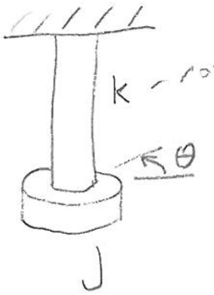
Another example : Torsional System



k - rotational stiffness (Nm/rad)

$M_{shaft} = k\theta$

$$-\Delta U = \Delta T$$

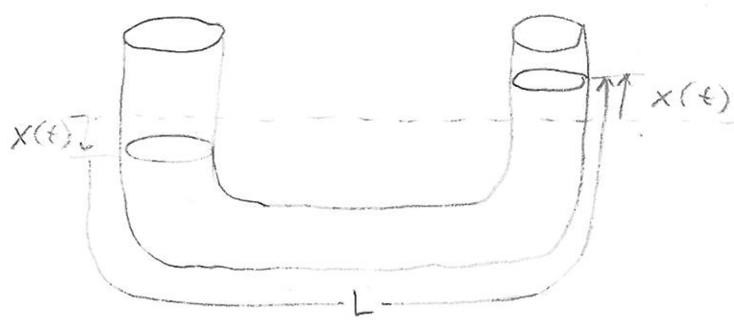
$$-\tfrac{1}{2}k\theta^2 = \tfrac{1}{2}J\dot{\theta}^2$$

taking derivative wrt time

$$-k\theta\cancel{\dot{\theta}} = J\cancel{\dot{\theta}}\ddot{\theta}$$

$$\boxed{J\ddot{\theta} + k\theta = 0}$$

Another Example : U-Tube manometer



$$-\Delta U = \Delta T$$

$$-\rho V g h = \tfrac{1}{2}m\dot{x}^2$$

$$-\cancel{\rho A}x g x = \tfrac{1}{2}\cancel{\rho A}L\dot{x}^2$$

$$-gx^2 = \tfrac{1}{2}L\dot{x}^2$$

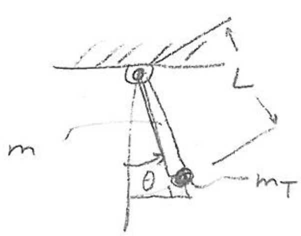
taking derivative wrt time

$$-g2\cancel{x}\cancel{\dot{x}} = \tfrac{L}{2}2\cancel{\dot{x}}\ddot{x}$$

$$\boxed{\ddot{x} + \frac{2g}{L}x = 0}$$

Problem 1.65

Pendulum arm of mass m
Mass at end of arm m_T



$$-\Delta U = \Delta T$$

$$-m_T g L(1-\cos\theta) - mg\tfrac{L}{2}(1-\cos\theta) = \tfrac{1}{2}m_T L^2\dot{\theta}^2 + \tfrac{1}{2}\left(\tfrac{1}{3}mL^2\right)\dot{\theta}^2 = 0$$

$$(m_T + \tfrac{m}{2})gL(1-\cos\theta) + (m_T + \tfrac{m}{3})\tfrac{1}{2}L^2\dot{\theta}^2 = 0$$

taking derivative wrt time :

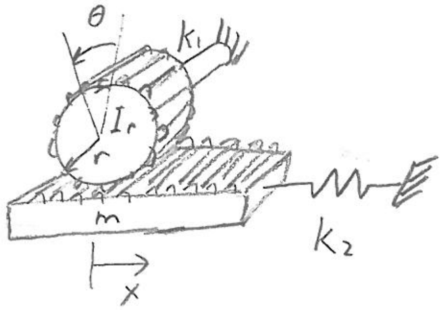
$$(m_T + \tfrac{m}{2})g\cancel{L}\sin\theta\cancel{\dot{\theta}} + (m_T + \tfrac{m}{3})L^{\cancel{2}}\cancel{\dot{\theta}}\ddot{\theta} = 0$$

taking $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{(m_T + \frac{m}{2})}{(m_T + \frac{m}{3})}\frac{g}{L}\theta = 0$$

Problem 1.67

Given: rack & pinion setup with torsional & lateral spring as shown.



Find: 1) "Equation of motion" → $x(t)$ or system differential equation?
- do both, just to be sure.

2) Natural frequency

Sol'n:

1) $-\Delta U = \Delta T$

torsional shaft, linear spring

$$-\left(\tfrac{1}{2}k_1\theta^2 + \tfrac{1}{2}k_2x^2\right) = \tfrac{1}{2}I_p\dot{\theta}^2 + \tfrac{1}{2}m\dot{x}^2$$ ← rack K.E.

pinion K.E.

2) θ and x are related ⇒ $x = r\theta$ or $\theta = \frac{x}{r}$ $\quad \dot{\theta} = \frac{\dot{x}}{r}$

- put all θ in terms of x

$$-\left(\tfrac{1}{2}k_1\left(\tfrac{x}{r}\right)^2 + \tfrac{1}{2}k_2x^2\right) = \tfrac{1}{2}I_r\left(\tfrac{\dot{x}}{r}\right)^2 + \tfrac{1}{2}m\dot{x}^2$$

3) Take derivative wrt to time

$$-\frac{k_1}{2r^2}2x\dot{x} - \frac{k_2}{2}2x\dot{x} = \frac{I_r}{2r^2}(2\dot{x})\ddot{x} + \frac{m}{2}(2\dot{x})\ddot{x}$$

4) Cancel $\dot{x}$

$$-\frac{k_1}{r^2}x - k_2x = \frac{I_p}{r^2}\ddot{x} + m\ddot{x}$$

5) Put into standard D.E. form

$$\boxed{\left(\frac{I_p}{r^2} + m\right)\ddot{x} + \left(\frac{k_1}{r^2} + k_2\right)x = 0}$$

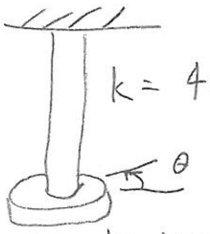
6) Solve for ω_n, $x(t)$

$$\boxed{\omega_n = \sqrt{\frac{\frac{k_1}{r^2} + k_2}{\frac{I_p}{r^2} + m}}}$$

- need initial conditions to get this.

$$x(t) = A\sin(\omega_n t + \phi)$$

Problem 1.73



$k = 400$ N·m/rad

$c = 20$ N·m·s/rad

$J = 1000$ m²·kg

$$J\ddot{\theta} + c\dot{\theta} + k\theta = 0$$

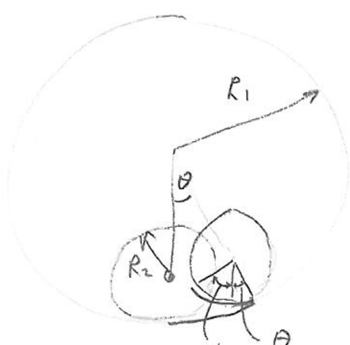
$$\omega_n = \sqrt{\frac{k}{J}} = \sqrt{\frac{400}{1000}} = 0.632 \text{ rad/s}$$

$$\zeta = \frac{c}{2\sqrt{kJ}} = \frac{20}{2\sqrt{400 \cdot 1000}} = 0.0158$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 0.632\sqrt{1-0.0158^2} = 0.632 \frac{\text{rad}}{\text{s}}$$

H.W.: 1.69, 1.70

Trick with 1.70



need to find relationship between θ & ϕ

Procedure for Using Energy Method to obtain system Equations

Step 1: $-\Delta U = \Delta T$

$\Delta U_g = mg(h_2 - h_1)$ - gravity

$\Delta U_k = \frac{1}{2} k (X_2^2 - X_1^2)$ - spring

$k = \frac{AE}{L}$ - bar

$k = \frac{Gd^4}{64nR^3}$ - coil spring

$k = \frac{3EI}{L^3}$ - cantilever

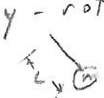
$\Delta U_{k_T} = \frac{1}{2} k (\theta_2^2 - \theta_1^2)$ - torsional spring $k = \frac{J_P G}{L}$ - bar torsion

polar moment of inertia m^4

$\Delta T_k = \frac{1}{2} m (\dot{x})^2$ - kinetic energy

$\Delta T_{k_r} = \frac{1}{2} J (\dot{\theta})^2$ - kinetic energy - rotational - J is mass moment of inertia $kg\,m^2$

$J = mL^2$



- parallel axis theorem

$J_1 = J_2 + mL^2$

Step 2: Get in terms of one independent variable

E.g. - solve for all θ in terms of x, or

- solve for all x in terms of θ

Step 3: take derivative wrt time

Step 4: cancel $\dot{x}$ or $\dot{\theta}$ terms

Step 5: put into nice differential equation form:

- put into same form as $m\ddot{x} + kx = 0$

Step 6: Solve for ω_n, etc. if required.