Homework 3, due Friday, September 12

Please turn in well-written solutions for the following:

(1) Use an ε argument to prove the following limit law:

Suppose $c \in \mathbb{R}$ and suppose f is a function such that $\lim_{x \to a} f(x) = A$. Then $\lim_{x \to a} (cf(x)) = cA$.

(2) Define $f : \mathbb{R} \backslash \{2\} \to \mathbb{R}$ by $f(x) = \dfrac{x^2 - 4}{x - 2}$. Use an ε argument to prove that $\lim_{x \to 2} f(x)$ exists.

(3) Define $f : \mathbb{R} \backslash \{0\} \to \mathbb{R}$ by $f(x) = \dfrac{|x|}{x}$.

(a) Use an ε argument to prove that $\lim_{x \to 0^+} f(x) = 1$.

(b) Use an ε argument to prove that $\lim_{x \to 0^-} f(x) = -1$.

(4) Use the Squeeze Theorem to prove that $\lim_{x \to 0} x \sin\left(\dfrac{1}{x}\right) = 0$.