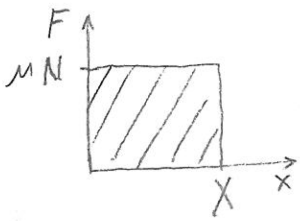
C_{eq} calculation for Coulomb Friction

Energy dissipated $E = Fd$ if force F is constant,

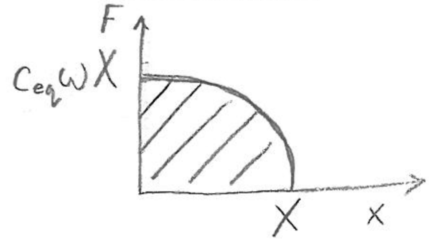
$E = \int_0^d F\,dx$ otherwise.

For each quarter cycle:

Actual



Equivalent



If $x(t) = X\sin\omega t$

then $F(t) = C_{eq}\dot{x}$

$= C_{eq}\omega X\cos\omega t$

and $t(x) = \dfrac{\sin^{-1}\left(\frac{x}{X}\right)}{\omega}$

$$\therefore F(x) \doteq C_{eq}\omega X\cos\left(\cancel{\omega}\left(\frac{\sin^{-1}\frac{x}{X}}{\cancel{\omega}}\right)\right)$$

$$= C_{eq}\omega X\sqrt{1-\left(\frac{x}{X}\right)^2}$$

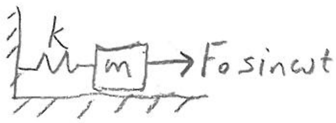
$$= C_{eq}\omega\sqrt{X^2 - x^2}$$

$$\mu N X = \frac{C_{eq}\omega\pi X^2}{4} \Leftarrow \int_0^X F(x)\,dx$$

$$\boxed{C_{eq} = \frac{4\mu N}{\pi\omega X}}$$

Coulomb Damping Problem

Given:



$m = 0.1\ kg$

$k = 10\ N/m$

$\mu = 0.05$

$F_o = 0.5\ N$

$\omega = 2\pi \cdot 1 = 6.28\ rad/s$

Find: X

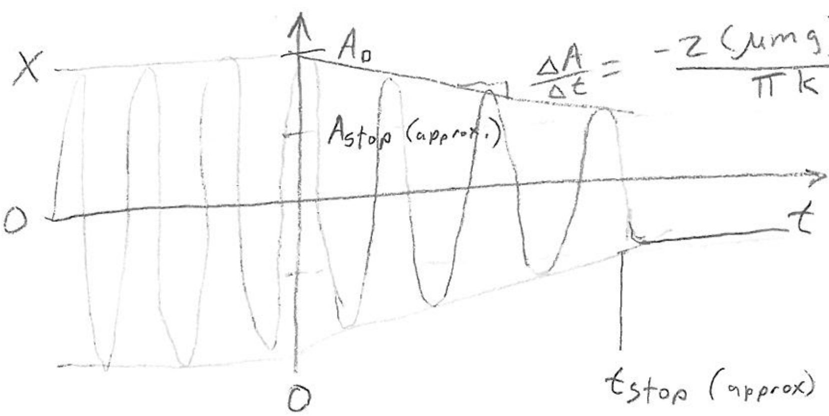
Sol'n: $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{0.1}} = 10\ rad/s$

$r = \frac{\omega}{\omega_n} = \frac{6.28}{10} = 0.628$

Approximate $C_{eq} = \frac{4\mu m g}{\pi \omega X}$ ← C_{eq} & X are both unknown, but this was solved using:

$$X = \frac{F_o}{k} \frac{\sqrt{1 - \left(\frac{4\mu m g}{\pi F_o}\right)^2}}{|1 - r^2|} = \frac{0.5}{10} \frac{\sqrt{1 - \left(\frac{4(0.05)(0.1)(9.81)}{\pi\ 0.5}\right)^2}}{|1 - 0.628^2|} = \underline{0.082\ m}$$

What if the harmonic force was turned off?



$\frac{\Delta A}{\Delta t} = \frac{-2(\mu m g)\omega_n}{\pi k}$

$A_{stop} = \frac{\mu m g}{k}$

$= \frac{(0.05)(0.1)(9.81)}{10}$

$= 0.0049\ m$

$A_o = X = 0.082\ m$

$$\frac{A_{stop} - A_o}{t_{stop} - 0} = \frac{-2\mu m g \omega_n}{\pi k}$$

$$\frac{0.0049 - 0.082}{t_{stop}} = \frac{-2(0.05)(0.1)(9.81)(10)}{\pi\ 10}$$

$t_{stop} = 2.47$ seconds is approximately how long it would continue oscillating.

$\boxed{\text{Homework } 2.\cancel{63}\ 73}$