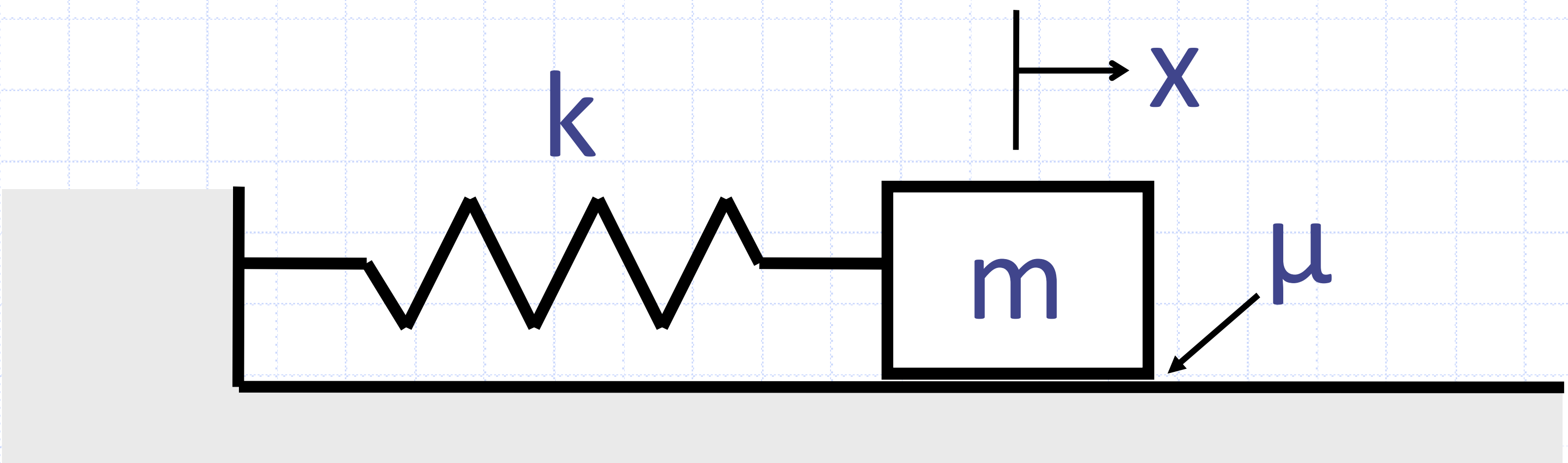
Coulomb Friction

Section 1.10

Coulomb Damping Problem (Problem 1.102)

- Given: A single degree-of-freedom system with mass of 2 kg, spring stiffness of 1000 N/m and a dynamic Coulomb friction coefficient of 0.153.
- How long will it take for the amplitude of vibrations to go down by 20 cm.



Coulomb Damping Problem

- Free Body Diagram & applying Newton's 2nd Law

Coulomb Damping Problem

- The system differential equation:

- How can we solve this?

1)

2)

Piece-wise Solution

- Assume $x_0 > 0$ and $v_0 = 0$, then, for $0 \le t \le T/2$:

$$m\ddot{x} + kx = \mu N$$

which has the solution:

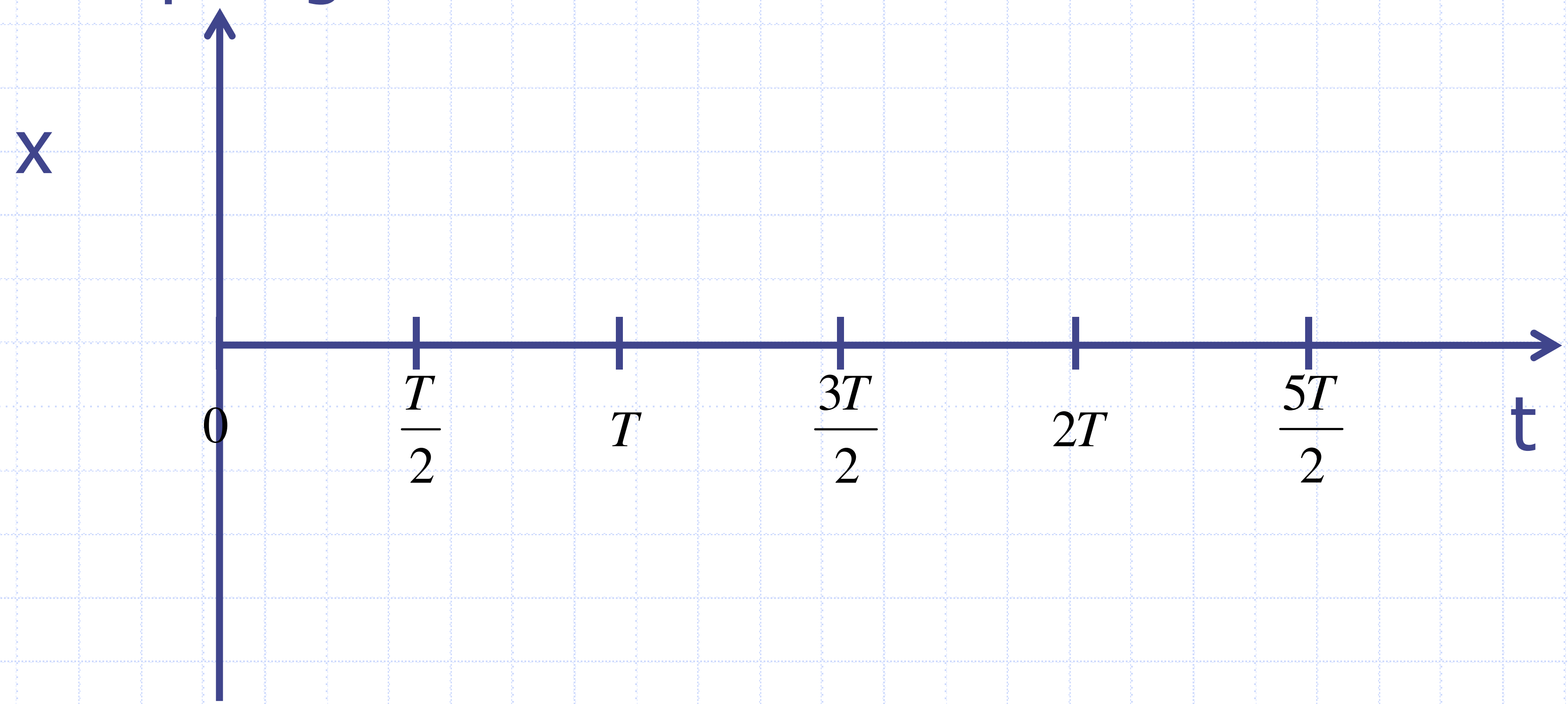
$$x_1(t) = A_1 \cos \omega_n t + B_1 \sin \omega_n t + \frac{\mu N}{k}$$

which, after accounting for initial conditions, is:

$$x_1(t) = \left(x_0 - \frac{\mu N}{k} \right) \cos \omega_n t + \frac{\mu N}{k}$$

Piece-wise Solution

- Graphing it:



- For T/2 ≤ t ≤ T:

$$m\ddot{x} + kx = -\mu N$$

Piece-wise Solution

which has the solution:

$$x_2(t) = A_2 \cos \omega_n t + B_2 \sin \omega_n t - \frac{\mu N}{k}$$

which, after accounting for initial conditions (at t = T/2), is:

$$x_2(t) = \left(x_0 - \frac{3\mu N}{k} \right) \cos \omega_n t - \frac{\mu N}{k}$$

similarly:

$$x_3(t) = \left(x_0 - \frac{5\mu N}{k} \right) \cos \omega_n t + \frac{\mu N}{k}$$

$$x_4(t) = \left(x_0 - \frac{7\mu N}{k} \right) \cos \omega_n t - \frac{\mu N}{k}$$

Piece-wise Solution

- Notice that the amplitude of the oscillations drops by $\frac{2\mu N}{k}$ every half period, therefore:

$$\frac{\Delta A}{T} = \frac{-4\mu N}{k}$$

$$\frac{\Delta A}{\Delta t} = \frac{\frac{-2\mu N}{k}}{T/2} = \frac{\frac{-2\mu N}{k}}{\pi/\omega_n} = \frac{-2\mu N \omega_n}{\pi k}$$

- The vibrations will stop when the spring force cannot overcome the friction force; when:

$\dot{x} = 0$ and $-\frac{\mu N}{k} \leq x \leq \frac{\mu N}{k}$