2$ Add *i* to  $j.V_{\lambda}$ fi  $j.\nu_i = 0;$ fi  $\forall i \in j.V_{\lambda}$ Send  $j.X_i, j.T_i$  $\langle A_3 \rangle :: recv_i(V) \longrightarrow$  $\forall k \in i.V$ if  $((j.T_k < i.T_k) \lor (k \notin j.V))$  $j.X_k = i.X_k; j.T_k = i.T_k; j.\nu_k = 1;$ fi elseif  $(j.T_k == i.T_k)$  $j.\nu_k = j.\nu_k + 1$ fi

Fig. 2. VCAST: Protocol Actions at vehicle j

every k intervals in each communication neighborhood. In the presence of channel interference, we note that nodes may occasionally duplicate the transmission of information of a vehicle within a neighborhood. But we expect this to cause a much smaller overhead when compared with all nodes transmitting. Finally, whenever information about a vehicle i is heard by a node j, it is added into the list j.V if the timestamp of the incoming record is more recent than  $j.T_i$ .

For the case where aggregate traffic information is to be propagated, each vehicle computes summary information such as density and average speed for the cell in which it resides based on information it possesses about vehicles within its communication range. This summary is then propagated in a distance-sensitive manner: information about a cell at distance  $d_c$  is forwarded at a rate of  $\frac{p}{d_c}$  Hz.

## C. Analysis

**Theorem III.1.** The average amount of data communicated per unit time by each node in VCAST to obtain information about all vehicles within a distance of R from itself is bounded by  $O(\frac{Rp}{r_h})$ .

Proof: Let *B* denote the average amount of data communicated per unit time, by each node in VCAST. We discretize distance in intervals of  $r_h$  (the communication range), as information forwarding rate decreases linearly after each time a vehicle's information is forwarded. The number of vehicles at a distance of at most  $hr_h$  away from a vehicle is bounded by  $O(\rho h^2 r_h^2)$ . The number of vehicles at a distance of at most  $(h-1)r_h$  away from a vehicle is bounded by  $O(\rho h^2 r_h^2)$ . As a result the number of vehicles between a distance of  $hr_h$  and  $(h-1)r_h$  away from a vehicle is bounded by  $O(\rho h^2 r_h^2)$ . As a result the number of vehicles between a distance of  $hr_h$  and  $(h-1)r_h$  away from a vehicle in terms of the communication range. Note that information about vehicles between a distance of  $hr_h$  and  $(h-1)r_h$  and  $(h-1)r_h$  is broadcast only at  $\frac{p}{h}$  Hz. Thus the total amount of data to be communicated about vehicles in the distance interval of *h* is  $O(\rho p r_h^2)$ . Note that this information is broadcast by at most one vehicle in every area  $\pi r_h^2$ . Hence, on average each node is responsible for communicating only  $O(\frac{\rho p r_h^2}{\rho r_h^2})$  bits about vehicles at distance interval *h* from itself. Summing up over all distance intervals and by noting that each vehicle broadcasts its own state at *p* Hz, we get:

$$B = O(p + \sum_{i=1}^{\frac{R}{r_h}} \frac{\rho p r_h^2}{\rho r_h^2}) = O(\frac{Rp}{r_h})$$

Note that this result makes an assumption that information about nodes at a distance of k communication hops is broadcast at most once every k intervals in each communication neighborhood. In the presence of channel contention and hidden terminals, the assumption may be violated causing duplicate transmissions of vehicular information within a communication neighborhood and an increase in the amount of

communication per node. The results of our simulation will include the impact of channel contention and duplicate transmissions.

Under a uniform distribution of vehicles in a 2-d region, it can be inferred from the results of the above theorem that the communication cost per node only grows as  $O(p\sqrt{N})$ . In contrast, without distancesensitive forwarding the average communication incurred per node is O(N), as discussed below.

Comparison with multi-hop broadcast protocols without distance-sensitivity: In order to analytically compare the bound on communication cost with multi-hop broadcast algorithms that do not incorporate distance-sensitivity in message forwarding, first consider an algorithm in which information is simply flooded without suppression of redundant messages. In this case, each node broadcasts information about all nodes in the region at a rate of p times every second yielding a communication cost per node that is bounded by  $O(\rho p R^2)$ . This is clearly not scalable with the size of the network, and such an approach is likely to yield severe channel contention thus reducing the achievable broadcast rate p. Several heuristics have been proposed to address this broadcast storm problem so that redundant broadcasts of the same message can be eliminated, i.e. information about every vehicle is broadcast exactly once in every communication neighborhood per interval. In this case, the average communication cost reduces by a factor of  $\rho \pi r_h^2$  over a naive flooding approach, yielding an average communication cost per node of  $O(p(\frac{R}{r_b})^2)$ , i.e., O(pN). Thus, by comparing with Theorem III.1, we see that the effective communication cost per node by applying distance-sensitive forwarding rules reduces by a factor of  $\sqrt{N}$ . Reducing the amount of communication incurred by each node, results in smaller channel contention and allows a higher broadcast rate at smaller distances. This is especially crucial when forwarding information over large areas because it ensures that information supply rates at smaller distances are not penalized for having to forward information over multiple hops.

We now obtain a bound on how outdated the state information possessed by a vehicle is because of distance-sensitive forwarding. We refer to this as *staleness* in the information possessed by a vehicle.

**Definition III.1** (Staleness(j,i)). The staleness S(j, i, t) in the state of vehicle *i* as possessed by vehicle *j* at time *t* is the time elapsed since the timestamp of the state of *i*  $(j.T_i)$ . Thus  $S(j, i, t) = t - j.T_i$ .

Note that the staleness with respect to a vehicle is initially equal to the message latency from the source when information about the vehicle is received, but continues to rise until the next update from the vehicle is received. The maximum staleness with respect to a vehicle thus occurs just before fresh information about the vehicle is received. Maximum staleness thus depends on the message latency as well as on the update interval. Staleness can further increase when messages containing information about a vehicle are lost. The effect of message losses will be quantified using simulations in Section IV-E.

**Theorem III.2.** The maximum staleness in the state of vehicle *i* at vehicle *j* in VCAST is bounded by  $O(\frac{d(i,j)^2}{pr_h^2}).$ 

Proof: Consider a vehicle j at a distance between  $hr_h$  and  $(h-1)r_h$  away from a vehicle i. Let us denote this by a distance-interval h from vehicle i in terms of the communication range  $r_h$ . Thus  $h = \lceil \frac{d(i,j)}{r_h} \rceil$ . Note that at a distance-interval of h-1 from a vehicle i, information about i is updated only once every h-1 broadcast intervals. As a result the maximum time before which j hears fresher information about i from some vehicle at distance interval h-1 away from i is bounded by  $\frac{h-1}{p}$  seconds. Likewise, the maximum time elapsed between a vehicle at distance-interval h-2 receiving information about i and a vehicle at distance-interval h-1 obtaining the same information is bounded by  $\frac{h-2}{p}$  seconds. Summing from a distance-interval of 1 to h, we get that the maximum latency in communicating the state of vehicle i to a vehicle j is bounded by  $O(\frac{d(i,j)^2}{pr_h^2})$ .

From Theorem III.2, we note that the staleness at a distance d from a vehicle does not depend upon the vehicular density or the region size, but only on the communication hop distance and the initial broadcast rate at the source. Hence, we expect that as the communication range increases the staleness should decrease. However, as the communication range increases, the interference within the vehicular transmission also increases and this can adversely affect the network reliability and system accuracy. In Section IV-E, we analyze the performance of our algorithm in large scale simulation and point out the impact of p and  $r_h$  on the staleness at different distances.

We now state the average communication rate and staleness when aggregate cell information is propagated instead of actual vehicle location information.

**Theorem III.3.** The amount of data communicated by each node in VCAST per unit time, when aggregate cell information is propagated, to obtain information about each cell at distance up to R from itself is bounded by  $O(\frac{Rp}{\rho r_c r_p^2})$ .

Proof: Let  $B_c$  denote the average amount of data communicated per unit time, by each node in VCAST when aggregate cell information is propagated. Recall that  $d_c(i, j)$  denotes the *cell distance* between vehicle *i* and cell *j*, i.e., the smallest number of cells traversed to reach cell *j* from *i*. The number of cells at a cell distance of at most  $d_c$  from a vehicle is bounded by  $O(\frac{\rho d_c^2 r_c^2}{\rho r_c^2})$ , i.e.  $O(d_c^2)$ . As a result the number of cells at exactly distance  $d_c$  away from a vehicle is bounded by  $O(d_c)$ . Information about cells at distance  $d_c$  away is broadcast at  $\frac{p}{d_c}$  Hz. Thus the total amount of data to be communicated about cells at distance  $d_c$  away is O(p). Note that this information is broadcast by at most one vehicle in every area  $\pi r_h^2$ . Hence, on average each node is responsible for communicating only  $O(\frac{p}{\rho r_h^2})$  bits about cells at distance  $d_c$  away from itself. Summing up over all cell distances from  $d_c = 1$  to  $d_c = \frac{R}{r_c}$ , we get:

$$B_c = O(\sum_{i=1}^{\frac{R}{r_c}} \frac{p}{\rho r_h^2}) = O(\frac{Rp}{\rho r_c r_h^2})$$

In comparison with the result from Theorem III.1, we note that the average communication rate reduces by a factor equal to the number of vehicles in each cell ( $\rho r_c r_h$ ) because only aggregate information about the cell is propagated with distance-sensitive rate as opposed to propagating information about each vehicle in the cell.

**Theorem III.4.** The staleness in the state of cell z at vehicle j in VCAST, when aggregate cell information is propagated, is bounded by  $O(\frac{d_c(j,z)d_h(j,z)}{p})$ , where  $d_h(j,z)$  denotes the communication hop distance between j and the closest vehicle to j in cell z.

*Proof:* Consider a vehicle j and cell z. The lowest broadcast rate about cell z that is available for vehicle j is  $\frac{d_c(j,z)}{p}$ . Also the number of communication hops between j and closest vehicle in z is  $d_h(j, z)$ . Hence, latency in forwarding information about cell z to vehicle j is bounded by  $\frac{d_c(j,z)d_h(j,z)}{p}$ . Also note that the subsequent update about cell z will be available within  $\frac{d_c(j,z)}{p}$  seconds. The result follows.

## **IV. PERFORMANCE**

We evaluate the performance of *VCAST* using simulations in ns-3, a discrete event simulator for wireless and mobile ad-hoc networks.

**Network models:** We use an IEEE 802.11b physical layer communication model with a DSSS rate of 11Mbps at each node. We first model vehicular traffic by considering vehicles to be in grids of different sizes (28x28, 25x25, 20x20 and 15x15) with uniform separation between the vehicles at all times (i.e., traveling with uniform speed). For the case of aggregated data forwarding, we have also simulated a network size of 3600 nodes which allows us to test over larger inter-vehicular distances. We consider inter-vehicular separations of 60m, 50m, 40m and 30m, thus simulating different densities. Note that a uniform separation of 45m would correspond to a headway time of 1.5s at a speed of 70 mph (and a proportionately lower headway time at lower speed), which are typically observed separations on roadways. We have chosen densities that create vehicular separation around this range. We also consider source broadcast rates of 1Hz, 5Hz and 10Hz, and communication ranges of 75m, 100m, 125m and 150m, which are within the range of expected transmission rate and range values of *Here I am* messages for intelligent transportation systems [1], [2]. These simulations characterize the performance of *VCAST* when it is used

to disseminate vehicular state information to a network of the corresponding size. The grid model allows us to emulate 2-d network traffic of different densities in a region and by simulating uniform velocity, the relative distance between vehicles is maintained during the course of the simulation allowing the clear characterization of distance versus information staleness. We use this model to compare *VCAST* with non distance-sensitive approaches and to systematically study the impact of density, source broadcast rates and communication range on the performance. Next, we consider a 2-d model of vehicles with non uniform mobility incorporated - the goal here is to study if time varying densities impact the performance and therefore we use a random way-point model to simulate mobility which generalizes possible vehicular traffic patterns.

**Measurement strategy:** Each vehicle's state information is assumed to be 10 bytes long. In any given slot, a vehicle may transmit a variable number of such records. At each vehicle, we measure the maximum staleness with respect to every other vehicle by measuring the time elapsed since the information originated at the source, just before fresh information about a vehicle is received. For information aggregation, we consider the cell size equal to the communication range of each vehicle and propagate only aggregate cell information instead of individual vehicle information, and measure the maximum staleness with respect to every cell in the region. All simulations are run for 20 seconds except simulations at 1Hz which are run for 40 second due to higher staleness values. We then group the maximum staleness based on pairwise distances between vehicle-vehicle and vehicle-cell respectively. The average of these measurements over multiple experiments are used in our evaluations.

## A. Impact of distance-sensitivity

The objective of this section is to quantify the performance gains achieved by using distance-sensitive forwarding as opposed to forwarding information about all vehicles at the same rate.

Scaling in number of nodes: In Fig. 3(a), we show the maximum staleness as a function of inter-vehicular distance for p = 10Hz at a network size of 225 nodes for both VCAST as well as non distance-sensitive

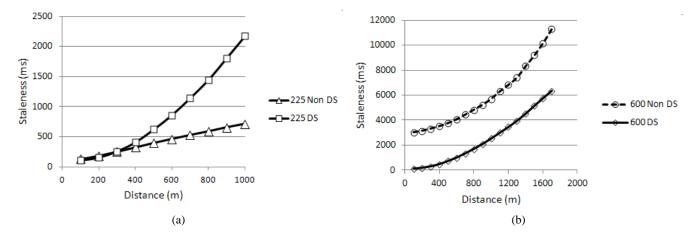


Fig. 3. Impact of network size: VCAST vs non distance-sensitive forwarding. (a) Maximum staleness vs pair-wise vehicular distance: p = 10Hz at 225 nodes (b) Maximum staleness vs pair-wise vehicular distance: p = 10Hz at 625 nodes

forwarding, i.e. information about all vehicles is forwarded at the source broadcast rate by every node. Here, we observe that the non distance-sensitive scheme is able to keep staleness low at all distances and the growth is linear with distance. On the other hand, VCAST has low staleness at small distances while the staleness is observed to grow as  $O(d^2)$  at higher distances, as expected. However, at a network size of 625 (see Fig. 3(b)), the non distance-sensitive approaches show much higher staleness even at smaller distances. However, VCAST is able to maintain low staleness at small distances while the staleness is observed to grow as  $O(d^2)$  at higher distances. Information within 400m is obtained at lower than 300ms using VCAST while it takes about 3 seconds in the case of a non distance-sensitive approach. The reason is that as the number of nodes increases, the channel contention increases at a much higher rate in the non distance-sensitive forwarding causing message losses and consequently increase in staleness (this is quantified in our analysis on message complexity). By reducing channel contention, VCAST is able to achieve scalability in number of nodes. In Fig. 4, we show the maximum staleness as a function of inter-vehicular distance for p = 10Hz at a network size of 225, 400, 625 and 784 nodes for VCAST. We observe that staleness values are preserved at corresponding inter-vehicular distances, irrespective of network size.

**Scaling in source broadcast rate:** In Fig. 5(a) and Fig. 5(b), we fix the network size to 784 nodes and vary the source broadcast rate. In Fig. 5(a), we observe that at s source broadcast rate of 1Hz, the non

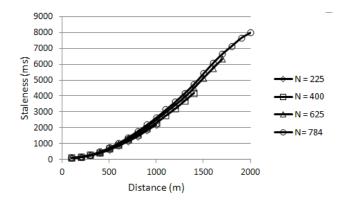


Fig. 4. Maximum staleness vs pair-wise vehicular distance for VCAST: p = 10Hz, network sizes of 225, 400, 625 and 784 vehicles, Communication range 100m, separation 50m: staleness values are preserved at corresponding inter-vehicular distances, irrespective of network size.

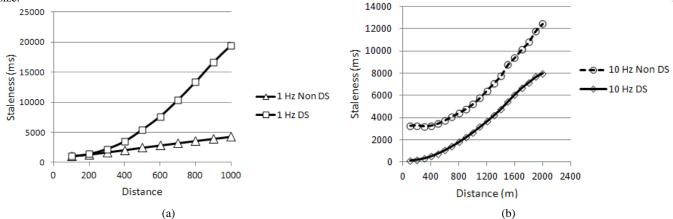


Fig. 5. Impact of source broadcast rate: VCAST vs non distance-sensitive forwarding. (a) Maximum staleness vs pair-wise vehicular distance: p = 1Hz at 784 nodes (b) Maximum staleness vs pair-wise vehicular distance: p = 10Hz at 784 nodes

distance-sensitive scheme is able to scale to large distances and has lower latencies at larger distances compared with VCAST since there is no staggered forwarding. However, when attempting to reduce the staleness at smaller distances, by increasing the source broadcast rate, non distance-sensitive forwarding fails. This is shown in Fig. 5(b) for broadcast rates of 10Hz. The non distance-sensitive approaches show much higher staleness even at smaller distances. On the other hand, VCAST is able to deliver lower staleness at smaller distances even at a network size of 784 nodes by increasing the source broadcast rate and progressively increasing the staleness at larger distances. This allows the network bandwidth to be utilized where it is needed and achieve scalability.

**Message complexity - the reason behind successful scaling in VCAST:** In Fig. 6(a) and Fig. 6(b), we show the number of vehicular records transmitted per second by every node for varying network sizes

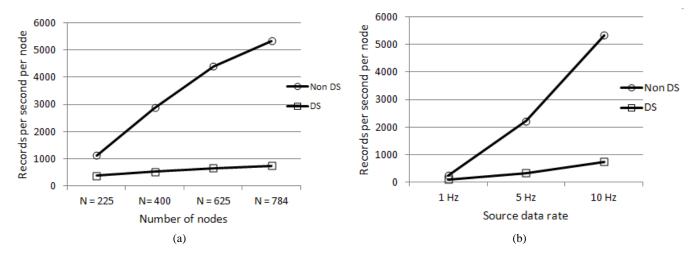


Fig. 6. Comparison of message complexity: VCAST vs non distance-sensitive forwarding. (a) Number of vehicular records transmitted per second per node at different network sizes for p = 10Hz and range 100m. (b) Number of vehicular records transmitted per second per node at different source broadcast rates for 784 nodes and range 100m

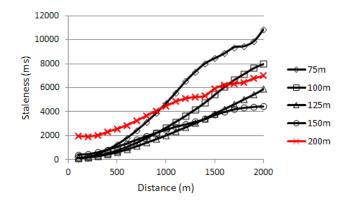


Fig. 7. Impact of communication range on VCAST: Maximum staleness vs pair-wise vehicular distance for VCAST: p = 10Hz, 784 vehicles, separation 50m, Communication range 75m, 100m, 125m, 150m, and 200m,

and varying source broadcast rates respectively. These figures highlight the significantly lower message complexity in *VCAST* which reduces the contention even as network size and broadcast rates grow.

## B. Impact of communication range, source broadcast rate and density

In this subsection, we evaluate the performance of *VCAST* under different communication ranges, source broadcast rate and network densities. The aim is to highlight the scalability of *VCAST* and identify the *tipping point* in terms of each of these parameters for *VCAST*, i.e., the point at which channel capacity is exceeded.

**Communication range:** In Fig. 7, we plot the maximum staleness in vehicle information against the inter-vehicular distance for a 784 node network with p = 10Hz at different communication ranges. As

range increases from 75m to 125m, we observe that staleness at larger distances decrease (due to fewer number of hops), while the staleness at smaller distances increase slightly due to increased contention. However, starting from a range of 150m, we observe that the channel contention within a single hop starts increasing and at a range of 200m, we observe a tipping point leading to higher message losses and much higher staleness at smaller distances (the corresponding plot is shown in red).

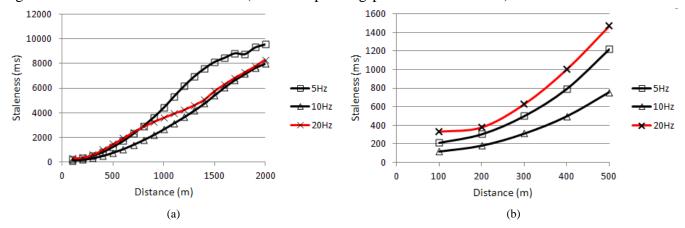


Fig. 8. Impact of source broadcast rate on VCAST: (a) Maximum staleness vs pair-wise vehicular distance with 784 vehicles, separation 50m, Communication range 100m, source broadcast rates of 5Hz, 10Hz and 20Hz (b) Zoomed in version at smaller distances

**Source broadcast rate:** In Fig. 8(a), we plot the maximum staleness in vehicle information against the inter-vehicular distance for a 784 node network with a communication range of 100m for different source broadcast rates. As rate increases from 5 Hz to 10Hz, we observe that staleness decreases as expected. However, with a rate of 20Hz, we observe that the channel contention starts increasing, and leading to disproportionate staleness values at smaller distances. Fig. 8(b) shows a zoomed in version of Fig. 8(a) at smaller distances. Here, we observe that staleness values for 20Hz are higher than those at 10Hz due to higher channel contention.

**Network density:** In Fig. 9(a), we plot the maximum staleness in vehicle information against the intervehicular distance for a 784 node network with a communication range of 100m and rate of 10Hz for different inter-vehicular separations. Note that in Fig. 9(a), the plots for separations of 50m, 40m and 30m only extend to 2, 1.7 and 1.2 Km respectively because these are the maximum inter-vehicular distances possible at the corresponding densities for a given network size of 784 vehicles. We observe that going

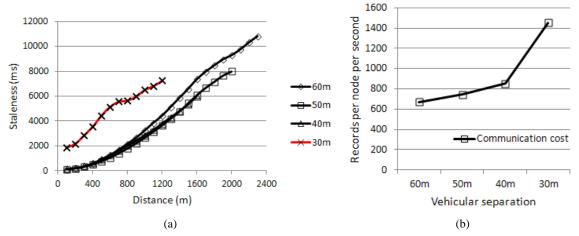
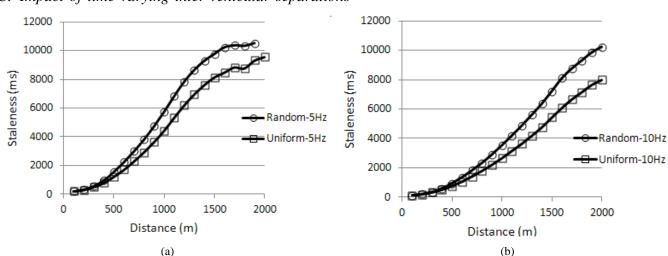


Fig. 9. Impact of network density on VCAST: (a) Maximum staleness vs pair-wise vehicular distance with 784 vehicles, Communication range 100m, source broadcast rat 10Hz, separation 60m, 50m, 40m and 30m (b) Number of vehicular records transmitted by each node per second with 784 vehicles, Communication range 100m, source broadcast rat 10Hz, separation 60m, 50m, 40m and 30m

from a separation of 60m to 40m, the staleness at larger distances decreases. This is because of fewer number of *hops* required to reach a given distance. However, with a separation of 30m, we observe that channel capacity is exceeded causing message losses and higher staleness values (the corresponding plot is shown in red). In Fig. 9(b), we have shown the corresponding message complexity at different intervehicular separations. Here we observe a drastic increase in messages per second, at a separation of 30m which explains the corresponding increase in message losses and information staleness.



C. Impact of time varying inter-vehicular separations

Fig. 10. Impact of time varying random mobility pattern on VCAST: (a) Maximum staleness vs pair-wise vehicular distance for uniform and random mobility with 784 vehicles, Communication range 100m, source broadcast rate 5Hz (b) Maximum staleness vs pair-wise vehicular distance for uniform and random mobility with 784 vehicles, Communication range 100m, source broadcast rate 5Hz (b) Maximum staleness vs pair-wise vehicular distance for uniform and random mobility with 784 vehicles, Communication range 100m, source broadcast rate 5Hz (b) Maximum staleness vs pair-wise vehicular distance for uniform and random mobility with 784 vehicles, Communication range 100m, source broadcast rate 10Hz

The aim of this subsection is to characterize the performance of VCAST in the presence of time varying

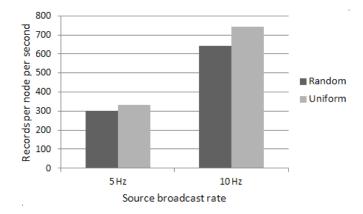


Fig. 11. Impact of time varying random mobility pattern on VCAST (comparison of communication cost): Number of vehicular records transmitted by each node per second at 5 Hz and 10 Hz source broadcast rates under uniform and random mobility patterns

inter-vehicular separations caused by non-uniform mobility patterns. We note that there are potentially several traffic mobility patterns that can arise in a real vehicular network scenario. Our goal here is to analyze the scalability of *VCAST* and one of the important underlying factors that can impact performance in the context of these mobility patterns is the time varying density. There could be instants when a vehicle has lot of neighbors within communication range and also instances when there are no neighbors. Therefore, in this simulation, we have chosen the *random 2d walk mobility pattern* with time varying speeds in the range of 20m/s to 40m/s, a potentially severe form of mobility that captures the essence of time varying separations and interference zone, caused in a traffic scenario.

In Fig. 10(a) and Fig. 10(b), we compare the information staleness graphs for the random and uniform mobility patterns at 5 Hz and 10 Hz source broadcast rates with 784 nodes. As seen from these figures, the graphs are quite similar, highlighting that random mobility does not significantly impact the performance. Staleness values at larger distances are observed to slightly higher for the case of random mobility. However, the reason for this is not higher communication cost but rather the fact that the severe random mobility scenario often creates sparse and disconnected regions within the network, thereby increasing the number of hops traversed between two vehicles at a given distance. This effect is more pronounced at larger distances. In Fig. 11, we compare the average communication cost for the random and uniform mobility scenarios. This graph highlights the fact that communication costs do not increase in the random mobility scenario - in fact a small decrease is observed in the average communication cost over several

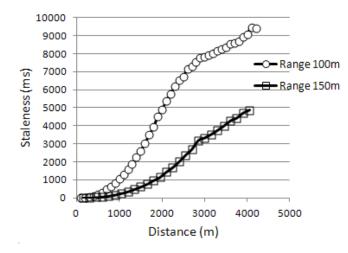


Fig. 12. Impact of aggregation on VCAST: Maximum staleness vs vehicle-cell distance with 3600 vehicles, source broadcast rat 10Hz, Communication range 100m and 150m

#### random patterns.

## D. Impact of aggregation

In this section, we evaluate *VCAST* when aggregate information about a cell is transmitted over multiple hops as opposed to individual vehicle information. The region being simulated is divided into square *cells* of equal size. The width of each cell is assumed to be equal to the single hop communication range. In this evaluation, we have increased the network size to 3600 nodes, thereby allowing us to evaluate staleness over larger distances. Fig. 12 shows the information staleness graphs with a communication range of 100m ad 150m, both with a source broadcast rate of 10Hz. As seen in these graphs, information up to 4 Km can be obtained in less than 5 seconds, without the need for any infrastructure - while still maintaining extremely low staleness values at smaller distances.

## E. Summary of results

Our experimental results show that by using distance-sensitive forwarding, *VCAST* is able to scale to larger network size as well as support much higher broadcast rates compared to non distance-sensitive techniques. The reason for scalability is the significantly lower message overhead which reduces the channel contention in the network. We also characterized the performance of *VCAST* under severe mobility

and studied information staleness when aggregated data is transmitted as opposed to individual vehicle information.

From all of the above experimental evaluations, we note that an ideal parameterization for *VCAST* is to set communication range to a small value and increase the source broadcast rate. By doing so, information at smaller distances can be obtained at high rates with low contention due to the small communication range. At the same time, information to larger distances can be transmitted with progressively increasing staleness. Moreover, by limiting the propagation of individual vehicular information up to distances of about 500 - 1000m (for utilization by safety applications) and only propagating aggregate information beyond that, we observe that information from several miles ahead can be obtained within a few seconds without the need for any communication infrastructure.

## V. CONCLUSIONS

We have presented an algorithm, *VCAST*, for obtaining individual vehicle location and aggregate traffic information over a multi-hop wireless vehicular network without the need for expensive road-side infrastructure, any special hardware or modification to vehicular transmission standards. To ensure scalability in forwarding information over multiple hops, traffic information is propagated at a rate that decreases linearly with distance from the source. By doing so, the required communication rate per node can be reduced when compared with schemes that do not utilize distance-sensitivity in information forwarding. This results in lower channel contention, thereby enabling higher source broadcast rates and better information quality at smaller distances while still being able to propagate information to large distances. Despite staggered forwarding, traffic information can be obtained with a staleness, which is a measure of error in the traffic information, that is bounded by  $O(d^2)$  where d is the communication hop distance from the source of the information.

The performance of VCAST was validated using extensive ns-3 simulation under different network sizes, network densities, source broadcast rates and communication ranges. The results of our evaluation showed

that by using distance-sensitive forwarding, *VCAST* is able to scale to larger network sizes as well as support much higher broadcast rates compared to non distance-sensitive techniques. The reason for scalability was shown to be the significantly lower message overhead which reduces the channel contention in the network. We also characterized the performance of *VCAST* under severe mobility and studied information staleness when aggregated data is transmitted as opposed to individual vehicle information.

In future work, we would like to integrate *VCAST* with control algorithms for vehicular safety and navigation that utilize information with distance-sensitive quality. We would like to design optimal control laws for vehicular acceleration under models of distance-sensitive information availability, that ensure the safety of the integrated control-communication system. We would also like to integrate our vehicular traffic mapping service with a navigation front-end for dynamic computation of alternate routes and evaluate the impact of distance sensitivity on the quality of navigation performance.

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