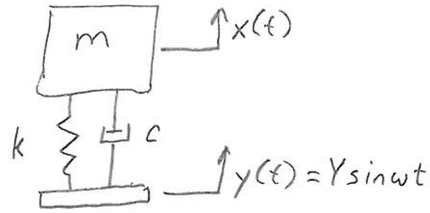
Bumpy Road Problem

Given:



$m = 1007$ kg

$k = 4 \times 10^5$ N/m

$c = 20 \times 10^3$ Ns/m

$$\omega = \frac{100 \frac{\cancel{km}}{\cancel{hr}}}{3600 \frac{s}{\cancel{hr}}} \times \frac{1000 \frac{\cancel{m}}{\cancel{km}}}{6 \frac{\cancel{m}}{cyc}} \times 2\pi \frac{rad}{\cancel{cyc}} = 29.1 \text{ rad/s}$$

$$Y = \frac{0.02}{2} = 0.01 \text{ m}$$

Find: X, F_T

Sol'n:

$$X = Y \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

$$= 0.01 \left[\frac{1 + 1.454^2}{(1-1.46^2)^2 + 1.454^2} \right]^{1/2}$$

$$= 0.00958 \text{ m}$$

$$= 9.58 \text{ mm}$$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{20 \times 10^3}{2\sqrt{4 \times 10^5 \times 1007}}$$

$$= 0.498$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4 \times 10^5}{1007}}$$

$$= 19.93 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = \frac{29.1}{19.93} = 1.460$$

$$2\zeta r = 2 \times 0.498 \times 1.460 = 1.454$$

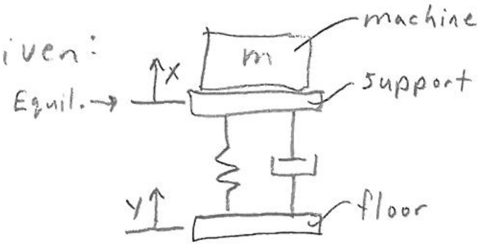
$$F_T = k r^2 X$$

$$= 4 \times 10^5 (1.46)^2 (0.00958)$$

$$= 8170 \text{ N}$$

Problem 2.~~37~~ 47

Given:



$$m = \frac{2000\,N}{9.81\ m/s^2} = 203.9\ kg$$

$$k = \frac{2000\,N}{0.05\,m} = 40\,000\ N/m$$

$$Y = 0.002\ m$$

$$r = 1$$

$$\zeta = 0.01$$

Find: $F(t)$, X

Sol'n: $$F_T = kYr^2\left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

$$= (40\times10^3)(0.002)(1)^2\left[\frac{1 + (2\times0.01\times1)^2}{(1-1)^2 + (2\times0.01\times1)^2}\right]^{1/2}$$

$$= 4000\ N$$

$$F(t) = F_T\cos(\omega t - \theta_1 - \theta_2)$$

$$\omega = \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40\,000}{203.9}} = 14.01\ rad/s$$

$$\theta_1 = \tan^{-1}\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right) = \tan^{-1}\left(\frac{2\times0.01\times14.01^2}{14.01^2 - 14.01^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\theta_2 = \tan^{-1}\left(\frac{\omega_n}{2\zeta\omega}\right) = \tan^{-1}\left(\frac{\cancel{14.01}}{2\times0.01\times\cancel{14.01}}\right) = \tan^{-1}(50) = 88.9° = 1.55\ rad.$$

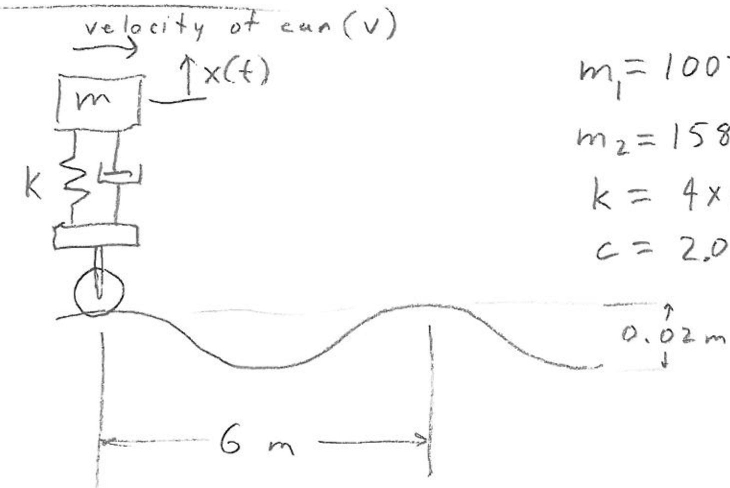
$$\boxed{F(t) = 4000\cos(14.01t - 3.12)}$$

$$X = Y\left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2} = 0.002\left[\frac{1 + (2\times0.01\times1)^2}{(1-1^2)^2 + (2\times0.01\times1)^2}\right]^{1/2} = 0.100\ m$$

$$\boxed{X = 10\ cm}$$

Problem 2.~~46~~ 56

Given:



$m_1 = 1007 + 200 = 1207$ kg

$m_2 = 1585 + 200 = 1785$ kg

$k = 4 \times 10^4$ N/m

$c = 2.0 \times 10^3 \frac{N \cdot s}{m}$

$Y = \frac{0.02}{2} = 0.01$ m

Find: X at $v = 20, 80, 100,$ and 150 km/hr

- Compare with results from Table 2.1

Sol'n: Step 1) $\omega = \frac{v\,(m/s)}{6} \times 2\pi$

at $20\frac{km}{hr}$ $\omega_1 = \frac{\left(\frac{20000}{3600}\right)}{6} \times 2\pi = 5.818$ rad/s

at $80\frac{km}{hr}$ $\omega_2 = 23.271$ rad/s

at $100\frac{km}{hr}$ $\omega_3 = 29.088$ rad/s

at $150\frac{km}{hr}$ $\omega_4 = 43.633$ rad/s

Step 2) car 1 $\omega_{n_1} = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{4\times10^4}{1207}} = 5.757$ rad/s

car 2 $\omega_{n_2} = \sqrt{\frac{k}{m_2}} = \sqrt{\frac{4\times10^4}{1785}} = 4.734$ rad/s

Step 3) car 1 $\zeta_1 = \frac{c}{2\sqrt{km_1}} = \frac{2.0\times10^3}{2\sqrt{40000 \times 1207}} = 0.1439$

car 2 $\zeta_2 = \frac{c}{2\sqrt{km_2}} = \frac{2.0\times10^3}{2\sqrt{4\times10^4 \cdot 1785}} = 0.1183$

Step 4) $r = \frac{\omega}{\omega_n}$

$r = \frac{\omega}{\omega_n}$	$20 \frac{km}{hr}$	80	100	150
Car 1	1.011 -was 0.923	4.042 -was 3.692	5.053 -was 4.615	7.579 -was 6.923
Car 2	1.229 -was 1.158	4.916 4.632	6.145 5.79	9.217 8.686

Step 5) $$X = Y \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

X(cm)	20	80	100	150
Car 1	3.57 -was 3.19	0.10 0.12	0.07 0.09	0.04 0.05
Car 2	1.77 -was 2.32	0.07 0.07	0.05 0.05	0.03 0.03

Adding the mass makes the natural frequency get smaller and makes r get larger, therefore the displacement amplitudes go down if r is already less than one, otherwise the amplitudes go up as the system gets closer to resonance (as with car 1 at 20kph). The increased mass also causes ζ to decrease, resulting in higher amplitudes near resonance (as with car 1 at 20kph). At higher values of r, the effect is less noticeable. With the heavier car, the effect is less noticeable.

Homework: 2.~~42~~ 52 & 2.~~43~~ 53