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MAE 423 – HEAT AND MASS TRANSFER
EXAM 3 - SPRING 2013

Name: Answer Key

You are allowed three sheets of notes.

1. A 20 mm diameter spherical ice cube (at 0° C) is placed in a drink (mostly water) which is at room temperature (20° C). What is the heat transfer rate if the ice cube is fully immersed without any shaking or stirring?

2

Given: $D = 0.02\ m$

$T_s = 0°C$

$T_\infty = 20°C$

Find: q

Assumptions: Natural convection

15 Solution: $q = \bar{h}_c A (T_\infty - T_s)$

$$\bar{h}_c = \frac{\overline{Nu}_D k_f}{D}$$

$$\overline{Nu}_D = 2 + 0.392 (Gr_D)^{1/4}$$

$$Gr_D = \frac{g\beta}{\nu^2}(T_\infty - T_s) D^3$$

$$= 0.551 \times 10^9 (20-0) 0.02^3$$

$$= 88.16 \times 10^3$$

$$\overline{Nu}_D = 2 + 0.392 (88.16 \times 10^3)^{1/4}$$

$$= 8.755$$

$T_f = \frac{T_s + T_\infty}{2}$

$= \frac{0 + 20}{2} = 10°C$

$\frac{g\beta}{\nu^2} = 0.551 \times 10^9 \frac{1}{Km^3}$

$k_f = 0.577 \frac{W}{mK}$

$\nu = 1.300 \times 10^{-6} \frac{m^2}{s}$

$\beta = 0.95 \times 10^{-4} \frac{1}{K}$

$$\bar{h}_c = \frac{8.755 \times 0.577}{0.02} = 252.6 \frac{W}{m^2 K}$$

$$A = \pi D^2 = \pi (0.02)^2 = 0.0012566\ m^2$$

$$q = 252.6 \times 0.0012566 \times (20-0)$$

$$\boxed{q = 6.35\ W}$$

2. A 5 mm diameter, 100 mm long straw is used to draw up the drink at a rate of 0.25 liters/minute. If the straw is at 20° C for its full length and the drink is at 10° C when it enters the straw, what is the temperature when it exits the straw? (Ignore entrance effects.)

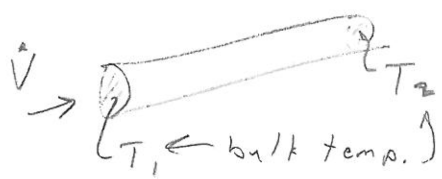
Given: Circular tube $D = 0.005\ m$

$L = 0.1\ m$

$\dot{V} = 0.25 \frac{\cancel{liter}}{\cancel{minute}} \times \frac{0.001\ m^3}{1\ \cancel{liter}} \times \frac{1\ min.}{60\ sec} = 4.167 \times 10^{-6}\ m^3/s$

15 $T_s = 20°C$

$T_1 = 10°C$



Find: T_2

Assumptions: Const. surface temp.

Solution: $q = \bar{h}_c A \left(\frac{T_1 - \cancel{T_s} - (T_2 - \cancel{T_s})}{\ln\left(\frac{T_1 - T_s}{T_2 - T_s}\right)} \right) = \dot{m} c_p (T_1 - T_2)$

$\bar{h}_c = \frac{\overline{Nu}_D k_f}{D}$

$\overline{Nu}_D = 3.36 \quad Pr > 0.6$

$Re_D = \frac{\bar{U} D}{\nu}$

$\bar{U} = \dot{V} / \left(\frac{\pi}{4} D^2\right) = \frac{4.167 \times 10^{-6}}{\frac{\pi}{4}(0.005)^2}$

$= 0.212\ m/s$

$Re_D = \frac{0.212 \times 0.005}{1.3 \times 10^{-6}}$

$= 815 < 2100$

$\therefore$ laminar

$\bar{h}_c = \frac{3.36 \times 0.577}{0.005}$

$= 387.7\ \frac{W}{m^2 K}$

Assume $T_f = 10°C$

(since we don't know T_2 we cannot calc. $T_f = \frac{T_1 + T_2}{2}$)

$\nu = 1.3 \times 10^{-6}\ m^2/s$

$Pr = 9.5$

$k_f = 0.577\ \frac{W}{mK}$

$\rho = 999.7\ \frac{kg}{m^3}$

$c_p = 4195\ J/kg\,K$

$\mu = 1296 \times 10^{6}\ \frac{N \cdot s}{m^6}$

$\mu_s (at\ 20°C) = 993 \times 10^{6}\ \frac{N \cdot s}{m^6}$

$A = \pi D L = \pi (0.005)(0.1) = 1.571 \times 10^{-3}\ m^2$

$$\dot{m} = \rho \dot{V} = 999.7 \times 4.167 \times 10^{-6} = 0.004166 \ \frac{kg}{s}$$

$$\frac{h_c A \cancel{(T_1 - T_2)}}{\ln \frac{T_1 - T_s}{T_2 - T_s}} = \dot{m} c_p \cancel{(T_1 - T_2)}$$

$$\frac{387.7 \times 1.571 \times 10^{-3}}{\ln \frac{10-20}{T_2 - 20}} = 0.004166 \times 4195$$

$$\boxed{T_2 = 10.34^\circ C}$$

- If we consider entrance effects, (which we should)

$$\overline{Nu_D} = 3.66 + \frac{0.0668 \, Re_D \, Pr \, D/L}{1 + 0.045 (Re_D \, Pr \, D/L)^{0.66}} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$= 3.66 + \frac{0.0668 \, (387.125)}{1 + 0.045 \times 387.125^{0.66}} \left(\frac{1296 \times 10^{-6}}{993 \times 10^{-6}}\right)^{0.14}$$

$$= 11.8$$

$$\bar{h}_c = \frac{\overline{Nu_D} \, k}{D} = \frac{11.8 \times 0.577}{0.005} = 1361.7$$

$$\frac{1361.7 \times 1.571 \times 10^{-3}}{\ln \frac{10-20}{T_2 - 20}} = 0.004166 \times 4195$$

$$\boxed{T_2 = 11.2^\circ C}$$

$x_{fully\text{-}dev,\,vel}$

$= 0.05 \, Re_D \, D$

$= 0.05 \times 815 \times 0.005$

$= 0.2 \text{ m} > 0.1$

$x_{fully\text{-}dev,\,temp}$

$= 0.05 \, Re_D \, Pr \, D$

$= 0.05 \times 815 \times 9.5 \times 0.005$

$= 1.94 \text{ m} > 0.1$

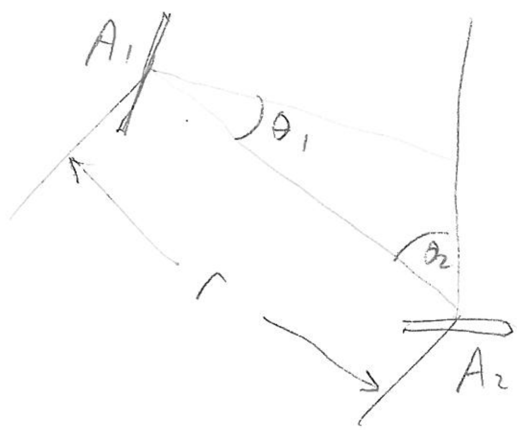
$Re_D \, Pr \, \frac{D}{L} = 815 \times 9.5 \times \frac{0.005}{0.1}$

$= 387.125$

$100 < 387.125 < 1500$

$\frac{\mu_b}{\mu_s} = \frac{1296 \times 10^{-6}}{993 \times 10^{-6}} = 1.305$

3. Name the five geometric factors (or parameters) that are important to radiation heat transfer between two planar faces.



$A_1, \theta_1, A_2, \theta_2, r$

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4. What is the "Reciprocity Theorem"?

$\epsilon_1 A_1 F_{1-2} = \epsilon_2 A_2 F_{2-1}$

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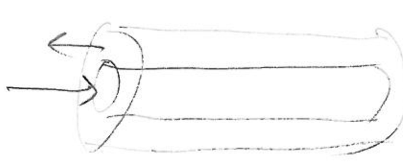
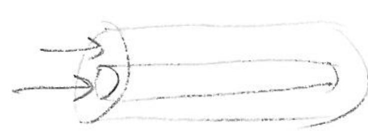
5. What is the difference between counter-current flow and parallel (or co-current) flow heat exchangers?





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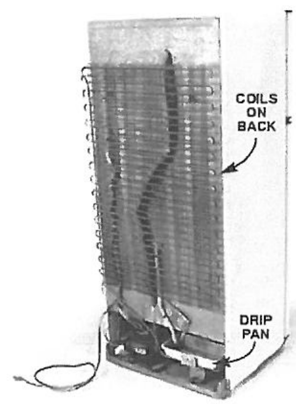
Counter-current

Parallel

6. Why is $T_f = \frac{T_\infty + T_s}{2}$ used for looking up fluid properties for external flow problems but not for flow in a duct?

1 For flow in a duct the bulk temperature is used. It reflects the avg temp. between the surface and the center of the duct.

7. Compute the length of tubing required for a refrigerator condenser heat exchanger, if:
   - the required heat transfer rate is 250 W
   - the refrigerant is condensing along the length of the tube at 45° C with a convection coefficient of 2000 W/m^{2}K and fouling factor of 0.0002 m^{2}K/W
   - the air is heated through natural convection from 20° C to 40° C with a convection coefficient of 10 W/m^{2}K and fouling factor of 0.0004 m^{2}K/W
   - the tube has an outside diameter of 10 mm and a wall thickness of 0.5 mm, with negligible thermal resistance
   - correction factor $F = 0.7$



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Given: $q = 250\ W$

$T_t = 45°C$ $\qquad T_{s,1} = 20°C \qquad T_{s,2} = 40°C$

$\bar{h}_{ci} = 2000\ \frac{W}{m^2K}$ $\qquad \bar{h}_{co} = 10\ \frac{W}{m^2K}$

$R_{di} = 0.0002\ \frac{m^2K}{W}$ $\qquad R_{do} = 0.0004\ \frac{m^2K}{W}$

15 $D_o = 0.01\ m$

$D_i = 0.01 - 2 \times 0.0005 = 0.009\ m$

$F = 0.7$

Find: L

Solution: $U_{do} = \dfrac{1}{\left(\frac{A_o}{A_i}\right)\frac{1}{h_{ci}} + \frac{A_o}{A_i}R_{di} + R_{do} + \frac{1}{h_{co}}}$

$\frac{A_o}{A_i} = \frac{\pi D_o L}{\pi D_i L} = \frac{D_o}{D_i} = \frac{0.01}{0.009} = 1.11$

$= \dfrac{1}{\frac{1.11}{2000} + 1.11 \times 0.0002 + 0.0004 + \frac{1}{10}}$

$= 9.884$

$\Delta T_1 = T_{s_1} - T_t = 20 - 45 = -25\ °C$

$\Delta T_2 = T_{s_2} - T_t = 40 - 45 = -5\ °C$

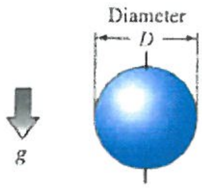
$q = U_{do} A_o F \dfrac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$

$250 = 9.884 \times (\pi \times 0.01 \times L)\left(\dfrac{-25 - (-5)}{\ln \frac{-25}{-5}}\right) \times 0.7$

$L = 92.6\ m$

- Surface Area of Sphere: $A = \pi D^2$
- 1 litre = 0.001 m^3

Sphere



$$\overline{Nu}_D = 2 + 0.392(Gr_D)^{1/4} \qquad 1 < Gr_D < 10^5$$

TABLE 13 Water at saturation pressure

Temperature, T			Density, ρ (kg/m^3)	Coefficient of Thermal Expansion, $\beta \times 10^4$ (1/K)	Specific Heat, c_p (J/kg K)	Thermal Conductivity, k (W/m K)	Thermal Diffusivity, $\alpha \times 10^6$ (m^2/s)	Absolute Viscosity, $\mu \times 10^6$ (N s/m^6)	Kinematic Viscosity, $\nu \times 10^6$ (m^2/s)	Prandtl Number, Pr	$\frac{g\beta}{\nu^2} \times 10^{-9}$ (1/K m^3)
°F	K	°C	$\times 6.243 \times 10^{-2}$ = (lb$_m$/ft^3)	$\times 0.5556$ = (1/R)	$\times 2.388 \times 10^{-4}$ = (Btu/lb$_m$ °F)	$\times 0.5777$ = (Btu/h ft °F)	$\times 3.874 \times 10^4$ = (ft^2/h)	$\times 0.6720$ = (lb$_m$/ft s)	$\times 3.874 \times 10^4$ = (ft^2/h)		$\times 1.573 \times 10^{-2}$ = (1/R ft^3)
32	273	0	999.9	−0.7	4226	0.558	0.131	1794	1.789	13.7	—
41	278	5	1000	—	4206	0.568	0.135	1535	1.535	11.4	—
50	283	10	999.7	0.95	4195	0.577	0.137	1296	1.300	9.5	0.551
59	288	15	999.1	—	4187	0.585	0.141	1136	1.146	8.1	—
68	293	20	998.2	2.1	4182	0.597	0.143	993	1.006	7.0	2.035
77	298	25	997.1	—	4178	0.606	0.146	880.6	0.884	6.1	—
86	303	30	995.7	3.0	4176	0.615	0.149	792.4	0.805	5.4	4.540
95	308	35	994.1	—	4175	0.624	0.150	719.8	0.725	4.8	—
104	313	40	992.2	3.9	4175	0.633	0.151	658.0	0.658	4.3	8.833
113	318	45	990.2	—	4176	0.640	0.155	605.1	0.611	3.9	—
122	323	50	988.1	4.6	4178	0.647	0.157	555.1	0.556	3.55	14.59
167	348	75	974.9	—	4190	0.671	0.164	376.6	0.366	2.23	—
212	373	100	958.4	7.5	4211	0.682	0.169	277.5	0.294	1.75	85.09
248	393	120	943.5	8.5	4232	0.685	0.171	235.4	0.244	1.43	140.0
284	413	140	926.3	9.7	4257	0.684	0.172	201.0	0.212	1.23	211.7
320	433	160	907.6	10.8	4285	0.680	0.173	171.6	0.191	1.10	290.3
356	453	180	886.6	12.1	4396	0.673	0.172	152.0	0.173	1.01	396.5
392	473	200	862.8	13.5	4501	0.665	0.170	139.3	0.160	0.95	517.2
428	493	220	837.0	15.2	4605	0.652	0.167	124.5	0.149	0.90	671.4
464	513	240	809.0	17.2	4731	0.634	0.162	113.8	0.141	0.86	848.5
500	533	260	779.0	20.0	4982	0.613	0.156	104.9	0.135	0.86	1076
536	553	280	750.0	23.8	5234	0.588	0.147	98.07	0.131	0.89	1360
572	573	300	712.5	29.5	5694	0.564	0.132	92.18	0.128	0.98	1766

TABLE 6.4 Summary of forced convection correlations for incompressible flow inside tubes and ducts[a,b,c]

System Description	Recommended Correlation	Equation in Text
Friction factor for laminar flow in long tubes and ducts	Liquids: $f = (64/\mathrm{Re}_D)(\mu_s/\mu_b)^{0.14}$ Gases: $f = (64/\mathrm{Re}_D)(T_s/T_b)^{0.14}$	(6.44) (6.45)
Nusselt number for fully developed laminar flow in long tubes with uniform heat flux, $\mathrm{Pr} > 0.6$	$\overline{\mathrm{Nu}}_D = 4.36$	(6.31)
Nusselt number for fully developed laminar flow in long tubes with uniform wall temperature, $\mathrm{Pr} > 0.6$	$\overline{\mathrm{Nu}}_D = 3.36$	(6.32)
Average Nusselt number for laminar flow in tubes and ducts of intermediate length with uniform wall temperature, $(\mathrm{Re}_{D_H}\mathrm{Pr}D_H/L)^{0.33}(\mu_b/\mu_s)^{0.14} > 2$, $0.004 < (\mu_b/\mu_s) < 10$, and $0.5 < \mathrm{Pr} < 16{,}000$	$\overline{\mathrm{Nu}}_{D_H} = 1.86(\mathrm{Re}_{D_H}\mathrm{Pr}D_H/L)^{0.33}(\mu_b/\mu_s)^{0.14}$	(6.42)
Average Nusselt number for laminar flow in short tubes and ducts with uniform wall temperature, $100 < (\mathrm{Re}_{D_H}\mathrm{Pr}D_H/L) < 1500$ and $\mathrm{Pr} < 0.7$	$\overline{\mathrm{Nu}}_{D_H} = 3.66 + \dfrac{0.0668\mathrm{Re}_{D_H}\mathrm{Pr}D/L}{1 + 0.045(\mathrm{Re}_{D_H}\mathrm{Pr}D/L)^{0.66}}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$	(6.41)
Friction factor for fully developed turbulent flow through smooth, long tubes and ducts	$f = 0.184/\mathrm{Re}_{D_H}^{0.2}(10{,}000 < \mathrm{Re}_{D_H} < 10^6)$	(6.56)
Average Nusselt number for fully developed turbulent flow through smooth, long tubes and ducts, $6000 < \mathrm{Re}_{D_H} < 10^7$, $0.7 < \mathrm{Pr} < 10{,}000$, and $L/D_H > 60$	$\overline{\mathrm{Nu}}_{D_H} = 0.027\,\mathrm{Re}_{D_H}^{0.8}\mathrm{Pr}^{1/3}(\mu_b/\mu_s)^{0.14}$ or Table 6.3 or the Gnielinski correlation, Eq. (6.65) for $\mathrm{Re}_D > 2300$	(6.61) (6.63)
Average Nusselt number for liquid metals in turbulent, fully developed flow through smooth tubes with uniform heat flux, $100 < \mathrm{Re}_D\mathrm{Pr} < 10^4$ and $L/D > 30$	$\overline{\mathrm{Nu}}_D = 4.82 + 0.0185\,(\mathrm{Re}_D\mathrm{Pr})^{0.827}$	(6.68)
Same as above, but in thermal entry region when $\mathrm{Re}_D\,\mathrm{Pr} < 100$	$\overline{\mathrm{Nu}}_D = 3.0\mathrm{Re}_D^{0.0833}$	(6.69)
Average Nusselt number for liquid metals in turbulent fully developed flow through smooth tubes with uniform surface temperature, $\mathrm{Re}_D\mathrm{Pr} > 100$ and $L/D > 30$	$\overline{\mathrm{Nu}}_D = 5.0 + 0.025(\mathrm{Re}_D\mathrm{Pr})^{0.8}$	(6.70)

[a] All physical properties in the correlations are evaluated at the bulk temperature T_b except μ_s, which is evaluated at the surface temperature T_s.

[b] $\mathrm{Re}_{D_H} = D_H\bar{U}\rho/\mu$, $D_H = 4A_c/P$, and $\bar{U} = \dot{m}/\rho A_c$.

[c] Incompressible flow correlations apply when average velocity is less than half the speed of sound (Mach number <0.5) to gases and vapors.

TABLE 6.3 Heat transfer correlations for liquids and gases in incompressible flow through tubes and pipes

Name (reference)	Formula[a]	Conditions	Equation
Dittus-Boelter [35]	$\overline{Nu}_D = 0.23 Re_D^{0.8} Pr^n$ $n \begin{cases} = 0.4 & \text{for heating} \\ = 0.3 & \text{for cooling} \end{cases}$	$0.5 < Pr < 120$ $6000 < Re_D < 10^7$	(6.60)
Sieder-Tate [16]	$\overline{Nu}_D = 0.027 Re_D^{0.8} Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$	$6000 < Re_D < 10^7$ $0.7 < Pr < 10^4$	(6.61)
Petukhov-Popov [36]	$\overline{Nu}_D = \frac{(f/8) Re_D Pr}{K_1 + K_2 (f/8)^{1/2} (Pr^{2/3} - 1)}$ where $f = (1.82 \log_{10} Re_D - 1.64)^{-2}$ $K_1 = 1 + 3.4f$ $K_2 = 11.7 + \frac{1.8}{Pr^{1/3}}$	$0.5 < Pr < 2000$ $10^4 < Re_D < 5 \times 10^6$	(6.63)
Sleicher-Rouse [37]	$\overline{Nu}_D = 5 + 0.015 Re_D^a Pr_s^b$ where $a = 0.88 - \frac{0.24}{4 + Pr_s}$ $b = 1/3 + 0.5e^{-0.6 Pr_s}$	$0.1 < Pr < 10^5$ $10^4 < Re_D < 10^6$	(6.64)

[a] All properties are evaluated at the bulk fluid temperature except where noted. Subscripts b and s indicate bulk and surface temperatures, respectively.