Institute of Technology
West Virginia University

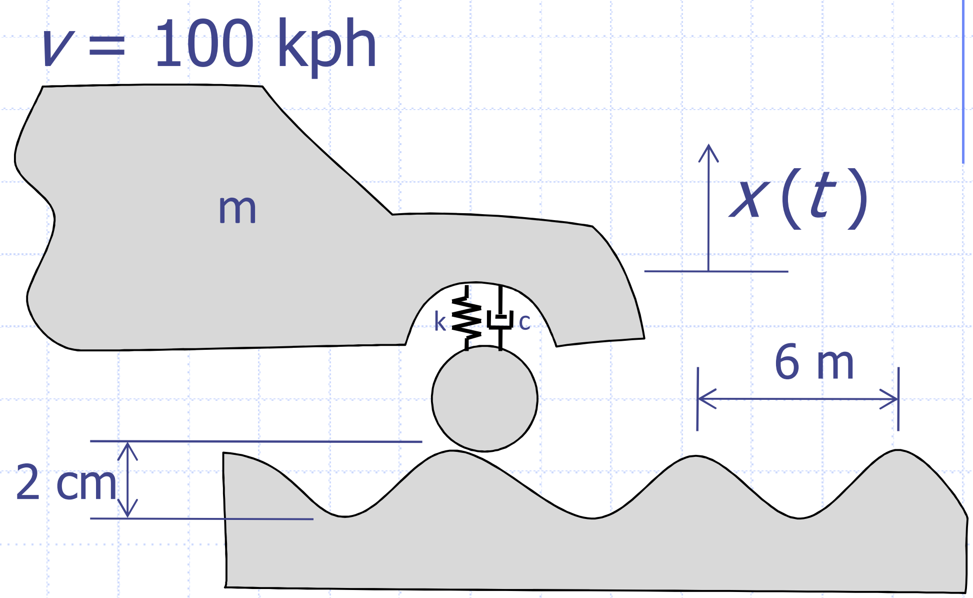
Base Excitation

Section 2.4

(Skipping Section 2.3)

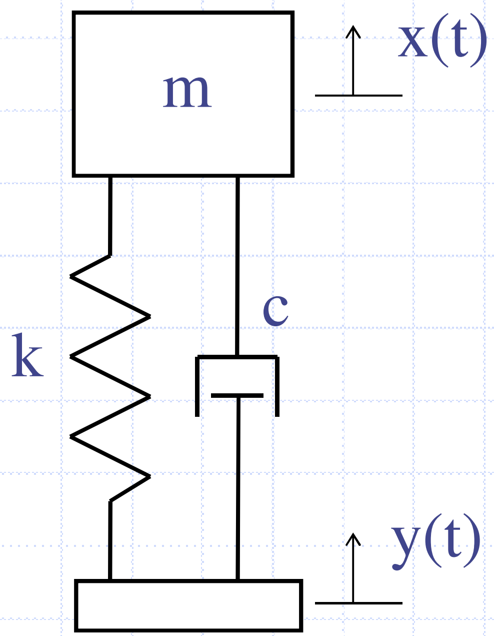
Vehicle traveling down bumpy road

- Given:
  - $m = 1007$ kg
  - $k = 4 \times 10^5$ N/m
  - $c = 20 \times 10^3$ Ns/m



- Find:
  - Amplitude of steady-state vibration: X
  - Amplitude of transmitted force: F_T

Equations of Motion for Moving Base



Moving Base

Free body diagram

Equations of Motion for Moving Base

If $$y(t) = Y \sin \omega t$$

and $$\dot{y}(t) = \omega Y \cos \omega t$$

then

$$m\ddot{x} + c\dot{x} + kx = cY\omega \cos \omega t + kY \sin \omega t$$

or

$$\ddot{x} + 2\varsigma\omega_n \dot{x} + \omega_n^2 x = 2\varsigma\omega_n \omega Y \cos \omega t + \omega_n^2 Y \sin \omega t$$

Solving the Differential Equation

- Solving in the same way as for $F_0\cos\omega t$ yields:

$$x_p(t) = \frac{2\varsigma\omega_n\omega Y}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\varsigma\omega_n\omega\right)^2}}\cos\left(\omega t-\theta_1\right)$$

$$+\frac{\omega_n^2 Y}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\varsigma\omega_n\omega\right)^2}}\sin\left(\omega t-\theta_1\right)$$

where: $$\theta_1 = \tan^{-1}\left(\frac{2\varsigma\omega_n\omega}{\omega_n^2-\omega^2}\right)$$

Solving the Differential Equation

$$x_p(t) = \omega_n Y \left[\frac{\omega_n^2 + (2\varsigma\omega)^2}{(\omega_n^2 - \omega^2)^2 + (2\varsigma\omega_n\omega)^2} \right]^{1/2} \cos(\omega t - \theta_1 - \theta_2)$$

where: $\theta_2 = \tan^{-1}\left(\frac{\omega_n}{2\varsigma\omega} \right)$

Solving the Differential Equation

- Final Equation (for **base excitation** without free vibration):

$$x_p(t) = X\cos(\omega t - \theta_1 - \theta_2)$$

where:

$$X = Y\left[\frac{1+(2\varsigma r)^2}{\left(1-r^2\right)^2+(2\varsigma r)^2}\right]^{1/2}$$

$$\theta_1 = \tan^{-1}\left(\frac{2\varsigma r}{1-r^2}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{1}{2\varsigma r}\right)$$

$$r = \frac{\omega}{\omega_n}$$

Plotting the "Displacement Transmissibility"

- Looking at just the steady-state vibration $x_p(t)$, we can plot the ratio of the amplitude X versus the amplitude Y as a function of base frequency ω.

How much force is transmitted?

$$F(t) = k(x - y) + c(\dot{x} - \dot{y}) = -m\ddot{x}(t)$$

$$-m\ddot{x}(t) = -m\frac{d^2 x_p(t)}{dt^2} = m\omega^2 Y\left[\frac{1+(2\varsigma r)^2}{(1-r^2)+(2\varsigma r)^2}\right]^{1/2}\cos(\omega t - \theta_1 - \theta_2)$$

- Final equation (for **transmitted force** due to steady-state vibration):

$$F(t) = F_T\cos(\omega t - \theta_1 - \theta_2)$$

$$F_T = kYr^2\left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2}\right]^{1/2}$$

Plotting the "Force Transmissibility"

- Looking at just the forced vibration $x_p(t)$, we can plot the ratio of the amplitude of the dynamic transmitted force F_T versus the static force kY as a function of base frequency ω.