Institute of Technology
West Virginia University

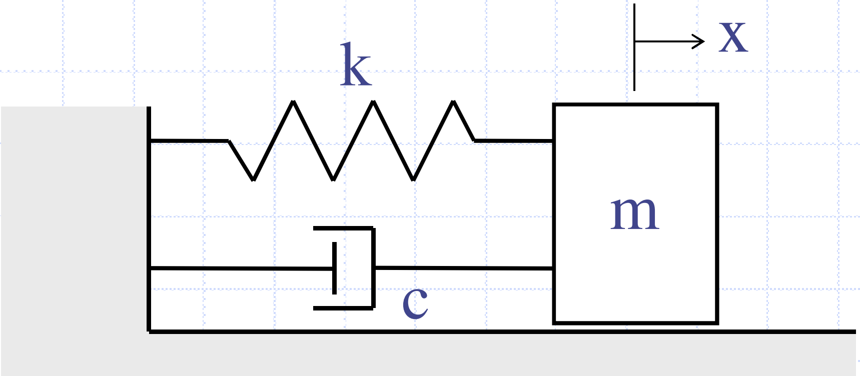
Viscous Damping

Section 1.3

Introduction

- **All objects** or systems **vibrate** at some level all the time.
- The vibration of **all objects** or systems is always **damped** by some amount by some natural mechanism, which is why free vibrations rarely last long (except in space)**.**
- **Kinetic energy** lost from damping is usually turned **into heat**.
- With **"viscous" damping**, the damping force is **proportional to the velocity** difference between the two bodies.

SDOF System with Viscous Damping



Design of a Simple Shock

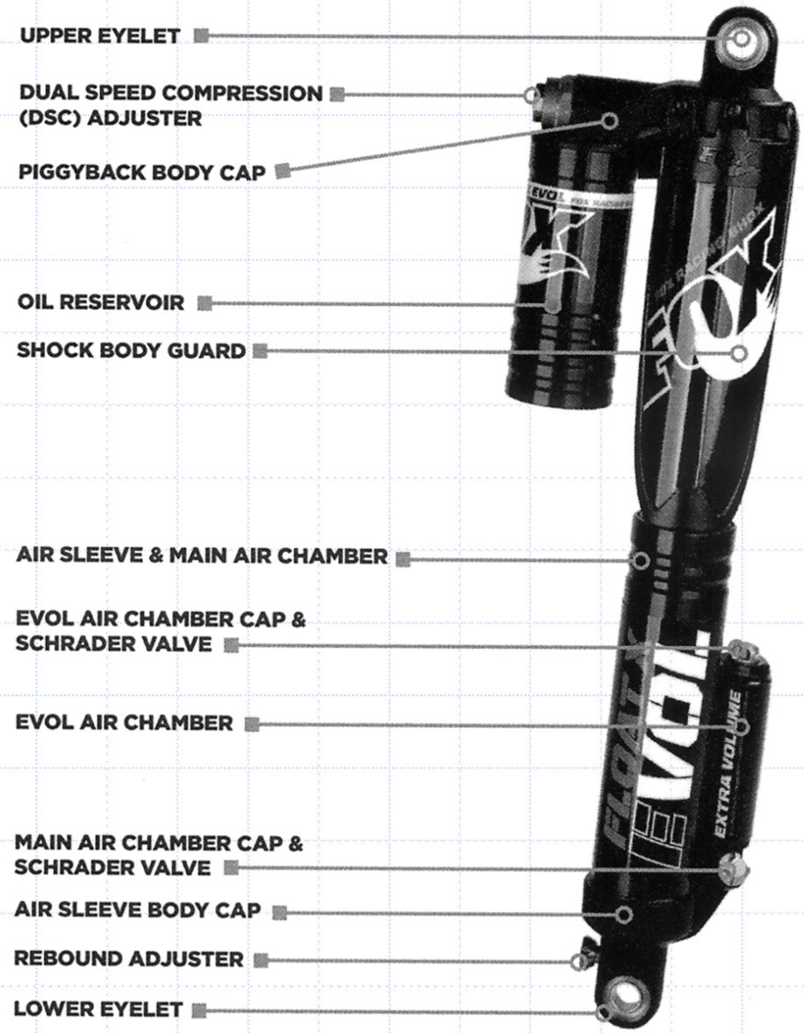
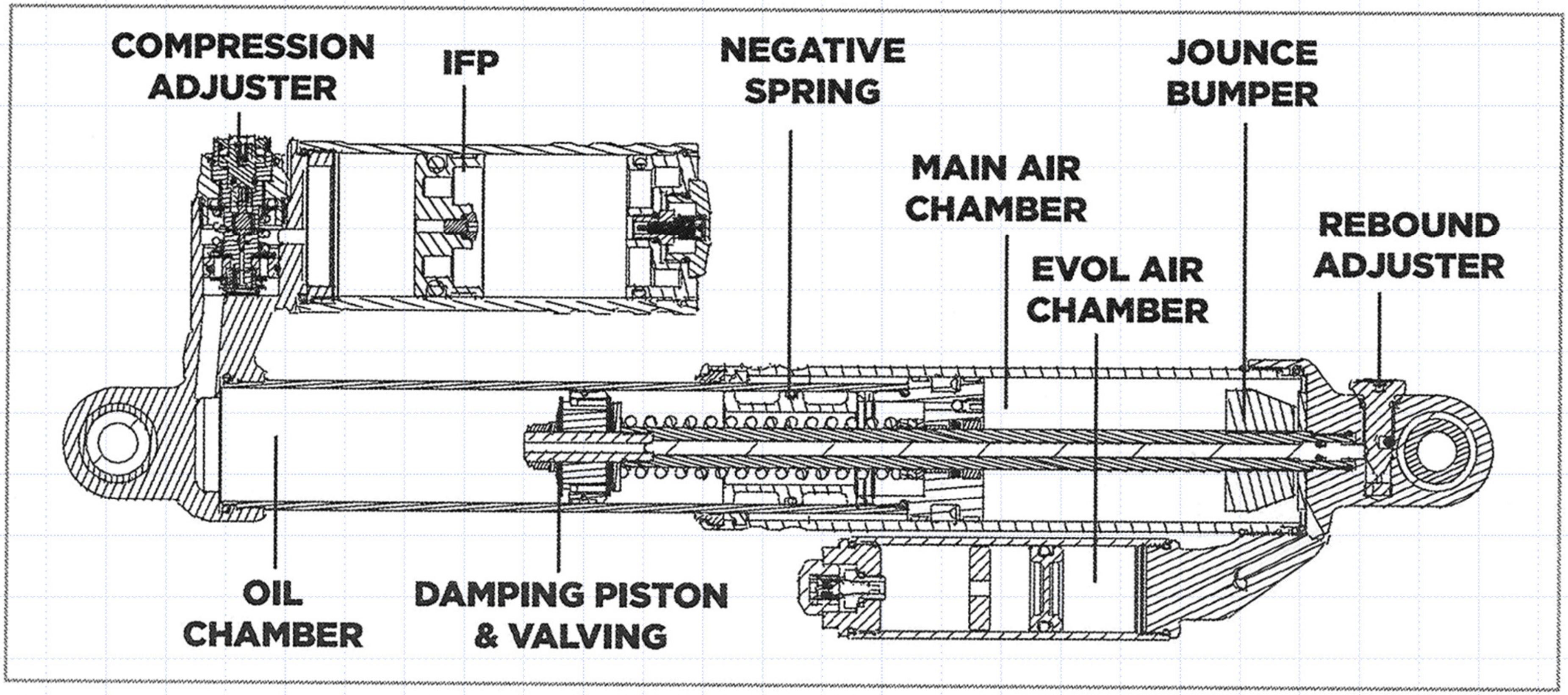


Image from Fox Racing Shox ATV FLOAT X EVOL Owner's Manual

Fox FLOAT X EVOL Shock

- Shock Cross-Section

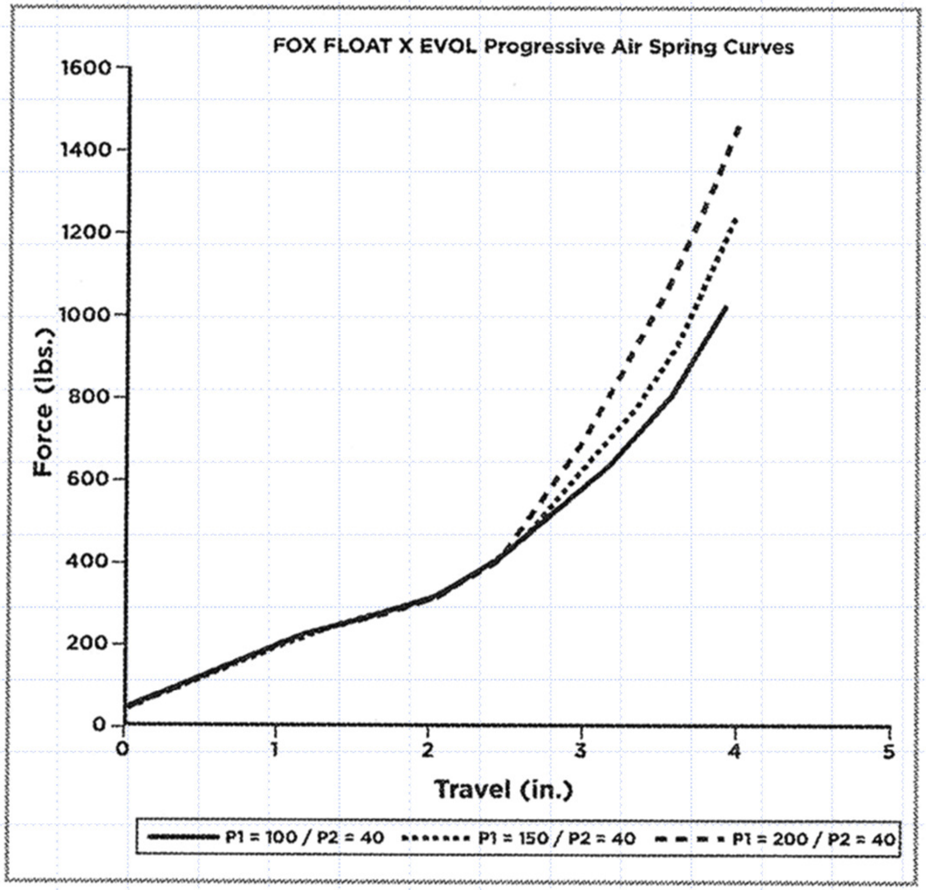


Cross-section of FLOAT X EVOL

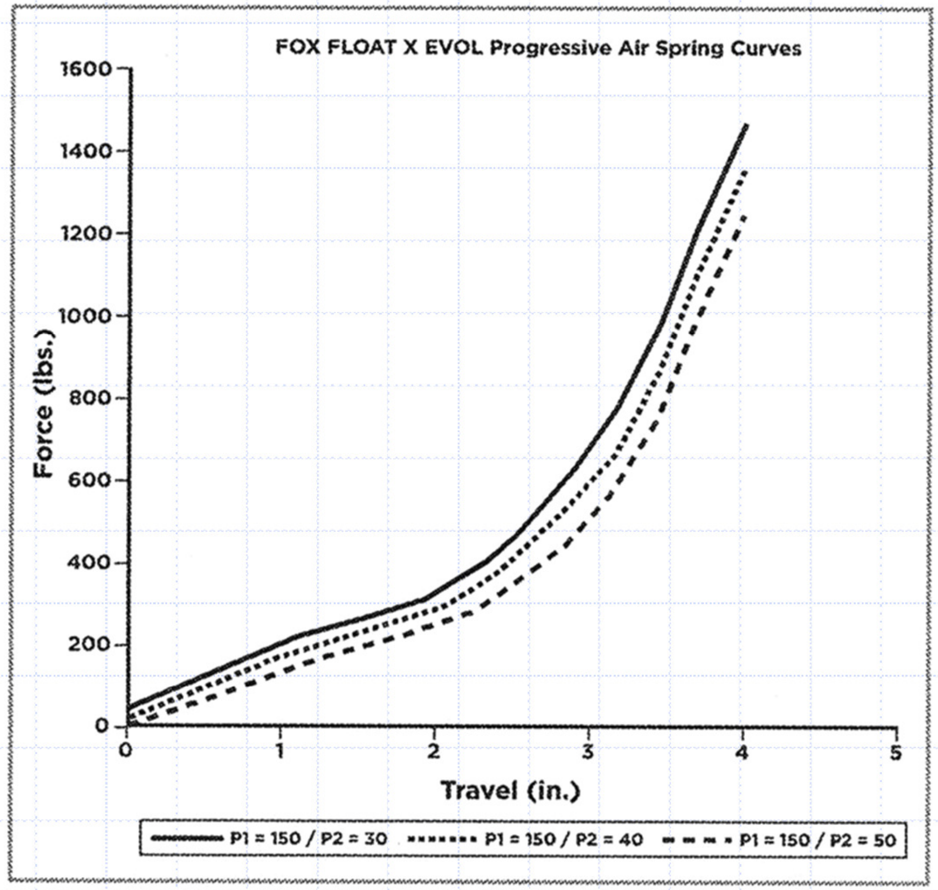
Image from Fox Racing Shox ATV FLOAT X EVOL Owner's Manual

Fox FLOAT X EVOL Shock

- Adjustable Spring Force Characteristics



Changing EVOL Air Chamber pressure adjusts the bottom-out resistance of the shock.

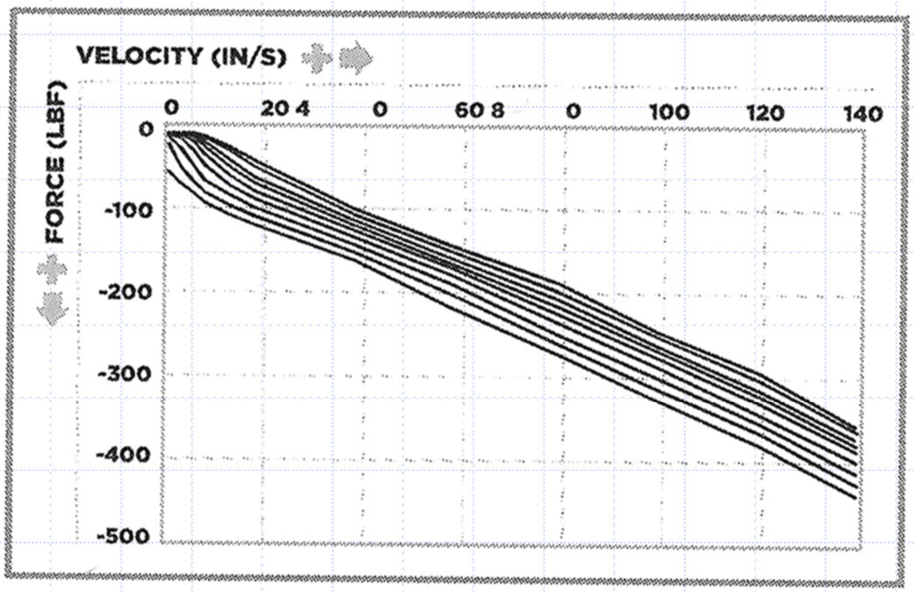


Changing MAIN Air Chamber pressure steadily adjusts the spring curve.

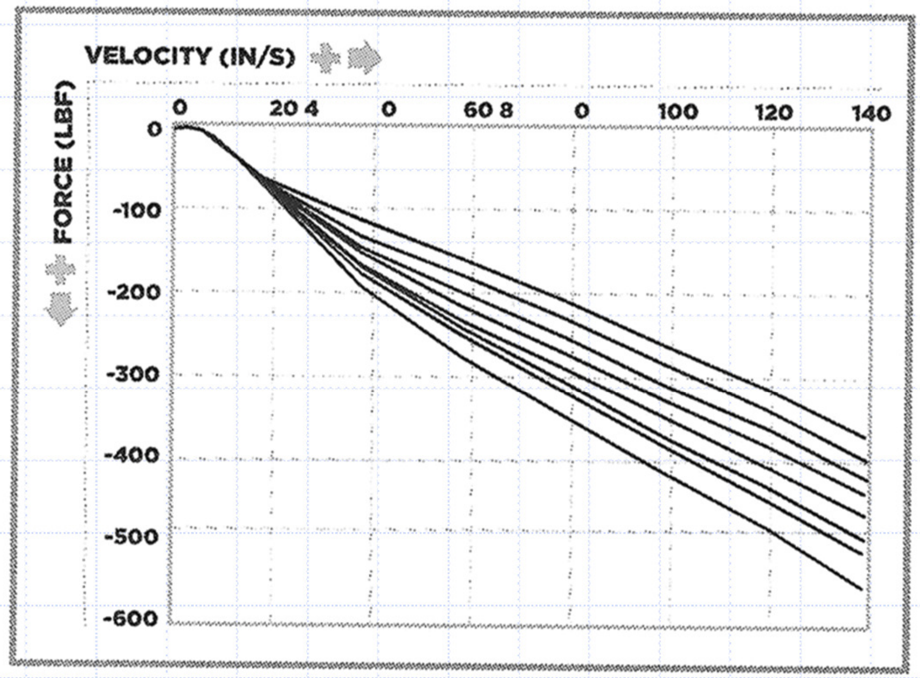
Images from Fox Racing Shox ATV FLOAT X EVOL Owner's Manual

Fox FLOAT X EVOL Shock

- Adjustable Compression Damping



Characteristic graph showing the affect of changing the LSC Adjuster.



Characteristic graph showing the affect of changing the HSC adjuster.

Images from Fox Racing Shox ATV FLOAT X EVOL Owner's Manual

Free Body Diagram

System Differential Equation:

Solving the Differential Equation

- The 3 cases:

The "Critical Damping Coefficient"

- Critical damping coefficient

  $c_{cr} =$

- Damping ratio

  $\zeta =$

- In terms of ζ and ω_n,

  $\lambda_{1,2} =$

Case 1) Underdamped motion

$$\varsigma < 1$$

$$\lambda_1 = -\varsigma\omega_n - \omega_n\sqrt{1-\varsigma^2}\,j = -\varsigma\omega_n - j\omega_d$$

$$\lambda_2 = -\varsigma\omega_n + \omega_n\sqrt{1-\varsigma^2}\,j = -\varsigma\omega_n + j\omega_d$$

$$\begin{aligned} x(t) &= a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t} \\ &= a_1 e^{(-\varsigma\omega_n - j\omega_d)t} + a_2 e^{(-\varsigma\omega_n + j\omega_d)t} \\ &= e^{-\varsigma\omega_n t}\left(a_1 e^{-j\omega_d t} + a_2 e^{+j\omega_d t}\right) \end{aligned}$$

$$x(t) = Ae^{-\varsigma\omega_n t}\sin\left(\omega_d t + \phi\right)$$

How to calculate A and Φ?

$$x(t) = Ae^{-\varsigma\omega_n t}\sin(\omega_d t + \phi)$$

$$x(0) = x_0 = A\sin(\phi) \qquad (1)$$

$$\dot{x}(t) = -\varsigma\omega_n Ae^{-\varsigma\omega_n t}\sin(\omega_d t + \phi) + \omega_d Ae^{-\varsigma\omega_n t}\cos(\omega_d t + \phi)$$

$$\dot{x}(0) = v_0 = -\varsigma\omega_n A\sin(\phi) + \omega_d A\cos(\phi) \qquad (2)$$

- Solving Eq. (1) and (2) for A and Φ yields:

$$A = \sqrt{\frac{(v_0 + \varsigma\omega_n x_0)^2 + (x_0\omega_d)^2}{\omega_d^2}}$$

$$\phi = \tan^{-1}\frac{x_0\omega_d}{v_0 + \varsigma\omega_n x_0}$$

Case 2) Overdamped motion

$$\varsigma > 1$$

$$\lambda_1 = -\varsigma\omega_n - \omega_n\sqrt{\varsigma^2 - 1}$$

$$\lambda_2 = -\varsigma\omega_n + \omega_n\sqrt{\varsigma^2 - 1}$$

$$x(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t}$$

$$= a_1 e^{\left(-\varsigma\omega_n - \omega_n\sqrt{\varsigma^2-1}\right)t} + a_2 e^{\left(-\varsigma\omega_n + \omega_n\sqrt{\varsigma^2-1}\right)t}$$

$$x(t) = e^{-\varsigma\omega_n t}\left(a_1 e^{-\omega_n\sqrt{\varsigma^2-1}t} + a_2 e^{\omega_n\sqrt{\varsigma^2-1}t}\right)$$

How to calculate a_1 and a_2?

$$x(t) = e^{-\varsigma\omega_n t}\left(a_1 e^{\left(-\varsigma\omega_n - \omega_n\sqrt{\varsigma^2-1}\right)t} + a_2 e^{\left(-\varsigma\omega_n + \omega_n\sqrt{\varsigma^2-1}\right)t}\right)$$

$$x(0) = x_0 = a_1 + a_2 \qquad (1)$$

$$\dot{x}(t) = -\varsigma\omega_n e^{-\varsigma\omega_n t}\left(a_1 e^{\left(-\varsigma\omega_n - \omega_n\sqrt{\varsigma^2-1}\right)t} + a_2 e^{\left(-\varsigma\omega_n + \omega_n\sqrt{\varsigma^2-1}\right)t}\right)$$

$$+ e^{-\varsigma\omega_n t}\left(\left(-\varsigma\omega_n - \omega_n\sqrt{\varsigma^2-1}\right)a_1 e^{\left(-\varsigma\omega_n - \omega_n\sqrt{\varsigma^2-1}\right)t} + \left(-\varsigma\omega_n + \omega_n\sqrt{\varsigma^2-1}\right)a_2 e^{\left(-\varsigma\omega_n + \omega_n\sqrt{\varsigma^2-1}\right)t}\right)$$

$$\dot{x}(0) = v_0 = -\varsigma\omega_n(a_1 + a_2) + a_1\left(-\varsigma\omega_n - \omega_n\sqrt{\varsigma^2-1}\right) + a_2\left(-\varsigma\omega_n + \omega_n\sqrt{\varsigma^2-1}\right) \qquad (2)$$

- Solving Equ. (1) and (2) for a_1 and a_2 yields:

$$a_1 = \frac{-v_0 + \left(-\varsigma + \sqrt{\varsigma^2-1}\right)\omega_n x_0}{2\omega_n\sqrt{\varsigma^2-1}}$$

$$a_2 = \frac{v_0 + \left(\varsigma + \sqrt{\varsigma^2-1}\right)\omega_n x_0}{2\omega_n\sqrt{\varsigma^2-1}}$$

Case 3) Critically damped motion

$$\varsigma = 1$$

$$\lambda_1 = \lambda_2 = -\omega_n$$

$$x(t) = (a_1 + a_2 t)e^{\lambda t}$$

$$x(t) = (a_1 + a_2 t)e^{-\omega_n t}$$

$$x(0) = x_0 = a_1 \qquad (1)$$

$$\dot{x}(t) = a_2 e^{-\omega_n t} + (a_1 + a_2 t)(-\omega_n e^{-\omega_n t})$$

$$\dot{x}(0) = v_0 = a_2 - \omega_n a_1 \qquad (2)$$

- Solving Equ. (1) and (2) for a_1 and a_2 yields:

$$a_1 = x_0$$

$$a_2 = v_0 + \omega_n x_0$$