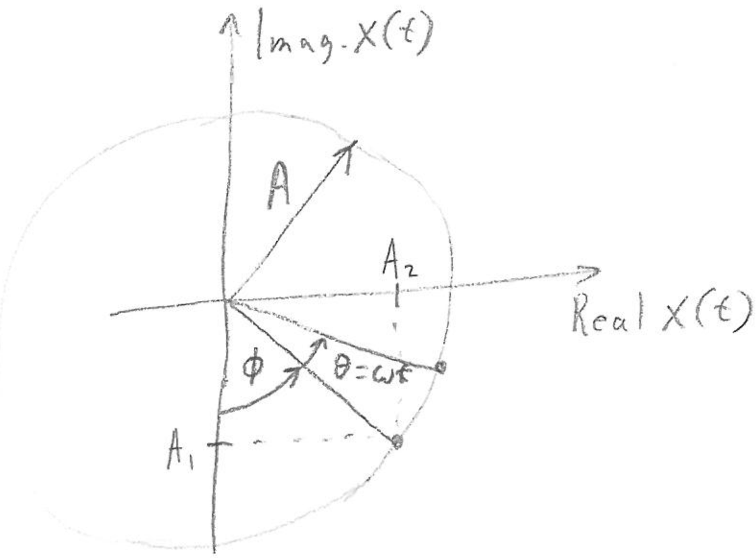
Euler's Relations - Explaining why $a_1 e^{j\theta} + a_2 e^{-j\theta} \Rightarrow A\sin(\theta+\phi)$

$$\left.\begin{aligned} e^{j\theta} &= \cos\theta + j\sin\theta \\ e^{-j\theta} &= \cos\theta - j\sin\theta \end{aligned}\right\} \text{Euler's relations}$$

$$\begin{aligned}
x &= a_1 e^{j\theta} + a_2 e^{-j\theta} \\
&= a_1(\cos\theta + j\sin\theta) + a_2(\cos\theta - j\sin\theta) \\
&= (a_{1R} + a_{1I}j)(\cos\theta + j\sin\theta) + (a_{2R} + a_{2I}j)(\cos\theta - j\sin\theta) \\
&= a_{1R}\cos\theta + a_{1I}j\cos\theta + a_{1R}j\sin\theta - a_{1I}\sin\theta + a_{2R}\cos\theta + a_{2I}j\cos\theta - a_{2R}j\sin\theta + a_{2I}\sin\theta \\
&= (a_{1R}\cos\theta - a_{1I}\sin\theta + a_{2R}\cos\theta + a_{2I}\sin\theta) + \underbrace{(a_{1I}\cos\theta + a_{1R}\sin\theta + a_{2I}\cos\theta - a_{2R}\sin\theta)j}_{\text{Ignore imaginary part}} \\
&= (a_{2I} - a_{1I})\sin\theta + (a_{1R} + a_{2R})\cos\theta \\
&= A_1\sin\theta + A_2\cos\theta \\
&= A\cos\phi\sin\theta + A\sin\phi\cos\theta \\
&= A\sin(\theta + \phi)
\end{aligned}$$



Problem 1.52

Given: $m = 1\ kg$
$c = 2\ kg/s$
$k = 10\ N/m$

Find: ζ, ω_n, damping condition

Sol'n
$$C_{cr} = 2\sqrt{km} = 2\sqrt{(10)(1)} = 6.32$$

$$\zeta = \frac{c}{C_{cr}} = \frac{2}{6.32} = 0.316 \quad \text{- under-damped}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{1}} = 3.16\ rad/s$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.16\sqrt{1-0.316^2} = 2.84\ rad/s$$

Problem 1.54

see printout

Given: $\omega_n = 2\ rad/s$, $\zeta = 0.1$, $V_0 = 0$
Find: plot response for $X_0 = 1, 5, 10$ & 100 mm
Sol'n: $X(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$
$\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\quad A = \sqrt{\frac{(V_0 + \omega_n\zeta X_0)^2 + (X_0\omega_d)^2}{\omega_d^2}}$ $\quad \phi = \tan^{-1}\frac{X_0\omega_d}{V_0 + \omega_n\zeta X_0}$

Problem 1.56

Given: $m = 100\ kg$
$K = 3000\ N/m$
$c = 300\ kg/s$

Find: ω_n, ζ, ω_d, oscillates?

Sol'n: $$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3000}{100}} = 5.48\ rad/s$$

$$\zeta = \frac{c}{2\sqrt{Km}} = \frac{300}{2\sqrt{3000 \cdot 100}} = 0.274$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 5.48\sqrt{1-0.274^2}$$
$$= 5.27\ rad/s$$

Yes, it oscillates.

Problem 1.59

Given $V_0 = 0$
$X_0 = 0.1\ m$

Find: form of response
- plot until dies out

Sol'n
$$A = \sqrt{\frac{(V_0 + \zeta\omega_n X_0)^2 + (X_0\omega_d)^2}{\omega_d^2}}$$

$$= \sqrt{\frac{(0 + 0.274 \times 5.48 \times 0.1)^2 + (0.1 \times 5.27)^2}{5.27^2}}$$

$$= 0.1040\ m$$

$$\phi = \tan^{-1}\frac{X_0\omega_d}{V_0 + \zeta\omega_n X_0} = \tan^{-1}\frac{(0.1)(5.27)}{0 + (0.274)(5.48)(0.1)} = \tan^{-1} 3.51$$

$$\phi = 74.1\ deg$$

see plot

Problem [redacted] 1.54

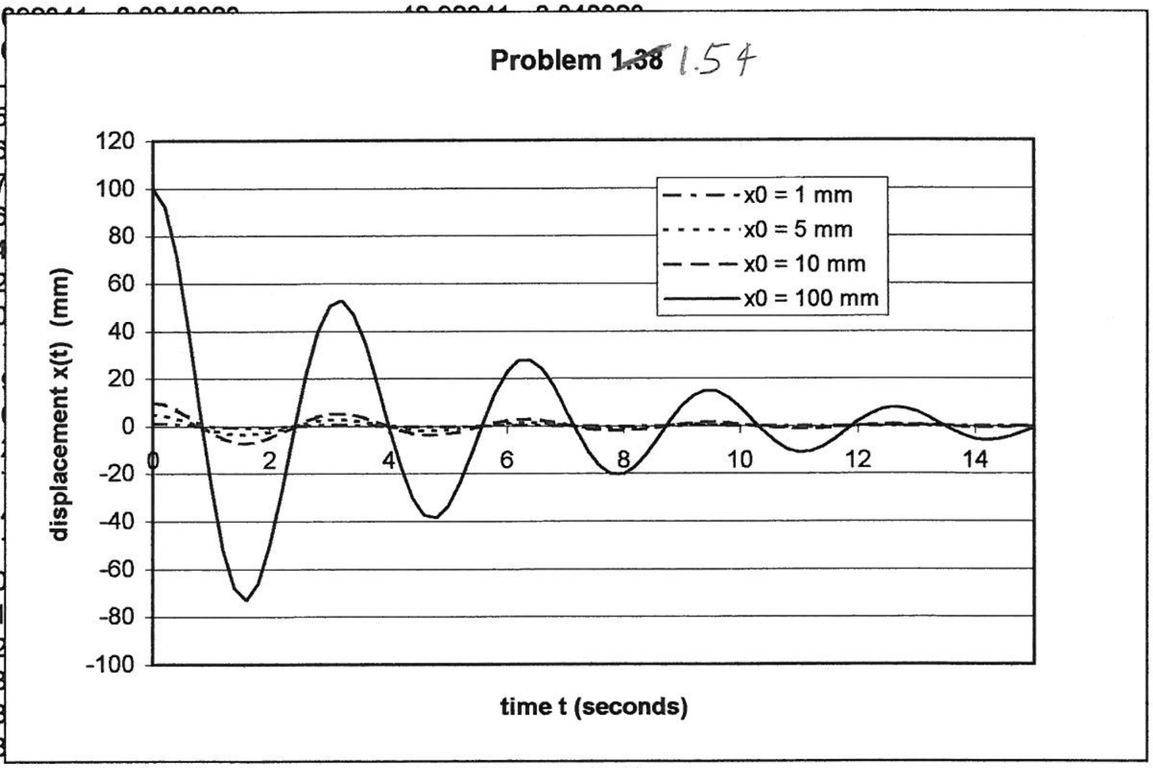
wn	2
z	0.1
wd	1.990

	x0=1 mm		x0=5 mm	
	x0 (m)	0.001	x0 (m)	0.005
	A (m)	0.001	A (m)	0.005
	phi (rad)	1.471	phi (rad)	1.471

t (sec)	x (mm)	x (m)	x (mm)	x (m)
0	1	0.001	5	0.005
0.2	0.923119	0.000923119	4.615595	0.004615595
0.4	0.712086	0.000712086	3.560429	0.003560429
0.6	0.409234	0.000409234	2.04617	0.00204617
0.8	0.067574	6.75744E-05	0.337872	0.000337872
1	-0.25807	-0.00025807	-1.29035	-0.001290351
1.2	-0.51952	-0.000519522	-2.59761	-0.002597608
1.4	-0.68205	-0.000682046	-3.41023	-0.003410228
1.6	-0.72859	-0.000728588	-3.64294	-0.003642942
1.8	-0.66101	-0.000661006	-3.30503	-0.003305028
2	-0.49833	-0.000498326	-2.49163	-0.002491628
2.2	-0.27254	-0.000272543	-1.36271	-0.001362715
2.4	-0.02277	-2.27669E-05	-0.11383	-0.000113834
2.6	0.21126	0.00021126	1.0563	0.0010563
2.8	0.39524	0.00039524	1.976199	0.001976199
3	0.505106	0.000505106	2.525528	0.002525528
3.2	0.529886	0.000529886	2.649429	0.002649429
3.4	0.472362	0.000472362	2.36181	0.00236181
3.6	0.34759	0.00034759	1.737952	0.001737952
3.8	0.179672	0.000179672	0.898362	0.000898362
4	-0.0026	-2.59684E-06	-0.01298	-1.29842E-05
4.2	-0.17046	-0.000170459	-0.85229	-0.000852293
4.4	-0.29955	-0.000299551	-1.49776	-0.001497755
4.6	-0.37327	-0.000373268	-1.86634	-0.001866341
4.8	-0.38468	-0.000384682	-1.92341	-0.001923411
5	-0.33685	-0.000336852	-1.68426	-0.001684258
5.2	-0.24159	-0.000241589	-1.20794	-0.001207943
5.4	-0.11699	-0.000116994	-0.58497	-0.000584969
5.6	0.015773	1.57728E-05	0.078864	7.8864E-05
5.8	0.135939	0.000135939	0.679694	0.000679694
6	0.22624	0.00022624	1.131201	0.001131201
6.2	0.275272	0.000275272	1.376359	0.001376359
6.4	0.278767	0.000278767	1.393836	0.001393836
6.6	0.239697	0.000239697	1.198486	0.001198486
6.8	0.167262	0.000167262	0.83631	0.00083631
7	0.075018	7.50176E-05	0.375088	0.000375088
7.2	-0.02152	-2.1517E-05	-0.10758	-0.000107585
7.4	-0.10736	-0.000107365	-0.53682	-0.000536824

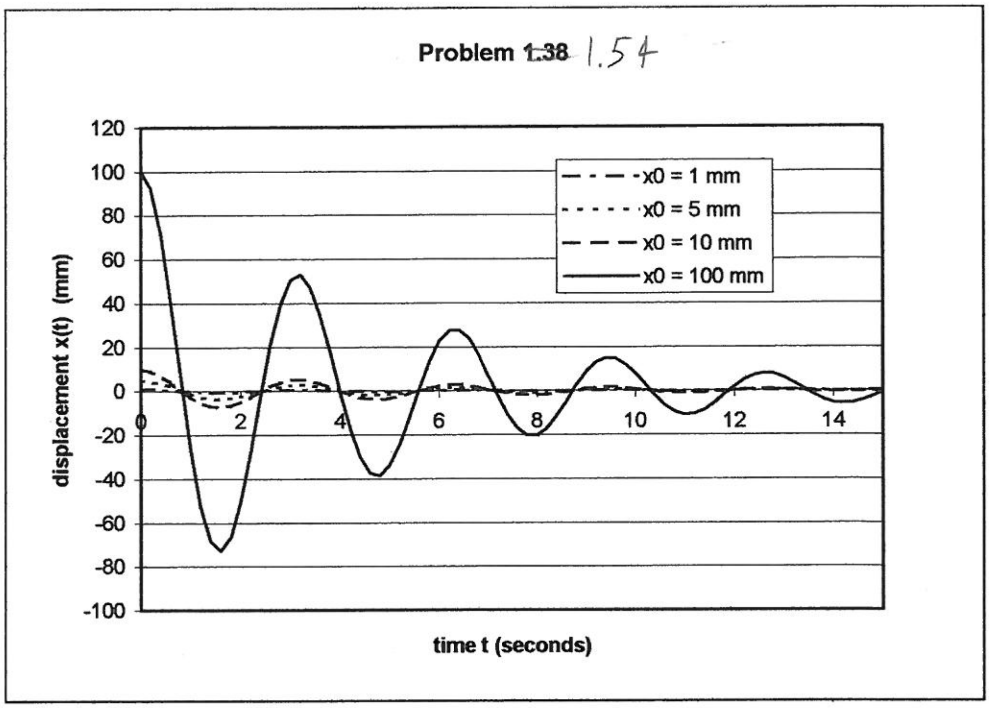
x0=10mm		x0=100mm	
x0 (m)	0.010	x0 (m)	0.100
A (m)	0.010	A (m)	0.101
phi (rad)	1.471	phi (rad)	1.471

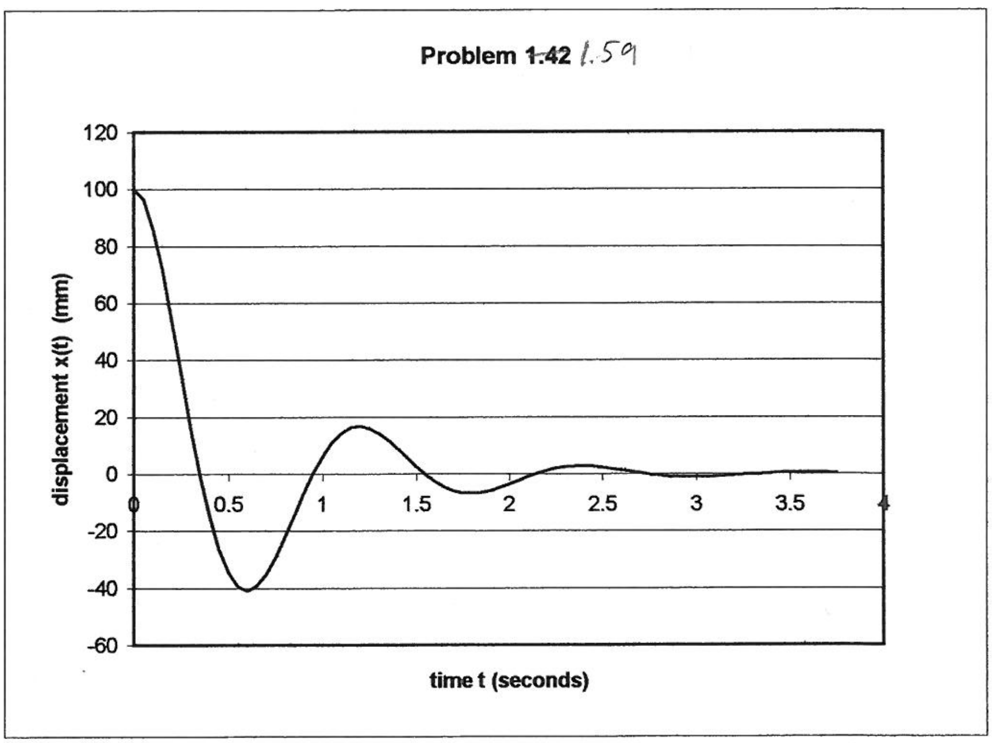
x (mm)	x (m)	x (mm)	x (m)
10	0.01	100	0.1
9.231191	0.0092312	92.31191	0.092312
7.120858	0.0071209	71.20858	0.071209



-2.41589	-0.0024159	-24.1589	-0.02416
-1.16994	-0.0011699	-11.6994	-0.0117
0.157728	0.0001577	1.57728	0.001577
1.359387	0.0013594	13.59387	0.013594
2.262401	0.0022624	22.62401	0.022624
2.752718	0.0027527	27.52718	0.027527
2.787672	0.0027877	27.87672	0.027877
2.396971	0.002397	23.96971	0.02397
1.67262	0.0016726	16.7262	0.016726
0.750176	0.0007502	7.501763	0.007502
-0.21517	-0.0002152	-2.1517	-0.00215
-1.07365	-0.0010736	-10.7365	-0.01074

Charts for Chapter 1.3 Problems





Problem 1.61

Given: $m = 150\ kg$

$k = 2000\ N/m$

$x_0 = 0$

$v_0 = 0.01\ m/s$

Die out amplitude 1% in 2 s.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{150}} = 3.65\ rad/s$$

Find: c for die out in 2 s.

Sol'n: $e^{-\zeta\omega_n t} = 0.01$

$$-\zeta\omega_n t = \ln 0.01$$

$$\zeta = \frac{-\ln 0.01}{\omega_n t} = \frac{-\ln 0.01}{3.65(2)} = 0.631$$

$$\zeta = \frac{c}{2\sqrt{km}}$$

$$c = \zeta 2\sqrt{km} = 0.631(2)\sqrt{2000 \cdot 150}$$

$$= 691\ N/(m/s)$$

$\boxed{\text{Homework 1.53, 1.58, 1.60}}$