Homework 9, due Friday, April 25

Please turn in solutions for the following problems:

(1) Each of the following functions has a singular point at $z = 0$. Decide if the singular point is isolated or not, and if it is isolated, determine if it is removable, essential, or a pole.

(a) $f(z) = \dfrac{3z^2 - 4}{z^3}$

(b) $f(z) = ze^{1/z}$

(c) $f(z) = \operatorname{Log}\left(\dfrac{1}{z}\right)$

(d) $f(z) = \dfrac{\sin(z)}{z}$

(e) $f(z) = \dfrac{\cos(z)}{z}$

(2) Find the residue at $z = 0$ of each of the following functions:

(a) $f(z) = e^{1/z^2}$

(b) $f(z) = \dfrac{\sin(z)}{z^3}$

(c) $f(z) = \dfrac{\cot(z)}{z^4}$ (for bonus points)

(3) Let C be the positively oriented circle $|z| = 3$. Compute each integral.

(a) $\displaystyle\int_C \frac{1}{z + z^3}\, dz$

(b) $\displaystyle\int_C \frac{e^z - 1}{z^2}\, dz$

(4) Let $f(z) = \dfrac{2z^2 + 1}{(z+1)(z^2+4)}$. Compute each integral:

(a) $\displaystyle\int_C f(z)\, dz$, where C is the positively oriented circle $|z - i| = 2$

(b) $\displaystyle\int_C f(z)\, dz$, where C is the positively oriented circle $|z| = 3$

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Page 239, problems 1, 2, 3
- Page 243, problems 1, 2
- Page 248, problems 1, 2, 3, 4, 5, 6
- Page 255, problems 1, 2, 3, 4