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Name: ______________________ You are allowed 3 sheets of notes.

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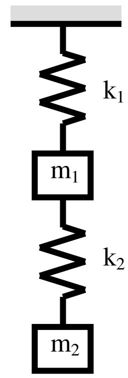
1. For the system shown on the right, solve for:
   a. system differential equation(s)
   b. natural frequencies
   c. mode shapes



$m_1 = 100$ kg
$m_2 = 1$ kg
$k_1 = 1000$ N/m
$k_2 = 100$ N/m

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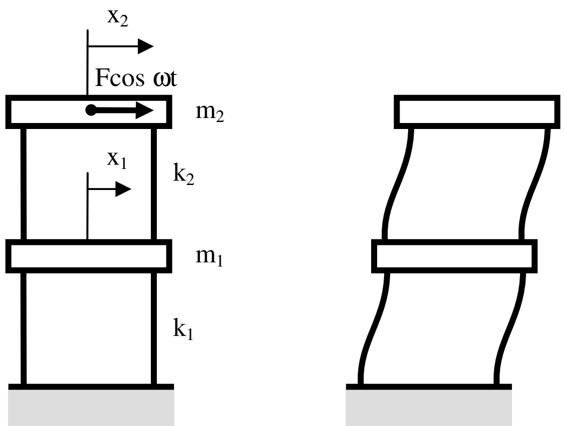
2. An air conditioning system is being installed on the roof of a two-story building. The fan causes a harmonic lateral force of 50 cos 20t N due to a rotating imbalance in the fan. If the masses of the floors and lateral stiffness of the support columns are as given, find:
   a. system differential equation(s)
   b. **x**(t)



$m_1 = 1x10^4$ kg
$m_2 = 3x10^4$ kg
$k_1 = 2x10^5$ N/m
$k_2 = 2x10^5$ N/m
F = 50 N
$\omega = 20$ rad/s

Use the "Direct Method" to compute the particular solution only. Note that you will need to solve a 2 dof linear system of equations. If you do not have a matrix calculator, in the first equation solve for x_1 as a function of x_2 (or vice versa) and substitute into the second equation.

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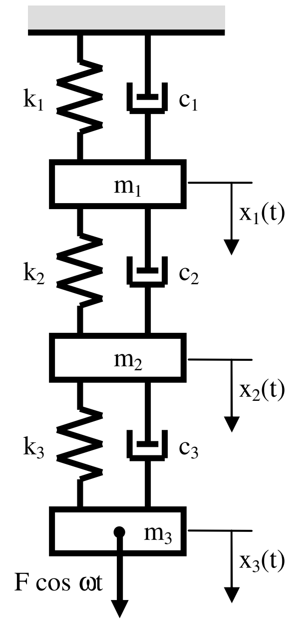
3. For the system shown on the right, using the "Direct Method," solve for:
   a. system differential equation(s)
   b. mass, damping and stiffness matrices
   c. single 6x6 matrix that can be used to solve for $\mathbf{X}_1$ and $\mathbf{X}_2$ in $\mathbf{x}(t) = \mathbf{X}_1 \cos \omega t + \mathbf{X}_2 \sin \omega t$.



$m_1 = 2$ kg
$m_2 = 2$ kg
$m_3 = 2$ kg
$c_1 = 20$ N s/m
$c_2 = 20$ N s/m
$c_3 = 20$ N s/m
$k_1 = 200$ N/m
$k_2 = 200$ N/m
$k_3 = 200$ N/m
$F = 10$ N
$\omega = 20$ rad/s

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4. (a) In a modal analysis, what are two ways of handling damping?

5 (b) What is a "node" in a mode shape?

(c) How is the modal analysis technique different than the direct method when solving for harmonic forced vibration?

5. A single-degree-of-freedom system has a mass of 2 kg, spring stiffness of 200 N/m and viscous damping of 20 N·s/m. Give:
   a) System differential equation
   b) Damping condition (underdamped, overdamped, or critically damped)
   c) Stability (stable or unstable)
   d) Correct equation to be used for $x(t)=\ldots$ and correct equations to be used for computing constants in this equation.
   e) Displacement as a function of time for initial conditions $x(0) = 10$ mm and $\dot{x}(0) = 0$.

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Problem 1 continued.

6. A single-degree-of-freedom system has a mass of 50 kg, spring stiffness of 500 N/m and viscous damping of 30 N·s/m. The floor supporting the mass (via the spring and damper) has a harmonic motion with amplitude 10 mm at a frequency of 1 Hz. Give:
    a) System differential equation
    b) Damping condition (underdamped, overdamped, or critically damped)
    c) Stability (stable or unstable)
    d) Correct equation to be used for $x(t)$=… and correct equations to be used for computing constants in this equation. (Assume all free vibration has been damped out.)
    e) Amplitude of the displacement
    f) Amplitude of the velocity
    g) Amplitude of the acceleration
    h) Amplitude of the transmitted force.

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Problem 2 continued.