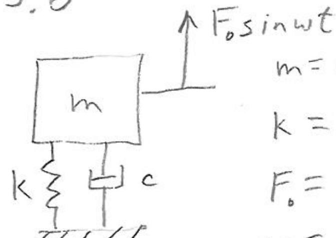
Problem 5.6

Given:



$m = 100\,kg$

$k = 700 \times 10^3\ N/m$

$F_0 = 350\ N$

$\omega = \dfrac{3000\ rpm}{60\ sec/min} \times 2\pi\ rad/rot = 314.2\ rad/s$

$\zeta = 0.2$

Find: (a) displacement amplitude X

(b) transmissibility ratio $T.R. = \dfrac{F_T}{F_0}$

(c) magnitude of transmitted force F_T

Sol'n: (a) For - SDOF system

- Forced vibration
- with damping

$$X = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$r = \frac{\omega}{\omega_n} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{700\times10^3}{100}} = 83.67\ rad/s$$

$$r = \frac{314.2}{83.67} = 3.755$$

$$X = \frac{350/700\times10^3}{\sqrt{(1-3.755^2)^2 + (2\times0.2\times3.755)^2}}$$

$$= 3.792\times10^{-5}\ m$$

(b)

$$\frac{F_T}{F_o} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

$$= \left[\frac{1 + (2 \times 0.2 \times 3.755)^2}{(1-3.755^2)^2 + (2\times 0.2 \times 3.755)^2}\right]^{1/2}$$

$$= 0.1369$$

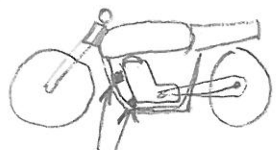
(c) $F_T = \left(\frac{F_T}{F_o}\right) F_o = 0.1369 \times 350 = 47.9 \text{ N}$

Homework 5, ~~8~~ 8

Isolation System Design Problem

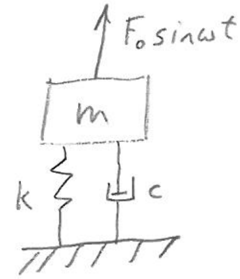
Problem: A dirt bike engine is currently rigidly mounted to the dirt bike frame, but is transmitting too much vibration. Design an isolation system to transmit as little vibration as possible.

Given:



- add "isolation system" (rubber mounts) here

approximate as SDOF system $\longrightarrow$ with harmonic applied force assuming frame is stationary

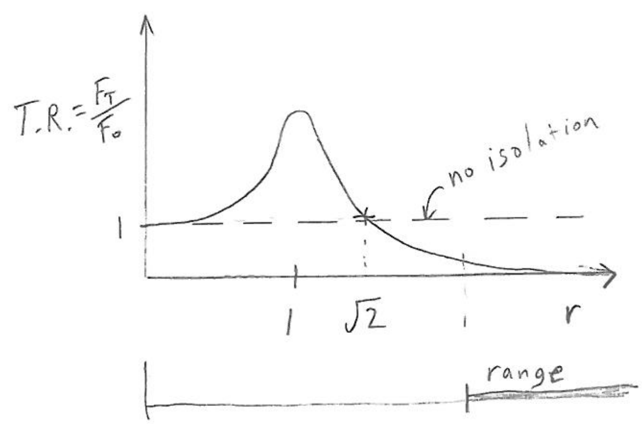


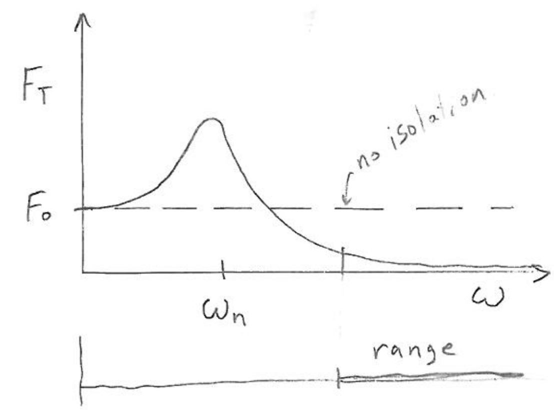
$m = 50$ kg

$F_o = 2$ N

$\omega = ?$

- Need to clarify the problem. Does the customer want to:
  - minimize the transmitted force at a given ω?
  - minimize the maximum transmitted force (considering all ω)?
  - minimize the maximum transmitted force for a specific frequency range?





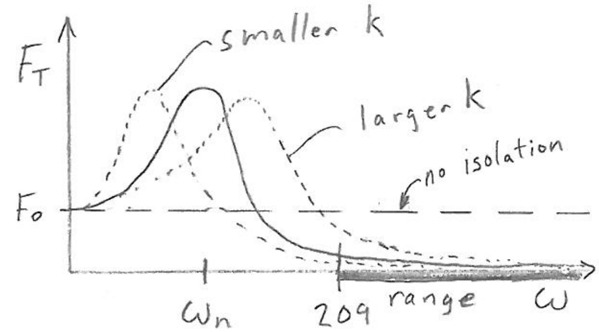
- assume customer wants to minimize maximum transmitted force for $\omega \geqslant \frac{2000 \text{ RPM}}{60 \text{ sec/min}} \times 2\pi \text{ rad/rev} = 209$ rad/s

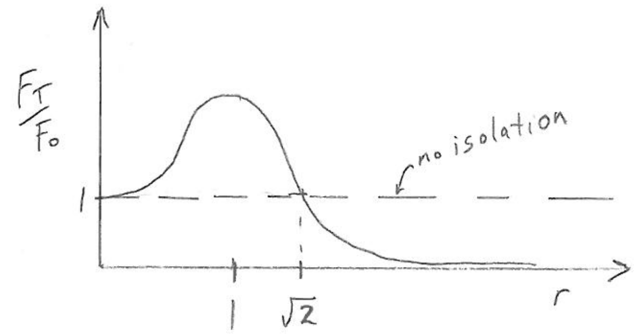
Find: (a) ideal k & c

(b) rubber bushing from catalog that best matches k & c requirement

(c) level of improvement: $\left(1-\frac{F_T}{F_o}\right)\times 100\%$ (vibration reduction percentage)

Sol'n: (a) 1. Choose k first





- Smaller k is better, but how small can k be?
  - What happens if k is too small?
    - Too much static deflection
    - Very large deflections can be initiated at low frequencies.
- Two possible approaches are:
  - Given X_{static}, solve for $k=\frac{mg}{X_{static}}$
  - Given % reduction (and estimate of ζ), solve for r & then k

    E.g. specify $\frac{F_T}{F_o}=0.1$ at 2000 RPM (90% reduction)

    then $\frac{F_T}{F_o}<0.1$ at greater than 2000 RPM (more than 90% improvement above 2000 RPM)
- Going with first approach, assume $X_{static}=2.5$ mm.

Then

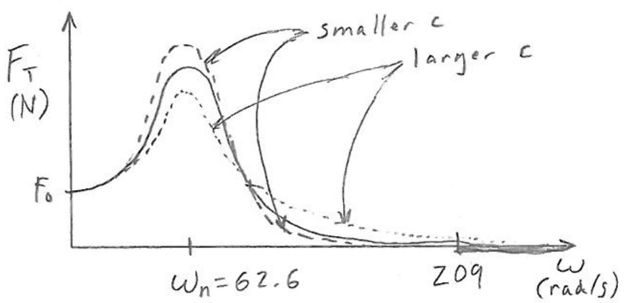
$$k=\frac{mg}{X_{static}}=\frac{(50)(9.81)}{(0.0025)}=196.2\times10^3\ N/m$$

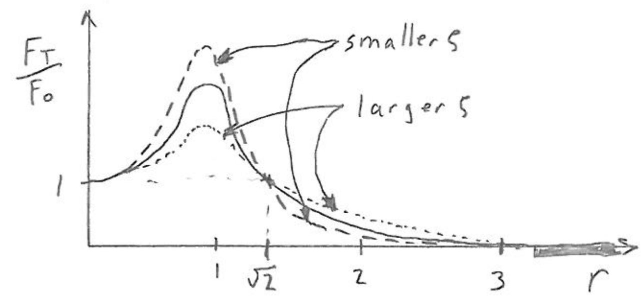
$$\boxed{k=196.2\ kN/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{196200}{50}} = 62.6 \text{ rad/s} \qquad (598 \text{ RPM})$$

$$r = \frac{\omega}{\omega_n} = \frac{209}{62.6} = 3.34$$

2. Next Choose c





- smaller c is better for $r > \sqrt{2}$
- larger c is better for $r < \sqrt{2}$
- Since we are interested in isolating for $r > 3.34$, smaller c is better. But what happens if c is too small?
  - Too much vibration amplitude at resonance.
  - Too much transmitted force at resonance.
- Let us assume that the transmitted force should not increase by more than 3 times (at resonance). $(r=1)$

$$\frac{F_T}{F_o} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2} = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2} = 3$$

Solving for ζ:

$$3 \times 2\zeta = \sqrt{1+4\zeta^2}$$
$$36\zeta^2 = 1 + 4\zeta^2$$
$$32\zeta^2 = 1$$
$$\zeta^2 = \frac{1}{32} \qquad \underline{\zeta = 0.177}$$

$$\zeta = \frac{c}{2\sqrt{km}} \qquad c = \zeta(2\sqrt{km}) = (0.177)(2\sqrt{196200 \times 50}) = 1109 \ \frac{Ns}{m}$$

$$\boxed{c = 1109 \ \frac{Ns}{m}}$$

(b) Looking at catalog for isolator with $k = 196.2 \frac{kN}{m}$ & $c = 1109 \frac{Ns}{m}$

Commercial isolators are often listed in terms of static load and deflection at static load. For us:

$$F_{static} = mg = 50 \times 9.81 = 490.5 \text{ N}$$

$$X_{static} = 2.5 \text{ mm} \quad \text{(we used this to get } k\text{)}$$

(Some listings also use engine weight & horse power.)

- Looking at "CBA Series Center-bonded mounts" from Lord Corporation of Erie, PA (www.lord.com)

$\boxed{\text{CBA12-100}}$ provides 445N at 2.3 mm

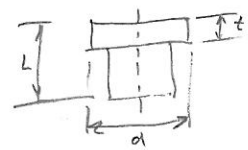
correction for highway $\frac{\text{Dyn}}{\text{Static}}$ rate from datasheet

$$k = \frac{445 \times 0.80 \times 1.08}{0.0023} = \underline{167 \frac{kN}{m}}$$

(next size up provides $k = 330$ kN/m)

$c = ?$ Damping values are much harder to get.

- dimensions are 1.25" dia × 1.44" length with 0.550" thk flange.



(c) Levels of improvement

- Using selected isolator

$$\omega_n = \sqrt{\frac{167000}{50}} = 57.8 \text{ rad/s} \quad (552 \text{ RPM})$$

$$r = \frac{\omega}{\omega_n} = \frac{209}{57.8} = 3.62$$

- assume $\zeta = 0.5$
- at $r = 3.62$

$$\frac{F_T}{F_0} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2} = \left[\frac{1 + (2 \times 0.5 \times 3.62)^2}{(1-3.62^2)^2 + (2 \times 0.5 \times 3.62)^2}\right]^{1/2}$$

$$= 0.297$$

∴ $\boxed{\text{at 2000 RPM \& above, improvement is greater than 70\%}}$ $(1 - 0.297) \times 100 = 70.3\%$

- at $r = 1$

$$\frac{F_T}{F_0} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2} = \left[\frac{1 + (2 \times 0.5 \times 1)^2}{\cancel{(1-1)^2} + (2 \times 0.5 \times 1)^2}\right]^{1/2} = 1.41$$

∴ $\boxed{\text{at resonance (552 RPM), transmitted force will increase by 41\%}}$

$(1 - 1.41) \times 100 = -41\%$

SOME EXAMPLES OF PRACTICAL APPLICATIONS OF VIBRATION ABSORPTION/ISOLATION:
(REF: Den Hartog: Mechanical Vibrations)

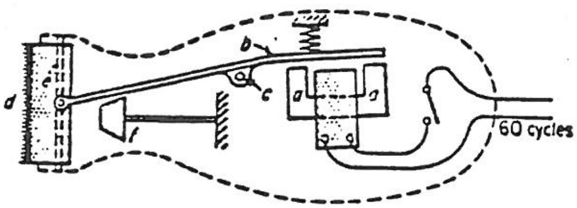


Fig. 3.9. Electric hair clipper with vibration absorber: a, magnet; b, armature tongue; c, pivot; d, cutter; e, guide for cutter; f, vibration absorber.

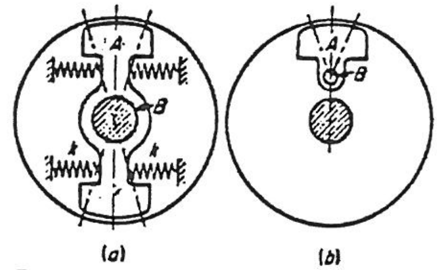


Fig. 3.10. Torsional dynamic vibration absorber (a) with mechanical springs and (b) with centrifugal springs.

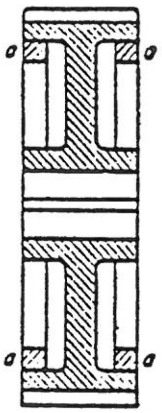


Fig. 3.16. Gear with sound-deadening rings inserted. These should be either shrunk or tack-welded in a few spots so as to allow some relative rubbing during the vibration.

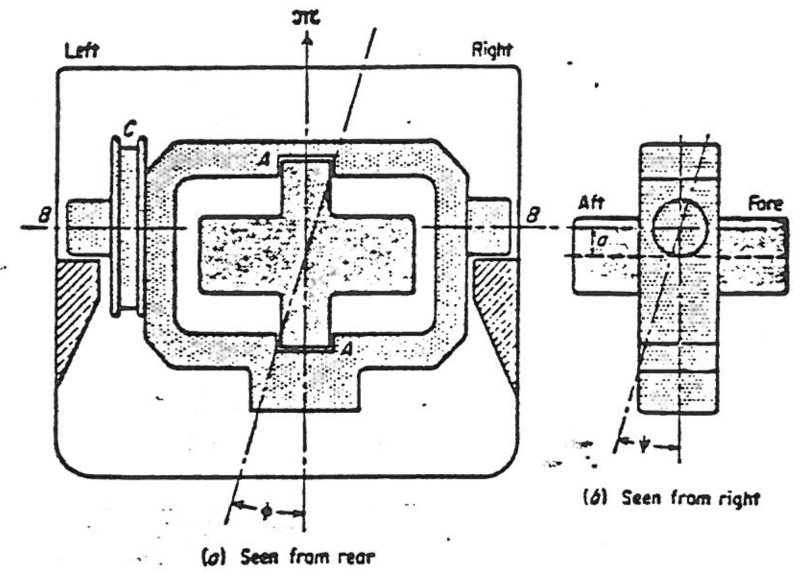


Fig. 3.20. Scheme of Schlick's anti-ship-rolling gyroscope. It operates by virtue of energy dissipation at the brake drum C.

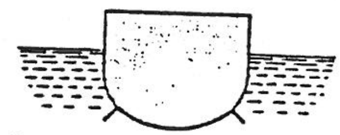


Fig. 3.22. Bilge keels, extending over more than half the length of a ship.

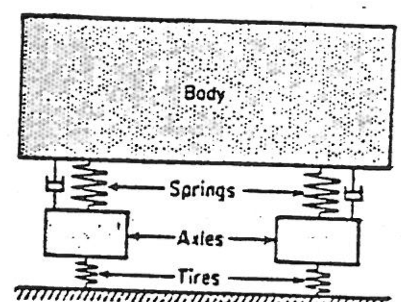


Fig. 3.23. Idealized scheme of conventional automobile with front and rear axles and shock absorbers.

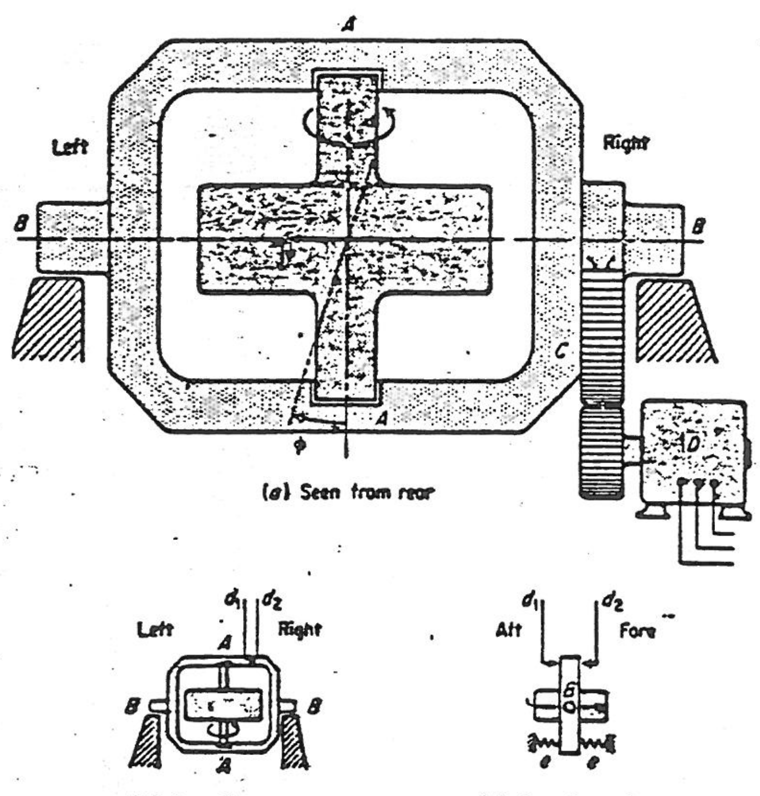


Fig. 3.21. Sperry's gyroscope for diminishing ship roll. The precession is forced by a motor D, which is controlled by a small pilot gyroscope shown in (b) and (c).

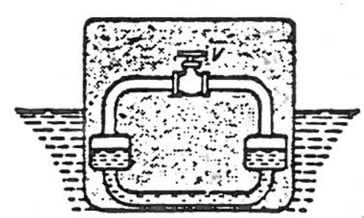


Fig. 3.18. Frahm antirolling tanks, old type.

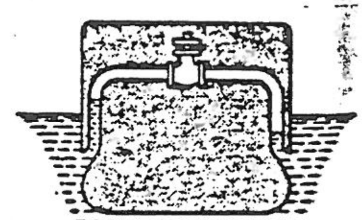


Fig. 3.19. Modern "blister" construction of Frahm's antirolling tanks.