Math 129: Pre-Calculus **Name (Print):** _____
Fall 2024
Practice Problems for Cumulative Algebra Exam

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1. Simplify the expression $\left(\dfrac{2x^3y^{-1}}{y^2}\right)^{-2}$ by eliminating any negative exponents.

2. Write the expression $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}}$ using rational exponents and simplify.

3. Write the expression $\dfrac{8}{\sqrt[3]{x^2}}$ with a rational denominator.

4. Simplify the sum $\sqrt{24} + \sqrt{56}$.

5. Find the product $(x + 3y)(2x - y)$ and simplify.

6. Find the product $(x + 2)(x^2 + 2x + 3)$ and simplify.

7. Factor $3x^3 - x^2 - 12x + 4$ completely.

8. Factor $8x^2 + 10x + 3$ completely.

9. Factor $6x^2 - 5x - 6$ completely.

10. Compute $\dfrac{x^2 + x - 6}{x^2 + 4x - 12} \div \dfrac{x + 3}{x - 1}$ and write your answer in lowest terms.

11. Compute $\dfrac{4x}{x + 2} - \dfrac{2 + 3x}{x + 2}$ and write your answer in lowest terms.

12. Compute $\dfrac{2x}{x^2 - 16} - \dfrac{3}{x^2 + 8x + 16}$ and write your answer in lowest terms.

13. Find the distance between the points $(6, -2)$ and $(-4, 5)$.

14. Find the midpoint of the segment that joins the points $(5, -1)$ and $(3, 5)$.

15. Find an equation of the circle of radius 5 centered at $(-2, 3)$.

16. Find the center and radius of the circle with equation $x^2 + (y - 2)^2 = 36$.

17. Find the center and radius of the circle with equation $x^2 + y^2 + 6y + 2 = 0$.

18. Find an equation of the line with slope 7 that passes through $(4, -1)$.

19. Find an equation of the line that passes through $(6, 2)$ and $(-4, 3)$.

20. Find an equation of the line that passes through $(3, -1)$ that is parallel to $y = 6x + 1$.

21. Find an equation of the line that passes through $(1, 1)$ that is perpendicular to $2x + y = 4$.

22. Solve the equation $6 - 4x = 10$.

23. Find all real and complex solutions to $x^2 + 14x = 32$.

24. Find all real and complex solutions to $2x^2 + 6x - 5 = 0$.

25. Find all real and complex solutions to $3x^2 - 2x + 1 = 0$.

26. Find all real solutions to $\dfrac{6}{x^2-1} - \dfrac{3}{2} = \dfrac{3}{x-1}$.

27. Find all real solutions to $\dfrac{2}{x+3} + \dfrac{3}{8} = \dfrac{5}{4x+12}$.

28. Find all real solutions to $x^6 - 2x^3 - 3 = 0$.

29. Find all real solutions to $x^{3/2} - 10x^{1/2} + 25x^{-1/2} = 0$.

30. Find all real solutions to $x^2\sqrt{x+3} = (x+3)^{3/2}$.

31. Find all real solutions to $x^5 - x^3 - 2x = 0$.

32. Solve the inequality $2 - 5x < 7$.

33. Solve the inequality $-4 < 2x - 4 \leq -2$.

34. Solve the equation $|8 - 3x| = 1$.

35. Solve the inequality $|4x + 1| \geq 21$.

36. Solve the inequality $x^2 + 5x + 6 > 0$.

37. Solve the inequality $2x^2 + x \geq 1$.

38. Consider the function $f(x) = x^2 - 4x$. Evaluate $f(x-3)$ and simplify.

39. Find the domain of the function $f(x) = \sqrt{4 - x^2}$.

40. Find the domain of the function $f(x) = \dfrac{x-1}{x^2 + 3x - 10}$

41. Find the domain of the function $f(x) = \dfrac{5x}{\sqrt{x-1}}$.

42. Find the average rate of change of the function $f(x) = 6x - x^2$ from $x = 1$ to $x = 4$.

43. If $f(x) = 3\sqrt{x-4}$ and $g(x) = x^2 - 1$, find the formula for $(f \circ g)(x)$.

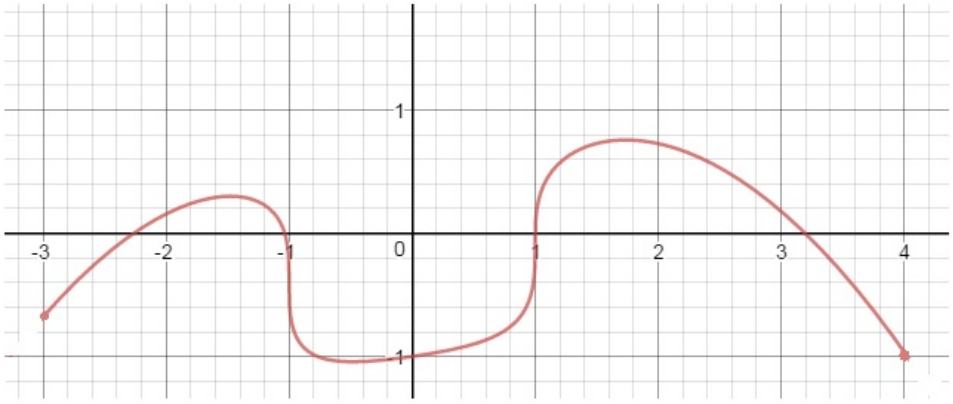
44. If $f(x) = 3\sqrt{x-4}$ and $g(x) = x^2 - 1$, find the formula for $(g \circ f)(x)$.

45. If $f(x) = 13x^{5/3} - 1$, find the formula for $f^{-1}(x)$.

46. If $f(x) = \dfrac{2x+1}{3x-7}$, find the formula for $f^{-1}(x)$.

47. Sketch a graph of the function $f(x) = \sqrt[3]{x^2 - 1}$ by making a table of values and plotting some points.

Consider the following graph of a function, $y = f(x)$.



48. Find the domain of f.

49. Find, approximately, the range of f.

50. Find, approximately, the intervals where f is increasing.

51. Find, approximately, the intervals where f is decreasing.

52. Find, approximately, the intervals on which $f(x) > 0$.

53. Find the approximate coordinates of any local maxima of f.

54. Find the approximate coordinates of any local minima of f.

55. Is f a one-to-one function?

56. Sketch the graph of $y = f(2 - x) + 1$.

57. Solve the inequality $\dfrac{x^2-9}{x^3+x^2-4x-4} > 0$.

58. Write the standard form of the quadratic function $f(x) = 2x^2 - 8x + 4$.

59. Find the coordinates of the vertex of the graph of $y = x^2 - 5x + 2$.

60. Find the maximum or minimum value of $f(x) = 3x^2 - 8x + 4$.

61. Determine the end behavior of the function $f(x) = 3x^4 - 4x^3 - 10x - 1$.

62. Consider the function $f(x) = x^4 + x^3 - 2x^2$. Find all real zeros of f, state their multiplicities, and sketch the graph of f.

63. Consider the function $f(x) = x - x^3$. Final all real zeros of f and their multiplicities, determine the end behavior of f, and sketch the graph of f.

64. Find the quotient and remainder of the division $\dfrac{x^4 - 2x^2 + 7x}{x^2 - x + 3}$.

65. Find the quotient and remainder of the division $\dfrac{x^2 - 5x + 4}{x - 3}$.

66. Let $f(x) = x^5 - 2x^4 - 9x^3 + 22x^2 + 4x - 24$. Suppose that you know that 2 is a zero of f of multiplicity 3. Use this information to completely factor f.

67. Find a polynomial of degree 3 with integer coefficients and zeros at $\frac{1}{2}$, -1, and 2.

68. Let $f(x) = \dfrac{1}{(x+2)^2}$. Find all zeros of f, vertical asymptotes of f, and horizontal asymptotes of f. Find the behavior of the graph near the vertical asymptotes, and use this to sketch a graph of f.

69. Let $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 8}$. Find all zeros of f, vertical asymptotes of f, and horizontal asymptotes of f. Find the behavior of the graph near the vertical asymptotes, and use this to sketch a graph of f.

70. Find the slant asymptote of $f(x) = \dfrac{x^3 + 3x + 4}{x^2 - 3x - 3}$.

71. Let $f(x) = 4e^{4-x}$. Use a calculator to find $f(-1)$, rounded to three decimal places.

72. Write the equation $\log_6(36) = 2$ in exponential form.

73. Write the equation $4^x = 20$ in logarithmic form.

74. Without a calculator, determine the value of $\ln\left(\dfrac{1}{e}\right)$.

75. Without a calculator, determine the value of $\ln(\sqrt{e})$.

76. Without a calculator, determine the value of $\log_8(4)$.

77. Let $f(x) = 7\log_3(x + 2)$. Use the change of base formula and a calculator to find $f(2)$, rounded to three decimal places.

78. Find the domain of the function $f(x) = \ln(8 - 2x)$.

79. Find the domain of the function $f(x) = \dfrac{1}{\log_2(x)}$.

80. Use the log laws to expand $\log_3\left(\dfrac{(x+4)\sqrt{2x}}{(x+1)^7}\right)$.

81. Write as a single logarithm using the log laws: $\ln(4x) - 2\ln(x - 1) - 6\ln(x + 2)$.

82. Solve the equation $3^{x-4} = 27$.

83. Solve the equation $e^{4x} + 4 = 9$.

84. Solve the equation $2^{2x} - 2^x - 12 = 0$.

85. Solve the equation $2^{x^2} \cdot (2^x)^2 \cdot \frac{1}{8} = 1$.

86. Solve the equation $4^{5x-3} = 3^{4x-5}$.

87. Solve the equation $\log_{10}(2x - 3) + 1 = 0$.

88. Solve the equation $\log_3(x^2 - 4) + \log_3(x) = \log_3(x - 2)$.

89. Solve the equation $\log_8(x + 5) - \log_8(x - 2) = 1$.

90. Solve the equation $2\ln(x + 2) + \ln(x - 2) = \ln x + \ln(x + 1) + \ln(x - 1)$.

91. Solve the system of equations

$$\begin{cases} x - 2y = 7 \\ 2x + 3y = -5 \end{cases}$$

92. Solve the system of equations

$$\begin{cases} x - 3y + z = -1 \\ 2x + 2y - 3z = 14 \\ 2z = -8 \end{cases}$$

93. Solve the system of equations

$$\begin{cases} x^2 - 2y = 6 \\ 2x^2 + 8y^2 = 32 \end{cases}$$

94. You invest \$500 into an account with an annual interest rate of 8% that compounds monthly. How much money will be in your account after 2 years?

95. You invest \$800 into an account with an annual interest rate of 10% that compounds continuously. How much money will be in your account after 3 years?

96. You and your friend are each investing \$1000 into bank accounts. Your account has an annual interest rate of 6% and compounds continuously. Your friend's account compounds monthly, but you don't know the interest rate. At the end of 1 year, you end up with the exact same amount of money in your account as your friend has in her account. Find the annual interest rate of your friend's account.