MATH 448 Bonus Homework, due Friday, September 13

Each of the following four problems can be completed for a 1% bonus applied to Exam 1, for a total of at most a bonus of 4%.

- The word game Spelling Bee gives players 7 distinct letters of the alphabet, with one of the 7 designated as the key letter. The object of the game is to make as many words as possible from the letters, but each word must include the key letter. Letters can be repeated to form the words. (For example, if the letters were A B C D E F with key letter G, then the word EGGED would be valid, but the word CAFE would not be valid because it doesn't include the G.) The game only accepts real words in the dictionary, but of course, when you get stuck, you resort to just inputting random strings of letters in the hopes that some of them form real words, and that's what these two problems are about.

    1. A "perfect pangram" is a 7-letter word that contains each of the 7 different given letters exactly once. You try to find a perfect pangram by inputting all 7-letter strings that contain each of the 7 letters exactly once. How many such strings are there?

    2. Next, you're trying to find 4-letter words that can be made from the 7 given letters. Once again, you input random 4-letter strings that contain the key letter. How many possible 4-letter strings are there that contain the key letter? (Remember, letters can be repeated!)

- You take an exam over and over until you pass it. You start with probability p of passing, and on each subsequent attempt, your chance of failing is cut in half. (So if initial probability of passing is 20%, then probability of failing first time is 80%, probability of failing second time is 40%, failing third time is 20%, etc). Let the random variable X be the number of times you have to take the exam.

    3. Write the formula for $P(X = n)$, giving your answer in terms of both p and n.

    4. Using $p = 0.2$, write an expression giving the expected value of X, with the sum written in summation notation, and use it to estimate $E(X)$ within 0.01.