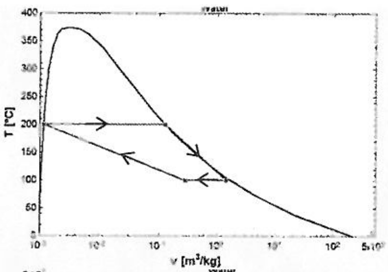
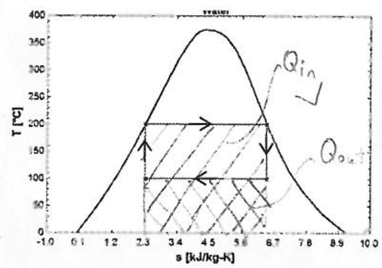
Name: Answer Key

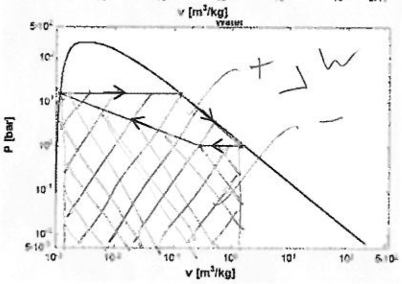
You are allowed three sheets of notes.

1. Fill in the blanks for each of the two (Carnot) cycles below.

(a)





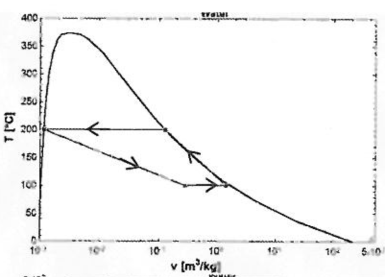


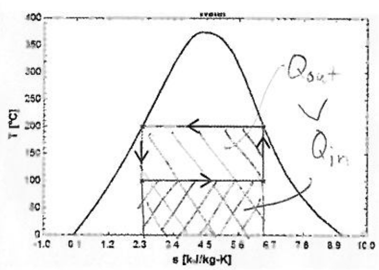
5 a) Heat engine or Heat pump/refrigerator (circle one)

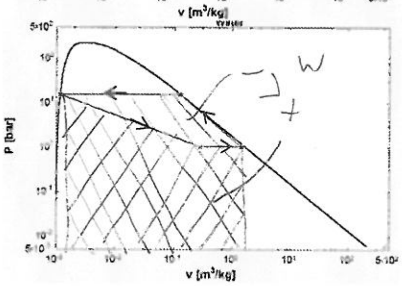
b) Efficiency or Coefficient of performance: $\eta = 21\%$

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{100+273}{200+273} = 0.21$$

(b)







a) Heat engine or Heat pump/refrigerator (circle one)

b) Efficiency or Coefficient of performance: _____

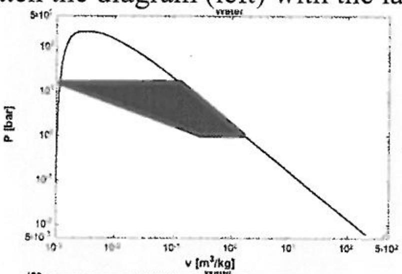
If heat pump $\beta = \frac{T_C}{T_H - T_C} = \frac{100+273}{200-100} = 3.73$

If refrigerator $\gamma = \frac{T_H}{T_H - T_C} = \frac{200+273}{200-100} = 4.73$

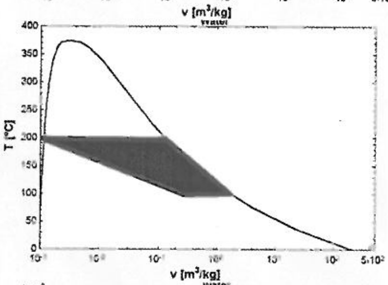
5

2. Match the diagram (left) with the label (right) for the shaded area.

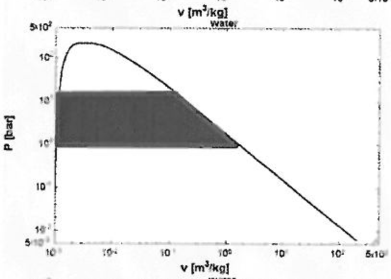
(a)



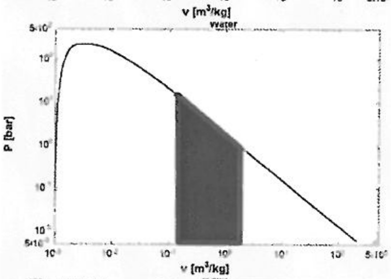
(b)



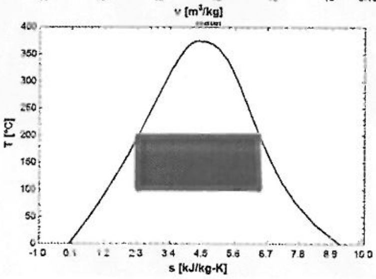
(c)



(d)



(e)



1. Net heat transfer per unit mass
2. Net work per unit mass
3. For closed systems
4. For open systems
5. Nothing we've studied

3. Fifty kJ of heat flows into a closed system at 50 °C and 100 kJ/K of entropy is produced ← σ irreversibly in the system. How much entropy is accumulated in the system?

Closed System

5 Given: $Q_{in} = 50$ kJ   Find: ΔS

$T_{in} = 50°C$

$\sigma = 100$ kJ/K

For constant T_b: $\int_1^2 \left(\frac{\delta Q}{T}\right)_b = \frac{Q}{T_b}$

Solution: $\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma = \left(\frac{Q_{in}}{T_{in}}\right)_{b_1} + \left(\frac{0}{T}\right)_{b_2} + \sigma = \frac{Q_{in}}{T_{in}} + \sigma$

Region of boundary (b_2) that is insulated ($Q=0$) if such a region exists.

Region of boundary with heat flowing in (b_1)

$= \frac{50}{50+273.15} + 100$

$\boxed{\Delta S = 100.15 \text{ kJ/K}}$

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4. Heat flows into a closed system at a rate of 50 kW at 500 °C and flows out at a rate of 30 kW at 20 °C. If the system is in steady state:
   a) At what rate is entropy produced irreversibly in the system? (What rate of)
   b) ~~How much~~ external work is done by the system?

5 Given: Closed System

$\dot{Q}_1 = 50$ kW $\dot{Q}_2 = -30$ kW

$T_1 = 500°C$ $T_2 = 20°C$

$\frac{dS}{dt} = 0$ (steady state)

Find: a) $\dot{\sigma}$

b) $\dot{W}$

Solution a) $\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$

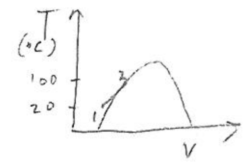
$0 = \frac{50}{500+273.15} - \frac{30}{20+273.15} + \dot{\sigma}$ $\boxed{\dot{\sigma} = 0.0377 \frac{\text{kW}}{\text{K}}}$

b) $\cancel{\frac{dE}{dt}}^{S.S.} = \dot{Q}_{net} - \dot{W} = \cancel{\frac{dKE}{dt} + \frac{dPE}{dt} + \frac{dU}{dt}}$ not helpful

$0 = (50-30) - \dot{W}$ $\boxed{\dot{W} = 20 \text{ kW}}$

5. One kg of water in a closed, expandable container is heated at atmospheric pressure (1.014 bar) from 20 °C to 100 °C. Find:

Find:
a) The change in specific volume. (Δv)
b) The change in internal energy. (Δu)
c) The change in enthalpy. (Δh)
d) The change in entropy. (Δs)
e) The amount of heat added. (Q)
f) The work done to expand the container. (W)



Given: $m = 1\ kg$ - closed system $V \neq$ constant

$Q > 0$ $\quad p = 1.014$ bar

$T_1 = 20°C \quad T_2 = 100°C$

Solution: Using saturated liquid properties for state 1 ($T_1 = 20°C$) (as approx.)
15 and sat. liquid properties for state 2 ($T_2 = 100°C$) from Table A-2:

a) $\Delta v = v_2 - v_1 = 1.0435\times10^{-3} - 1.0018\times10^{-3} = 0.0417\times10^{-3}\ \frac{m^3}{kg}$

b) $\Delta u = u_2 - u_1 = 418.94 - 83.95 = 334.99\ \frac{kJ}{kg}$

c) $\Delta h = h_2 - h_1 = 419.04 - 83.96 = 335.08\ \frac{kJ}{kg}$

d) $\Delta s = s_2 - s_1 = 1.3069 - 0.2966 = 1.0103\ \frac{kJ}{kg\cdot K}$

e) $Q - W = m\left(u_2 - u_1 + \cancel{\frac{V_2^2 - V_1^2}{2}} + \cancel{g(z_2 - z_1)}\right)$

$Q - 0.004228 = 1(334.99)$

$Q = 335\ kJ$

or

$Q = m(h_2 - h_1) = 1(335.08) = \underline{335\ kJ}$

f) $W = m\int_1^2 p\,dv = \underset{\text{for const. } p}{m p \Delta v}$

$= 1(1.014\times10^5)(0.0417\times10^{-3})$

$= 4.228\ J$

$= \underline{0.004228\ kJ}$

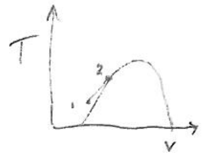
f) $W = m(\Delta h - \Delta u) = 1(335.08 - 334.99) = 0.09\ kJ$

off due to rounding in Δh & Δu.

15

6. One kg of water in a closed, expandable container is heated at atmospheric pressure (1.014 bar) from 20 °C to 100 °C. Using interpolated values from Table A-19 (c_p = 4.186 kJ/kg·K @ 60 °C, ρ = 997.4 kg/m^3 @ 20 °C, and ρ = 958.0 kg/m^3 @ 100 °C), find:

Find:
a) The change in specific volume. (Δv)
b) The change in internal energy. (Δu)
c) The change in enthalpy. (Δh)
d) The change in entropy. (Δs)
e) The amount of heat added. (Q)
f) The work done to expand the container. (W)



Given: $m = 1\ kg$ closed system, expandable, heated

$P_1 = P_2 = 1.014$ bar

$T_1 = 20°C$ $\quad \rho_1 = 997.4\ kg/m^3$ $\quad c_p = 4.186\ \frac{kJ}{kg \cdot K}$

$T_2 = 100°C$ $\quad \rho_2 = 958.0\ kg/m^3$

Solution: a) $\Delta v = v_2 - v_1 = \frac{1}{\rho_2} - \frac{1}{\rho_1} = \frac{1}{958.0} - \frac{1}{997.4} = 4.123 \times 10^{-5} \frac{m^3}{kg}$

b) $\Delta u = c_v \Delta T \approx c_p \Delta T = 4.186\,(100-20) = 334.9 \frac{kJ}{kg}$

for low pressure liquids

c) $\Delta h = c_p \Delta T = 4.186\,(100-20) = 334.9 \frac{kJ}{kg}$

$c = c_v = c_p$ for incompressible liquids

d) $\Delta s = c \ln \frac{T_2}{T_1} = c_p \ln \frac{T_2}{T_1} = 4.186 \ln \frac{100+273.15}{20+273.15} = 1.010 \frac{kJ}{kg \cdot K}$

e) $Q = m(h_2 - h_1) = m\Delta h = 1 \times 334.9 = 334.9\ kJ$

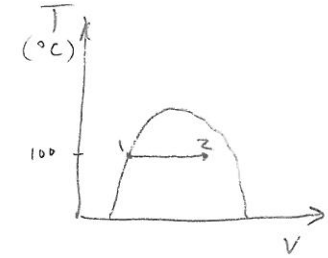
for constant p

f) $W = mp\Delta v = 1 \times 1.014 \times 10^5 \times 4.123 \times 10^{-5} = 4.181\ J = 0.004181\ kJ$

convert to Pa

7. One kg of water in a closed, expandable container at 100 °C is heated at atmospheric pressure (1.014 bar) with 2000 kJ. Find:

Find:
a) The change in specific volume. (Δv)
b) The change in internal energy. (Δu)
c) The change in enthalpy. (Δh)
d) The change in entropy. (Δs)
e) The work done to expand the container. (W)



Given: $m = 1\ kg$ closed system, expandable

$T_1 = 100°C$

$P_1 = P_2 = 1.014\ bar$

$Q = 2000\ kJ$

Solution: $Q - W = m\left((u_2 - u_1) + \cancel{\frac{V_2^2 - V_1^2}{2}} + g(\cancel{z_2 - z_1})\right)$

For constant pressure: $W = mp\Delta v = m(p_2 v_2 - p_1 v_1)$ $h = u + pv$

15

$Q = m(h_2 - h_1)$

$2000 = 1 \times (h_2 - 419.04)$

$h_2 = 2419.04\ \frac{kJ}{kg}$

$X_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{h_2 - h_f}{h_{fg}}$

$= \frac{2419.04 - 419.04}{2257.0}$ ← from Table A-2, T = 100°C

$= 0.8861$

For State 1

$T_1 = 100°C$

$P_1 = 1.014\ bar$

$x_1 = 0$

$v_1 = 1.0435 \times 10^{-3}\ m^3/kg$

$u_1 = 418.94\ \frac{kJ}{kg}$

$h_1 = 419.04\ \frac{kJ}{kg}$

$s_1 = 1.3069\ \frac{kJ}{kg \cdot K}$

From Table A-2

Easier way:

$\Delta v = \Delta x (v_g - v_f)$

$\Delta u = \Delta x (u_g - u_f)$

$\Delta h = \frac{Q}{m}$

$\Delta x = \frac{\Delta h}{h_{fg}}$

$\Delta s = \Delta x (s_g - s_f)$

a) $\Delta v = v_2 - v_1 = 1.4826 - 1.0435 \times 10^{-3}$
$= 1.482\ m^3/kg$

b) $\Delta u = u_2 - u_1 = 2268.7 - 418.94$
$= 1849.8\ \frac{kJ}{kg}$

c) $\Delta h = h_2 - h_1 = 2419.04 - 419.04$
$= 2000\ \frac{kJ}{kg}$

d) $\Delta s = s_2 - s_1 = 6.666 - 1.3069$
$= 5.359\ \frac{kJ}{kg \cdot K}$

e) $W = mp\Delta v = 1 \times 1.014 \times 10^5 \times 1.482$
$= 150 \times 10^3\ J$
$= 150\ kJ$

For State 2

$T_2 = 100°C$

$p_1 = 1.014\ bar$

$X_2 = 0.8861$

$v_2 = v_f + x_2 (v_g - v_f)$
$= 1.0435 \times 10^{-3} + 0.8861(1.673 - 1.0435 \times 10^{-3})$
$= 1.4826\ m^3/kg$

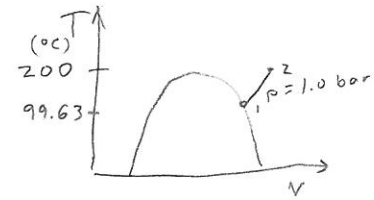
$u_2 = u_f + x_2 (u_g - u_f)$
$= 418.94 + 0.8861(2506.5 - 418.94)$
$= 2268.7\ \frac{kJ}{kg}$

$h_2 = 2419.04\ \frac{kJ}{kg}$ from calc on left

$s_2 = s_f + x_2 (s_g - s_f)$
$= 1.3069 + 0.8861(7.3549 - 1.3069)$
$= 6.666\ \frac{kJ}{kg \cdot K}$

8. One kg of saturated water vapor in a closed, expandable container at 1.0 bar pressure is heated to 200 °C. Find:

Find:
a) The change in specific volume. (Δv)
b) The change in internal energy. (Δu)
c) The change in enthalpy. (Δh)
d) The change in entropy. (Δs)
e) The amount of heat added. (Q)
f) The work done to expand the container. (W)



Given: $m = 1$ kg closed system, expandable

$P_1 = P_2 = 1.0$ bar $x_1 = 1$

$T_2 = 200°C$

Solution: From Table A-4, $P_1 = 1.0$ bar

$T_{sat} = 99.63°C$

15 $v_1 = v_{sat} = 1.694 \ m^3/kg$

$u_1 = u_{sat} = 2506.1 \ kJ/kg$

$h_1 = h_{sat} = 2675.5 \ kJ/kg$

$s_1 = s_{sat} = 7.3594 \ \frac{kJ}{kg \cdot K}$

For $T_2 = 200°C$, $P_2 = 1.0$ bar, from Table A-4

$v_2 = 2.172 \ m^3/kg$

$u_2 = 2658.1 \ \frac{kJ}{kg}$

$h_2 = 2875.3 \ kJ/kg$

$s_2 = 7.8343 \ kJ/kg \cdot K$

a) $\Delta v = v_2 - v_1 = 2.172 - 1.694 = 0.478 \ m^3/kg$

b) $\Delta u = u_2 - u_1 = 2658.1 - 2506.1 = 152.0 \ kJ/kg$

c) $\Delta h = h_2 - h_1 = 2875.3 - 2675.5 = 199.8 \ kJ/kg$

d) $\Delta s = s_2 - s_1 = 7.8343 - 7.3594 = 0.4749 \ \frac{kJ}{kg \cdot K}$

e) $Q = m \Delta h = 1 \times 199.8 = 199.8 \ kJ$

f) $W = m p \Delta v = 1 \times 1.0 \times 10^5 \times 0.478 = 47800 \ J = 47.8 \ kJ$

or

$W = m(\Delta h - \Delta u) = 1 \times (199.8 - 152.0) = 47.8 \ kJ$

9. One kg of air in a closed, expandable container is heated from ~~20 °C~~ 295 K to ~~200 °C~~ 505K at 101400 Pa pressure. Using the properties $\bar{R} = 8.3144598$ kJ/kmol·K, $M = 28.97$ kg/kmol, $c_p = 1.013$ kJ/kg·K, $c_v = 0.726$ kJ/kg·K, $k = 1.395$, find:

Find: a) The change in specific volume. (Δv)
b) The change in internal energy. (Δu)
c) The change in enthalpy. (Δh)
d) The change in entropy. (Δs)
e) The amount of heat added. (Q)
f) The work done to expand the container. (W)

Given: $m = 1$ kg closed system, expandable, heated

$T_1 = 295$ K $P_1 = P_2 = 101400$ Pa $\bar{R}, M, c_p, c_v, k$

$T_2 = 505$ K

Assumption: ideal gas, constant c_p & c_v

15 Solution: $R = \frac{\bar{R}}{M} = \frac{8.3144598}{28.97} = 0.2870 \frac{kJ}{kg \cdot K} = 287.0 \frac{J}{kg \cdot K}$

a) $pv = RT$ $v_1 = \frac{RT_1}{P_1} = \frac{287.0 \times 295}{101400} = 0.8350 \ m^3/kg$

$v_2 = \frac{RT_2}{P_2} = \frac{287.0 \times 505}{101400} = 1.4293 \ m^3/kg$

$\Delta v = v_2 - v_1 = 1.4293 - 0.8350 = 0.5943 \ m^3/kg$

b) $\Delta u = c_v \Delta T = 0.726 (505 - 295) = 152.5 \frac{kJ}{kg}$

c) $\Delta h = c_p \Delta T = 1.013 (505 - 295) = 212.7 \frac{kJ}{kg}$

d) $\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$ ← use this version, since we are given P_1 & P_2 directly in the problem statement

$= 1.013 \ln \frac{505}{295} \cancel{- 0.2870 \ln \frac{101400}{101400}}^{\,0}$

$= 0.545 \frac{kJ}{kg \cdot K}$

e) $Q = m \Delta h = 1 \times 212.7$ ← const pressure, $\Delta PE = \Delta KE = 0$

$= 212.7$ kJ

f) $W = \underbrace{m p \Delta v}_{\text{constant pressure}} = 1 \times 101400 \times 0.5943 = 60.3 \times 10^3 \text{ J} = 60.3 \text{ kJ}$

or

$W = m(\Delta h - \Delta u) = 1(212.7 - 152.5) = 60.2$ kJ

10. One kg of air in a closed, expandable container is heated from 295 K to ~~475~~ 505 K at 1.014 bar pressure. Using the properties:

Given:
- 295 K: h_1 = 295.17 kJ/kg, u_1 = 210.49 kJ/kg, s_1^o = 1.68515 kJ/kg·K
- ~~475~~ 505 K: h_2 = ~~477.37~~ 508.17 kJ/kg, u_2 = ~~341.01~~ 363.21 kJ/kg, s_2^o = ~~2.16682~~ 2.22973 kJ/kg·K

$R = 0.287058 \frac{kJ}{kg\cdot K}$

find:

a) The change in specific volume. (Δv)
b) The change in internal energy. (Δu)
c) The change in enthalpy. (Δh)
d) The change in entropy. (Δs)
e) The amount of heat added. (Q)
f) The work done to expand the container. (W)

Given: $m = 1$ kg closed system, expandable, heated

$T_1 = 295$ K $P_1 = P_2 = 1.014$ bar

$T_2 = 505$ K

15

Assumption: ideal gas

Solution: a) $\Delta v = 0.5943 \ m^3/kg$ (see previous question)

b) $\Delta u = u_2 - u_1 = 363.21 - 210.49 = 152.7 \frac{kJ}{kg}$

c) $\Delta h = h_2 - h_1 = 508.17 - 295.17 = 213.0 \frac{kJ}{kg}$

d) $\Delta s = s_2^o - s_1^o - R \ln\frac{P_2}{P_1}$

$= 2.22973 - 1.68515 - \cancel{0.287058 \ln\frac{1.014}{1.014}}^{\,0}$

$= 0.54458 \frac{kJ}{kg\cdot K}$

e) $Q = m\Delta h = 1 \times 213.0 = 213$ kJ

f) $W = mp\Delta v = 1 \times 1.014 \times 10^5 \times 0.5943 = 60.3 \times 10^3 \ J = 60.3$ kJ

11. Liquid water flows at atmospheric pressures into a container from one source at a rate of 3 kg/s at 20 °C and from another source at 2 kg/s at 10 °C. Water flows out of the container at a rate of 5 kg/s at 30 °C. If no work is being done and the system is in steady state:
    a) At what rate does heat enter the system according to the energy rate balance?
    b) If the heat enters at 40°C, at what rate does the heat enter the system according to the entropy rate balance, in the ~~isentropic~~ case? internally reversible
    c) Is there a logical way to reconcile these values?

Given: Liquid water $\quad p = 1.014$ bar

15 $\dot{m}_{1_{in}} = 3\ \frac{kg}{s} \qquad \dot{m}_{2_{in}} = 2\ kg/s \qquad \dot{m}_{3_{out}} = 5\ kg/s$

$T_1 = 20°C \qquad T_2 = 10°C \qquad T_3 = 30°C$

$\dot{W} = 0$

$\frac{dE}{dt} = 0 \qquad \frac{dS_{cv}}{dt} = 0$

Find: a) $\dot{Q}$ - from energy rate balance

b) $T_b = 40°C$, $\dot{\sigma}_{cv} = 0$, $\dot{Q}$ - from entropy rate balance

c) Q_s Reconcilable

Assumptions: $\Delta KE = \Delta PE = 0$

Solution: a) $\cancel{\frac{dE}{dt}} = \dot{Q} - \cancel{\dot{W}} + \sum_i \dot{m}_i (h_i + \cancel{g z_i} + \cancel{\frac{V_i^2}{2}}) - \sum_e \dot{m}_e (h_e + \cancel{g z_e} + \cancel{\frac{V_e^2}{2}})$

$0 = \dot{Q} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$

$0 = \dot{Q} + 3 \times 83.96 + 2 \times 42.01 - 5 \times 125.79$ $\qquad$ h's for sat. liq. from Table A-2

$\dot{Q} = 293$ kJ

b) $\cancel{\frac{dS_{cv}}{dt}} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \cancel{\dot{\sigma}_{cv}}$

$0 = \frac{\dot{Q}}{T_b} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3$

$0 = \frac{\dot{Q}}{40+273} + 3 \times 0.2966 + 2 \times 0.1510 - 5 \times 0.4369$

$\dot{Q} = 311$ kJ

c) To design a reversible process requires both equations to be satisfied simultaneously, which is not necessarily easy. If we used the $\dot{Q}$ from (a), we would need to accept some irreversibility. If we used the $\dot{Q}$ from (b), we would need to adjust at least one of the temperatures or two of the flow rates.

12. A pump pumps 1 kg/s of liquid water at 20 °C from atmospheric pressure (1.014 bar) to a pressure of ~~20~~ 25 bar.

a) What temperature is the pressurized water leaving the pump, in the isentropic case?

b) If there are internal irreversibilities, will the temperature be higher or lower?

Given: pump $\dot{m} = 1$ kg/s

liquid water $T_1 = 20°C$

$P_1 = 1.014$ bar $\quad P_2 = 25$ bar

Find: a) T_{2s}

b) $T_2 > T_{2s}$ or $T_2 < T_{2s}$?

c) $\dot{W}_{cvs}$

Solution: a) For isentropic pump process $s_{2s} = s_1$

For sat. liquid water at $T_1 = 20°C$, from Table A-2

$s_1 = 0.2966 \frac{kJ}{kg \cdot K}$

For comp. liquid water at $P_2 = 25$ bar, $s_{2s} = 0.2966 \frac{kJ}{kg \cdot K}$, interpolating from Table A-5,

$T_{2s} = 20.036 °C$

b) If $s_{2s} > s_1$, looking at Table A-5, we see that $T_2 > T_{2s}$

c) $\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_{2s} - h_1$

From Table A-2, for $T_1 = 20°C$, P_1 is low

$h_1 \approx h_f = 83.96 \frac{kJ}{kg}$

Interpolating from Table A-5, for $P_2 = 25$ bar, $s_{2s} = 0.2966$,

$h_{2s} = 86.45 \frac{kJ}{kg}$

$\left(-\frac{\dot{W}_{cv}}{1}\right)_s = 86.45 - 83.96$

$\dot{W}_{cvs} = -2.49$ kW

Work is entering system.