

MATH 448 BONUS HOMEWORK 3, DUE WEDNESDAY, OCTOBER 30

Each of the following four problems can be completed for a 1% bonus applied to Exam 3, for a total of at most a bonus of 4%.

All of these problems concern a random number generator, which is capable of generating as many random numbers as you wish according to any specified probability distribution that you wish. You can assume the numbers are generated independent of each other. In each of the problems, you are asked about the random variable  $X$ , which is defined to be the maximum of all the numbers generated in each question.

1. Suppose you generate 3 random numbers that are uniformly distributed on the interval  $[0, 1]$ . Find the cumulative distribution function for  $X$  by computing  $P(X \leq x)$  for any  $x$ .
2. Once again, you generate 3 random numbers that are uniformly distributed on the interval  $[0, 1]$ . Compute  $E(X)$ .
3. Now, suppose you generate  $n$  different random numbers that are uniformly distributed on the interval  $[0, 1]$ . Compute  $E(X)$ , giving your answer in terms of  $n$ .
4. Finally, suppose you generate  $n$  different random numbers that are exponentially distributed with parameter  $\lambda = 1$ . Compute  $E(X)$ , giving your answer in terms of  $n$ .