Section 5.4: Distributions Related to the Normal Distribution

We won't focus too much on these distributions, except to point out their use in the statistics part of the class. Note that all the tables given for these distributions give critical points in the body of the table rather than probabilities. We'll talk a bit more about how to use these tables when we come across the applications of these random variables in later chapters.

The Chi-Square Distribution.
Suppose $X_1, X_2, \ldots, X_\nu \sim N(0,1)$ are independent. Define $X = X_1^2 + X_2^2 + \cdots + X_\nu^2$. Then X is *chi-square distributed with ν degrees of freedom*, and we write $X \sim \chi^2_\nu$. Then $\mathrm{E}(X) = \nu$ and $\mathrm{Var}(X) = 2\nu$. We won't worry about writing down the density and distribution functions of this random variable. To estimate probabilities, we will use the table on page 789 of the textbook.

The t-distribution.
Suppose $X \sim N(0,1)$ and $Y \sim \chi^2_\nu$ are independent. Define $t_\nu = \dfrac{X}{\sqrt{\frac{Y}{\nu}}}$. Then t_ν is *t-distributed with ν degrees of freedom.*
Note that as $\nu \to \infty$, this becomes a standard normal. Again, for our purposes, we will use the table on page 790 of the textbook to estimate probabilities.

The F-Distribution.
Suppose that $X_1 \sim \chi^2_{\nu_1}$ and $X_2 \sim \chi^2_{\nu_2}$ are independent. Define $F_{\nu_1,\nu_2} = \dfrac{X_1/\nu_1}{X_2/\nu_2}$. Then F_{ν_1,ν_2} is *F-distributed with degrees of freedom ν_1 and ν_2*. To estimate probabilities, we will use the table on pages 791-793 of the textbook.