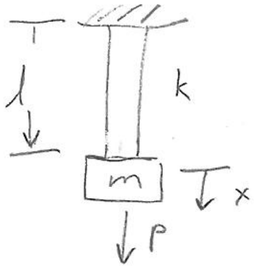
Chapter 1.5 Stiffness

The Stiffness of a spring depends on the material and geometry of the spring. The stiffness is given by $k = \frac{\Delta P}{\Delta X}$ for linear motion and $k = \frac{\Delta M}{\Delta \theta}$ for rotational motion.

E.g. for a straight bar with axial motion:

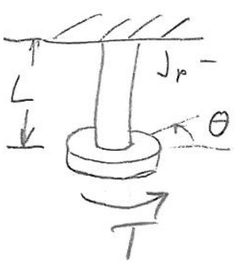


$$\delta = \frac{PL}{AE} \quad \text{or} \quad \frac{P}{\delta} = \frac{AE}{L}$$

$P_1 = 0 \quad x_1 = 0$
$P_2 = P \quad x_2 = \delta$

$$k = \frac{AE}{L}$$

For a twisting rod



J_p – polar moment of inertia of rod

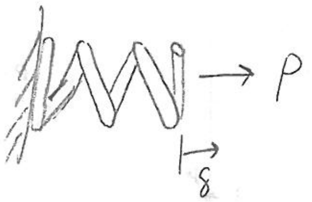
$\theta_1 = 0 \quad M_1 = 0$
$\theta_2 = \theta \quad M_2 = T$

$$\theta = \frac{TL}{J_p G}$$

$$\frac{T}{\theta} = \frac{J_p G}{L} \quad \text{or} \quad k = \frac{J_p G}{L}$$

G is shear modulus for material

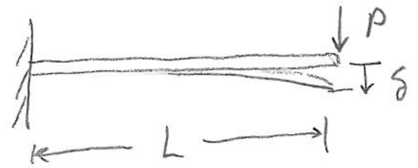
Coil spring



$$k = \frac{P}{\delta} = \frac{G d^4}{64 n R^3}$$

d is diameter of wire
$2R$ is diameter of turns
n is number of turns

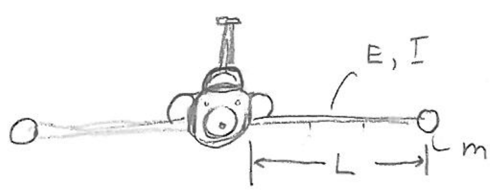
Cantilever Beam



$$k = \frac{P}{\delta} = \frac{3EI}{L^3}$$

I is moment of inertia
about neutral axis

E.g. aircraft with wing-tip fuel tanks
E.g. F-5 freedom fighter



Assume $m = 10\,\text{kg}$ empty
$m = 1000\,\text{kg}$ full

$I = 5.2 \times 10^{-5}\ m^4$
$E = 6.9 \times 10^{9}\ N/m^2$
$L = 2\ m$

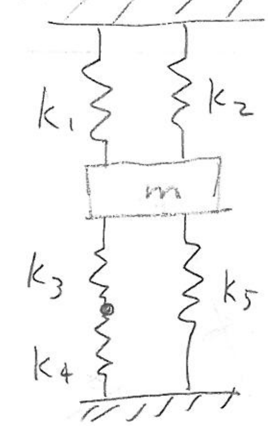
$$\omega_{n_{full}} = \sqrt{\frac{3EI}{mL^3}} = \sqrt{\frac{(3)(6.9\times10^9)(5.2\times10^{-5})}{1000(2)^3}} = 11.6 \text{ rad/s}$$

$$\omega_{n_{empty}} = \sqrt{\frac{3EI}{mL^3}} = \sqrt{\frac{(3)(6.9\times10^9)(5.2\times10^{-5})}{10\,(2)^3}} = 115 \text{ rad/s}$$

- Text gives stiffness for a variety of other spring types.

Problem 1.87

Given



$k_1 = k_5 = k_2 = 100\ N/m$

$k_3 = 50\ N/m$

$k_4 = 1\ N/m$

Find:

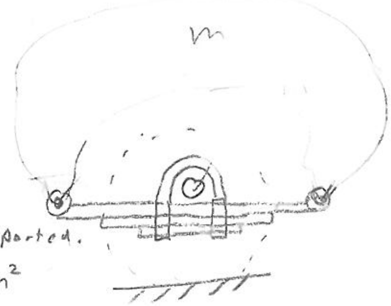
$k_{eq} = ?$

$$k_{eq} = k_1 + k_2 + \frac{1}{1/k_3 + 1/k_4} + k_5$$

$$= 100 + 100 + \frac{1}{1/50 + 1/1} + 100 = 300.98\ N/m$$

Problem 1.93

Given: Leaf spring



Let's make it simply supported.

$E_1 = 2\times10^{11}\ N/m^2$

$\omega_{n_1} \leftarrow$ a specific natural frequency

mass of spring is negligible compared to m (mass of device)

Find: ω_{n_2} for $E_2 = 7.1\times10^{10}\ N/m^2$

Sol'n:

$$\omega_{n_1} = \sqrt{\frac{k_1}{m}} \qquad k_1 = \underbrace{\frac{48E_1I}{L^3}}_{\text{for simply supported beam}} = C\,E_1$$

C is a constant that accounts for the geometry

$$\omega_{n_1} = \sqrt{\frac{CE_1}{m}} \rightarrow \frac{\omega_{n_1}^2}{E_1} = \frac{C}{m}$$

$$\omega_{n_2} = \sqrt{\frac{CE_2}{m}} \rightarrow \frac{\omega_{n_2}^2}{E_2} = \frac{C}{m}$$

$$\frac{\omega_{n_1}^2}{E_1} = \frac{\omega_{n_2}^2}{E_2} \rightarrow \omega_{n_2} = \omega_{n_1}\sqrt{\frac{E_2}{E_1}} = \omega_{n_1}\sqrt{\frac{7.1\times10^{10}}{2\times10^{11}}} = 0.596\,\omega_{n_1}$$

ω_n decreases by 40%

Homework 1.82 1.88