

Math 155: Calculus I
Spring 2025
Practice Problems for Final Exam

Name (Print): _____

1. Compute $\lim_{x \rightarrow 2} \frac{x^2 - 2}{x^3 + 4x + 3}$.
2. Compute $\lim_{x \rightarrow 0} e^x$.
3. Compute $\lim_{x \rightarrow 4} \log_2(x)$.
4. Compute $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x^2 - 9}$.
5. Compute $\lim_{x \rightarrow -1} \frac{\sqrt{2x+3} - 1}{x + 1}$.
6. Compute $\lim_{x \rightarrow 0} \frac{(x+2)^3 - 8}{x}$.
7. Compute $\lim_{x \rightarrow 0} \left(\frac{1}{x\sqrt{x+1}} - \frac{1}{x} \right)$.
8. Compute $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$.
9. Compute $\lim_{x \rightarrow 0^+} \ln(x)$.
10. Compute $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$.
11. Compute $\lim_{x \rightarrow \infty} \frac{2x^3 - 4x + 18}{x^3 + 20x^2 + 11x + 13}$.
12. Compute $\lim_{x \rightarrow -\infty} \frac{5x^2 + 10x - 1}{x^3 + 2}$.

13. Compute $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 1}{x^3 - 1}$.

14. Compute $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin(x)}$.

15. Compute $\lim_{x \rightarrow \infty} \frac{\ln(x)}{\sqrt{x}}$.

16. Compute $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{\sqrt{x}}$.

17. Compute $\lim_{x \rightarrow -1} \frac{x^9 + 1}{x^5 + 1}$.

18. Compute $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x}$.

19. Compute $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$.

20. Compute $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{\sin(x) + e^x - 1}$.

21. Compute $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$.

22. Compute $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$.

23. Compute $\lim_{x \rightarrow \infty} x^{1/x}$.

24. Compute $\lim_{x \rightarrow 0^+} (1-x)^{1/x}$.

25. Use the Squeeze Theorem to find $\lim_{x \rightarrow 0} x^2 \sin(\frac{\pi}{x})$.
26. Let $f(x) = \frac{x^2 - 16}{2x^2 + 8x}$. Find any discontinuities and determine whether they are removable, jump, or infinite.
27. Let $f(x) = \frac{x^2 + 2x + 1}{x^2 - 1}$. Find any discontinuities and determine whether they are removable, jump, or infinite.
28. Let $f(x) = x^2 + 3x - 2$. Use the limit definition of the derivative to compute $f'(4)$.
29. Let $f(x) = \frac{2}{x + 3}$. Use the limit definition of the derivative to compute $f'(x)$.
30. Let $f(x) = x \ln(x^2)$. Find $f'(x)$.
31. Let $f(x) = e^{-x^2}$. Find $f'(x)$.
32. Let $f(x) = \sin(x)e^{-\cos(x)}$. Find $f'(x)$.
33. Let $f(x) = \frac{6x}{\ln(x)}$. Find $f'(x)$.
34. Let $f(x) = \sqrt{\sec(x)}$. Find $f'(x)$.
35. Let $f(x) = \ln(x) \ln(\ln(x))$. Find $f'(x)$.
36. Let $f(x) = e^x e^{(e^x)}$. Find $f'(x)$.
37. Let $f(x) = \tan(e^x) e^{-x}$. Find $f'(x)$.
38. Let $f(x) = \cosh(x^2)$. Find $f'(x)$.

39. Find the equation of the line tangent to the curve $y = e^{\sin(x)}$ at the point $(0, 1)$.
40. Let $f(x) = xe^x$. Find $f''(x)$.
41. Consider the curve $x - 2y = \ln(xy)$. Find $\frac{dy}{dx}$.
42. Consider the curve $(1 + x)^2 y^2 = e^x$. Find the equation of the tangent line to this curve at the point $(0, -1)$.
43. Let $f(x) = \int_1^{\ln(x)} e^t dt$. Find $f'(x)$.
44. The radius of a spherical bubble is increasing at a rate of 2 mm/min. At what rate is the volume increasing at the moment when the radius is 40 mm? You may use the fact that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.
45. Two people start from the same point. One walks east at 3 mph and the other walks south at 2 mph. How fast is the distance between the people changing after a half hour?
46. Use a linear approximation to the function $f(x) = \ln(x)$ at $a = 1$ to estimate the number $\ln(0.95)$.
47. Use a linear approximation to the function $f(x) = e^x$ at $a = 0$ to estimate the number $e^{-1/2}$.

48. Consider the function $f(x) = x^2e^x$. Find the intervals on which f is increasing or decreasing, find the local maximum and minimum values of f , find the intervals of concavity of f , and find the inflection points of f .
49. Consider the function $f(x) = \frac{x^2 - 3x - 10}{x^2 - 4}$. Find the intervals on which f is increasing or decreasing, find the local maximum and minimum values of f , find the intervals of concavity of f , find the inflection points of f , find any vertical and horizontal asymptotes of f , and sketch the graph of f .
50. A box with a square base and an open top must have a volume of 1000 cm^3 . Find the dimensions of the box that minimize the amount of material used.
51. A cylindrical box must have a volume of 1000 cm^3 . The side of the box is to be made of a material that costs $\$0.10$ per cm^2 and the circular bases of the box are to be made of a material that costs $\$0.50$ per cm^2 . Find the dimensions of the box that minimize the cost of the box. You may use the facts that the volume of a cylinder of radius r and height h is $\pi r^2 h$ and that the surface area of the side of the cylinder is $2\pi r h$.
52. Compute $\int_0^1 e^{2x} dx$.
53. Compute $\int_2^4 4 - \frac{6}{x^2} dx$.
54. Compute $\int_0^\pi \sqrt{x} - \cos(3x) dx$.
55. Compute $\int_0^1 \frac{4}{x^2 + 1} dx$.
56. Compute $\int_0^1 10^x dx$.
57. Compute $\int_0^1 \sinh(x) dx$.
58. Find $\int \frac{1}{x \ln(x) \ln(\ln(x))} dx$.
59. Find $\int \frac{x^2 - 2x + 1}{x} dx$.
60. Find $\int 1 + x^2 + \frac{1}{1 + x^2} dx$.

61. Find $\int \frac{1}{(x+3)^2} dx$.

62. Find $\int \frac{x}{(x+3)^2} dx$.

63. Find $\int \frac{(\tan^{-1}(2x))^2}{4x^2+1} dx$.

64. Find $\int (x+1)\sqrt{x-2} dx$.

65. Find $\int xe^{-x^2} dx$.

66. Find $\int \cot(x) dx$.

67. Find $\int \frac{e^x - 1}{e^x - x} dx$.

68. Find $\int \frac{1+x}{1+x^2} dx$.

69. Find $\int \sinh^2(x) \cosh(x) dx$.

70. Find the area of the region bounded by $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, and $x = 2$.
71. Find the area of the region bounded by $y = \sin(x)$ and $y = e^x$ from $x = 0$ to $x = \pi/2$.
72. Find the volume of the solid whose base is the region bounded by $y = 2^x$ and $y = 1 + 2x - x^2$ and whose cross-sections perpendicular to the x -axis are squares.
73. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = \cos^{-1}(x)$ about the y -axis.
74. Find the volume of the solid obtained by rotation the region between $y = \ln(x)$ and the x -axis from $x = 1$ to $x = 3$ about the x -axis.
75. Find the work done in moving a box from $x = 0$ to $x = 1$ meters if the force acting on it is $F(x) = 3x^2$ N.
76. A spring with spring constant 12 kg/s^2 is stretched from its natural length of 1 m to 110 cm . Compute the work done.
77. A tank is a right circular cone with base radius 3 m and height 6 m , and it is filled with water up to depth 5 m . Compute the work required to pump the water out of the tank. Use 9800 N per m^3 as the density for water.
78. Find the mass of a one-dimensional rod that is 3 feet long (starting at $x = 0$) and has a density function of $\rho(x) = e^{x/2} \text{ lb/ft}$.

79. Find the average value of the function $f(x) = \frac{\ln(x)}{x}$ on the interval $[1, e]$.
80. Find the average value of the function $f(x) = \sec^2(x)e^{\tan(x)}$ on the interval $[0, \pi/4]$.
81. Let $f(x) = \frac{x\sqrt{x}}{e^{\sin(x)}}$. Use logarithmic differentiation to find $f'(x)$.
82. Find the equation of the tangent line to the curve $y = \frac{(x+1)^6 \tan^{-1}(x)}{\cos^{-1}(x)}$ at the point $(0, 0)$.
(Hint: Use logarithmic differentiation.)
83. Let $f(x) = \ln(x) \left(\frac{1}{x^2} \right)$. Use logarithmic differentiation to find $f'(x)$.
84. Let $f(x) = \sin \left(\frac{x}{2} \right)^{\sin(x)}$. Use logarithmic differentiation to find the derivative, and use it to find the equation of the tangent line to the curve $y = f(x)$ at the point $(\pi, 1)$.
85. Let $f(x) = x^7 + 2x^5$. Find $(f^{-1})'(3)$.
86. You have a bowl that is a perfect half-sphere of radius 4 in. Water begins to enter the bowl at a rate of $1 \text{ in}^3/\text{min}$. You want to find the rate at which the water level is rising when the water level is 2 in. To do so, you need to do the following steps:
- (a) First, you want a formula for the volume of water in the bowl when the water level is at a height h . But you notice that this volume happens to be the volume of the solid obtained by rotating the region bounded by the y -axis, the line $y = -4 + h$, and the curve $y = -\sqrt{16 - x^2}$ about the y -axis. Find this volume, in terms of h .
 - (b) Set up a related rates problem using your volume formula from part (a) to compute the solution.