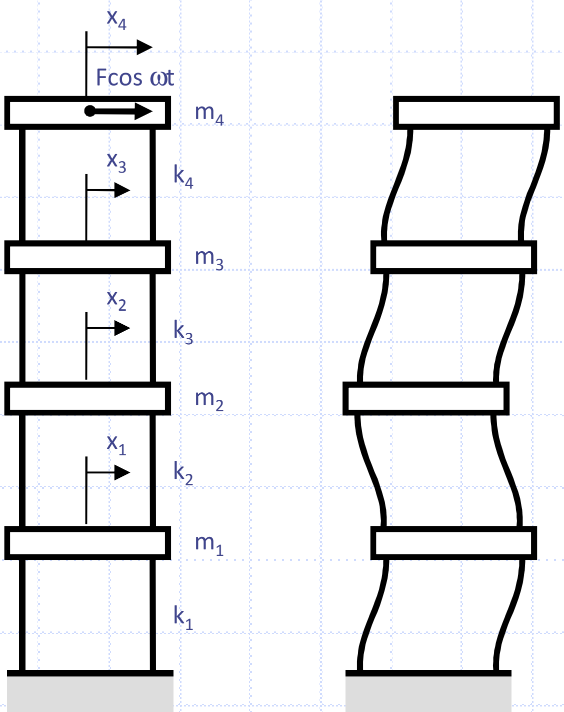
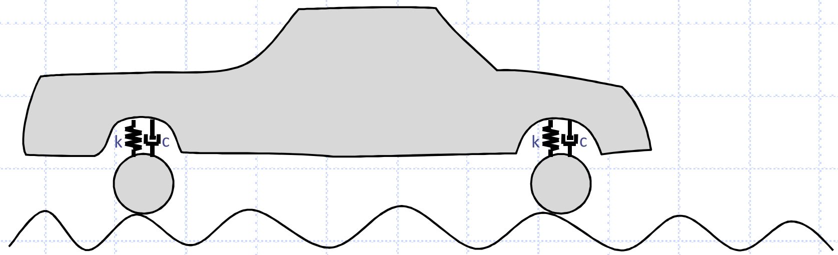
Multi-Degree-of-Freedom Systems

Section 4.1

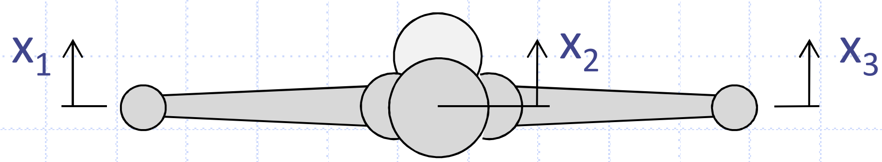
Examples of Multi-Degree-of-Freedom (MDOF) Systems



Swaying Building



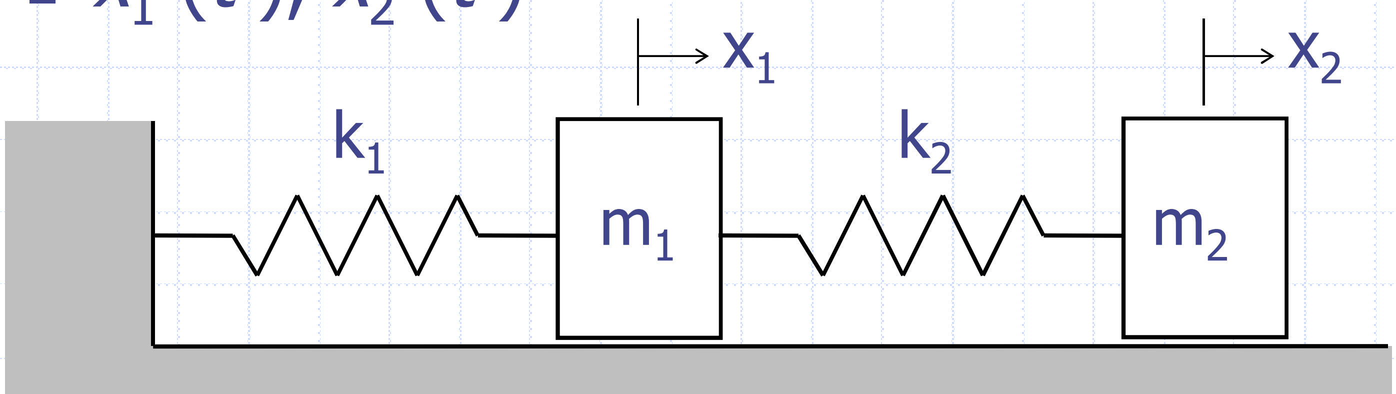
Moving Vehicle



Flying Aircraft

Two DOF Spring-Mass System

- Given:
  - $m_1 = 9$ kg
  - $m_2 = 1$ kg
  - $k_1 = 24$ N/m
  - $k_2 = 3$ N/m
  - $x_{1,0} = 1$ mm
  - $x_{2,0} = 0$ mm
  - $v_{1,0} = 0$ m/s
  - $v_{2,0} = 0$ m/s
- Find:
  - ω_{n1}, ω_{n2}
  - $x_1(t), x_2(t)$



Two DOF Spring-Mass System

- Sol’n:

Step 1: Derive system differential equations

Mass 1

Mass 2

Two DOF Spring-Mass System

- Write out coupled equations as matrix equation

With variables:

With numbers:

Solving Differential Equation

Step 2: Solve matrix differential equation

Assume a solution: $\bar{x}(t) = \bar{u}\sin(\omega_n t + \phi)$

$$\bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} u_1 \sin(\omega_n t + \phi) \\ u_2 \sin(\omega_n t + \phi) \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \sin(\omega_n t + \phi)$$

$$\ddot{\bar{x}}(t) = \begin{bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} =$$

Solving Differential Equation

Substitute back into system differential equation:

Simplify:

$$\bar{u} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ is a trivial solution; we are not interested in it.

Solving Differential Equation

If we could solve for $\left(-\omega_n^2\overline{M}+\overline{K}\right)^{-1}$ then we could get

$$\overline{u}=\left(-\omega_n^2\overline{M}+\overline{K}\right)^{-1}\overline{0}=\overline{0}=\begin{bmatrix}0\\0\end{bmatrix}$$

but this is still the trivial solution!

The only way to get a non-trivial solution would be if it were impossible to solve for $\left(-\omega_n^2\overline{M}+\overline{K}\right)^{-1}$

When is a matrix not invertible??

Solving Differential Equation

When the determinant is zero!!

Need to find ω_n that satisfy: $\det\left(-\omega_n^2\overline{M}+\overline{K}\right)=0$

Recall that the determinant of a matrix is computed as follows:

Solving Differential Equation

Putting in the numbers:

Mode 1: ω_{n1} =

Substituting ω_{n1} into equation from slide 7:

Mode 1

These equations are **not linearly independent**. There is no single solution. Instead there is a set of solutions. But each solution must satisfy:

We could use a scaling factor (*s*):

This is the **"Mode Shape"**.

Mode 1

Usually *s* is chosen so that the largest u value is 1:

This is the **"Normalized Mode Shape"**.

Therefore, the equation of motion, considering only the first mode is:

Mode 2: ω_{n2} =

Substituting ω_{n2} into equation from slide 7:

Mode 2

- The normalized mode shape is:

- The equation of motion, considering only the second mode is:

Both Modes Together

- The overall equation of motion, considering both modes is:

- The unknowns (A_1, Φ_1, A_2, Φ_2) must be determined from the initial conditions.

Solving for A_1, Φ_1, A_2, Φ_2

- Applying the initial conditions:

$$\bar{x}(t) =$$

$$\dot{\bar{x}}(t) =$$

$$\bar{x}(0) =$$

$$\dot{\bar{x}}(0) =$$

Solving for A_1, Φ_1, A_2, Φ_2

- Need to solve the nonlinear equations:

Solving for A_1, Φ_1, A_2, Φ_2

Solving for A_1, Φ_1, A_2, Φ_2

- Therefore, the overall equation of motion is: