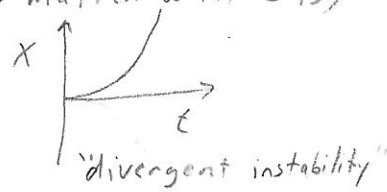
Chapter 1.8 Stability

- If m and k are positive, and c=0, the amplitudes of the vibration will remain constant → the system is well behaved (the system is stable)

- if m, c, and k are positive, the amplitudes of vibration will decrease; the vibrations will die out → the system is well behaved (the system is stable)

- if m is positive and k is negative (no matter what c is)

$$x(t) = A\sinh\omega_n t + B\cosh\omega_n t$$

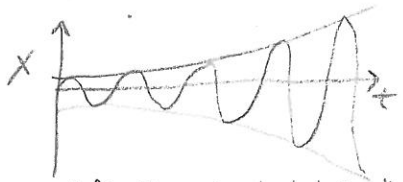


"divergent instability"

x(t) quickly goes to ±∞.

- the system is not well behaved (the system is unstable)

- if m and k are positive and c is negative, the motion grows without bound, with oscillations.



"flutter instability"

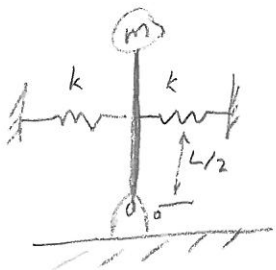
$$x = Ae^{-\zeta\omega_n t}\sin(\omega_n t + \phi)$$

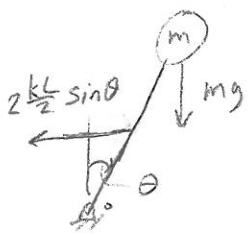
However $-\zeta\omega_n > 0$

- In general, if c<0 or k<0, the system is unstable.

E.g. Inverted Pendulum

$$\Sigma M_o = I\ddot{\theta}$$





$$kL\sin\theta\left(\frac{L}{2}\right) - mgL\sin\theta = -mL^2\ddot{\theta}$$

if $\sin\theta \approx \theta$

$$mL^2\ddot{\theta} + \left(\frac{kL^2}{2} - mgL\right)\theta = 0$$

Unstable if $mg > \frac{kL}{2}$

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