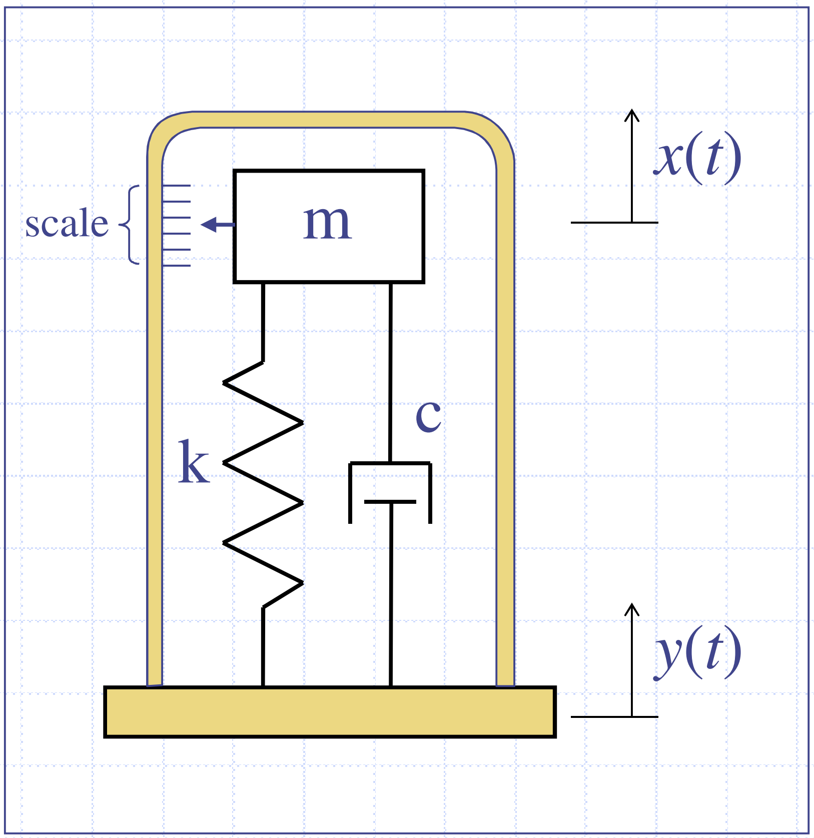
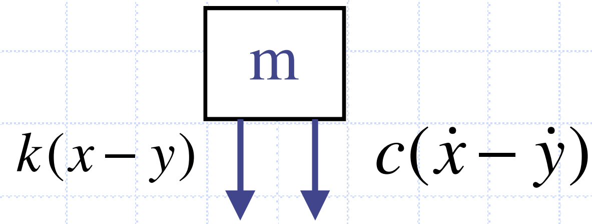
Measurement Devices

Section 2.6

Equations of Motion for Vibration Measurement Devices

- We can't directly observe x(t); but we can directly observe z(t) = x(t) – y(t)!!





$$\sum F = -k(x-y) - c(\dot{x}-\dot{y})$$

$$= m\ddot{x} = m(\ddot{x}-\ddot{y}) + m\ddot{y}$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

If $\quad y(t) = Y\cos\omega t$

and $\quad \ddot{y}(t) = -\omega^2 Y\cos\omega t$

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y\cos\omega t$$

Solving the Differential Equation

- Final Equation (for **motion relative to harmonically moving base** without free vibration):

$$z_p(t) = Z\cos(\omega t - \theta)$$

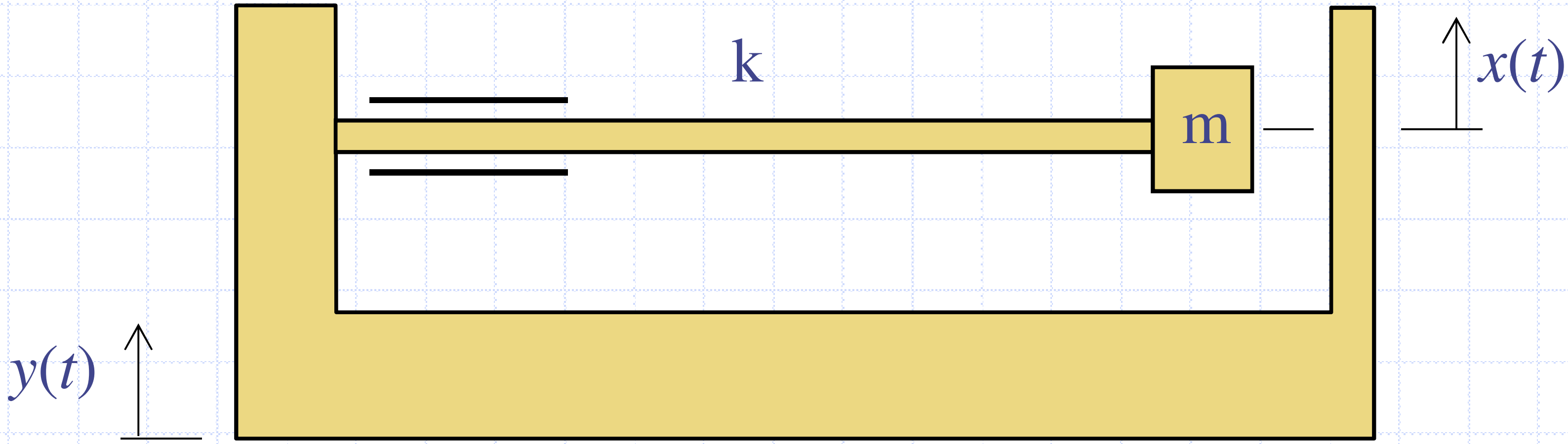
where:

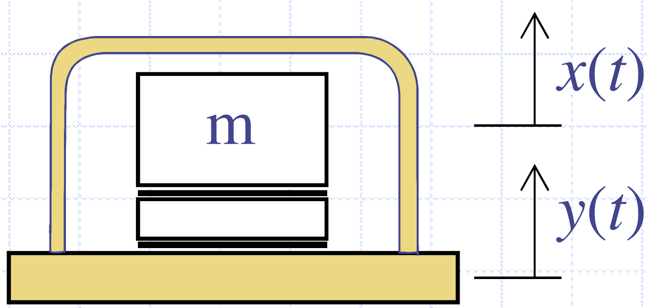
$$Z = \frac{Yr^2}{\sqrt{(1-r^2)^2 + (2\varsigma r)^2}}$$

$$\theta = \tan^{-1}\left(\frac{2\varsigma r}{1-r^2}\right)$$

$$r = \frac{\omega}{\omega_n}$$

Other Implementations





Plotting the Frequency Response

- What is the relationship between the observed vibration amplitude (Z) and the measured vibration amplitude (Y)?

Plotting the Frequency Response

- What is the relationship between the observed vibration amplitude (Z) and the base acceleration amplitude ($\omega^2 Y$)?

$$\frac{Z}{\omega^2 Y} = \frac{1/\omega_n^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$