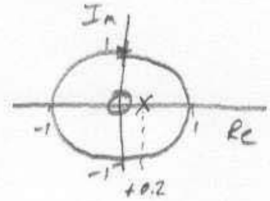
Plot the poles and zeros. Determine the stability of the following discrete-time systems.

$H(z) = \dfrac{z}{z - 0.2}$

zeros = 0

poles = 0.2



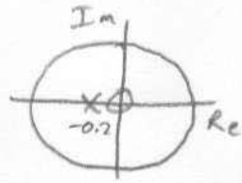
Stable

All poles lie within the unit circle

$H(z) = \dfrac{z}{z + 0.2}$

zero = 0

pole = -0.2



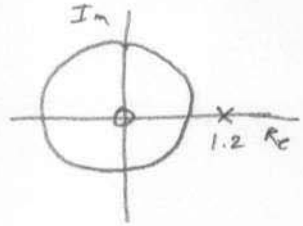
Stable

All poles lie within the unit circle

$H(z) = \dfrac{z}{z - 1.2}$

zero = 0

pole = 1.2



Unstable

A pole is outside the unit circle

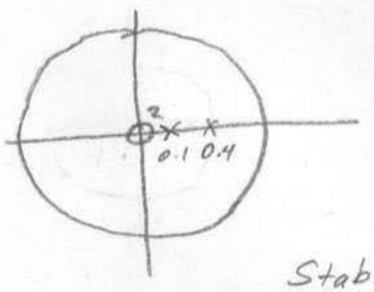
$H(z) = \dfrac{z^2}{z^2 - 0.6z + 0.08}$

zeros = 0, 0

$$\text{poles} = \frac{0.6 \pm \sqrt{0.36 - 0.32}}{2} =$$

$$= \frac{0.6 \pm \sqrt{0.04}}{2} = 0.3 \pm 0.1$$

$$= 0.1,\ 0.4$$

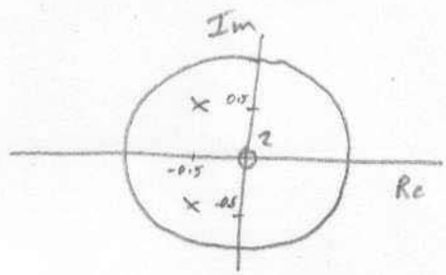


Stable Both poles lie within the unit circle

$H(z) = \dfrac{z^2}{(z + 0.5 + j0.5)(z + 0.5 - j0.5)}$

zeros = 0, 0

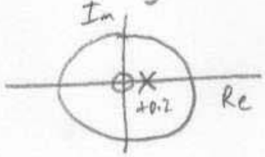
poles = $-0.5 \pm j0.5$

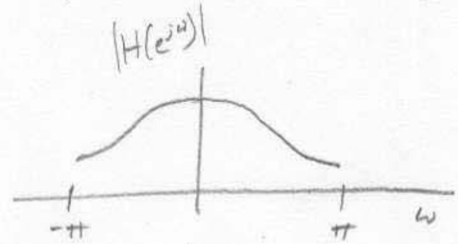


Stable Both poles lie within the unit circle

Use the pole-zero plots of the following systems to sketch the magnitude frequency response (only if the system is stable). Determine the type of filtering function

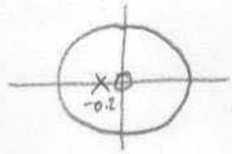
$$H(z) = \frac{z}{z - 0.2}$$

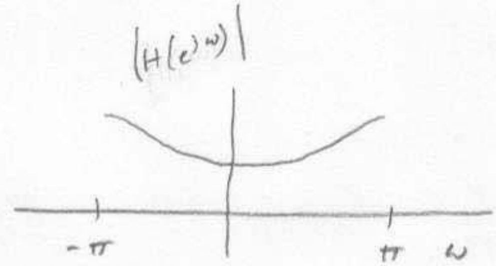




Lowpass Filter (look from 0 to π)

$$H(z) = \frac{z}{z + 0.2}$$



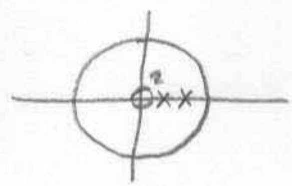


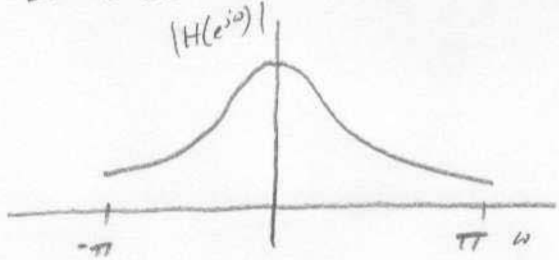
Highpass Filter

$$H(z) = \frac{z}{z - 1.2}$$

Unstable, because the pole lies outside the unit circle

$$H(z) = \frac{z^2}{z^2 - 0.6z + 0.08} = \frac{z^2}{(z - 0.1)(z - 0.4)}$$



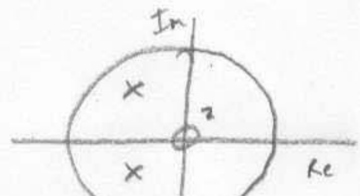


The pole at 0.4 dominates

Lower corner frequency than the first frequency response plot

Lowpass Filter

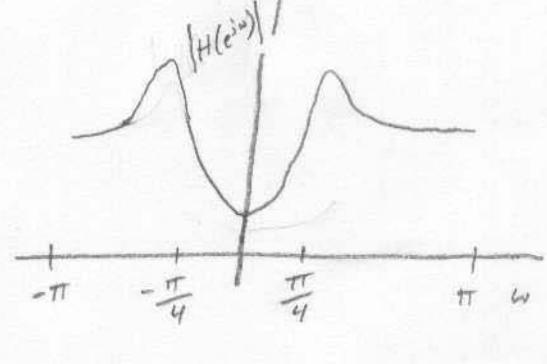
$$H(z) = \frac{z^2}{(z + 0.5 + j0.5)(z + 0.5 - j0.5)}$$



magnitude of poles $= \sqrt{(0.5)^2 + (0.5)^2} = 0.7071$

angle of poles $= \tan^{-1} \frac{0.5}{0.5} = \frac{\pi}{4}$

Highpass Filter



Find the frequency response for the following discrete-time system.

$$H(z) = \frac{z}{z+0.9}$$

Determine the steady-state response to the following input signal

$$x[n] = 1 + 5\cos\left(\frac{\pi}{4}n\right) + 10\cos\left(\frac{\pi}{2}n\right)$$

Sketch the magnitude frequency response over $-\pi$ to π

$$H(e^{jw}) = \frac{e^{jw}}{e^{jw}+0.9}$$

Magnitude Response

$$|H(e^{jw})|^2 = H(e^{jw})\,H^*(e^{jw}) = \frac{e^{jw}}{e^{jw}+0.9}\;\frac{e^{-jw}}{e^{-jw}+0.9}$$

$$= \frac{1}{(e^{jw}+0.9)(e^{-jw}+0.9)} = \frac{1}{1+0.9e^{jw}+0.9e^{-jw}+0.81} =$$

$$= \frac{1}{1.81 + 0.9\underbrace{(e^{jw}+e^{-jw})}_{2\cos(\omega)}} = \frac{1}{1.81+1.8\cos(\omega)}$$

$$|H(e^{jw})| = \sqrt{\frac{1}{1.81+1.8\cos(\omega)}} = \frac{1}{\sqrt{1.81+1.8\cos(\omega)}}$$

$$\angle H(e^{jw}) = \angle(e^{jw}) - \angle(e^{jw}+0.9)$$ (can plug this directly into a calculator)

$$= \angle(\cos w + j\sin w) - \angle(\cos w + 0.9 + j\sin w) =$$

$$= \tan^{-1}\left(\frac{\sin(\omega)}{\cos(\omega)}\right) - \tan^{-1}\left(\frac{\sin(\omega)}{0.9+\cos(\omega)}\right)$$

ω	$\lvert H(e^{j\omega})\rvert$	$\angle H(e^{j\omega})$
0	0.5263	0
$\frac{\pi}{4}$	0.5695	0.3709 rad
$\frac{\pi}{2}$	0.7433	0.7328 rad

Frequencies of interest (from the input)

$\omega = 0, \frac{\pi}{4}, \frac{\pi}{2}$

$\frac{\pi}{4} - 0.4145 = 0.3709$

$\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{0.9+0}\right) = 0.7328$

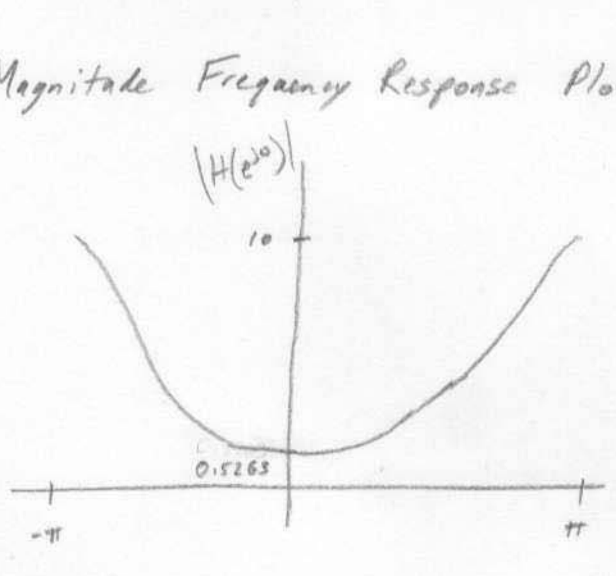
Steady-state output

$$y[n] = (1)\lvert H(e^{j0})\rvert \cos\left(0n + \angle H(e^{j0})\right) + (5)\lvert H(e^{j\frac{\pi}{4}})\rvert \cos\left(\tfrac{\pi}{4}n + \angle H(e^{j\frac{\pi}{4}})\right) +$$

$$+ (10)\lvert H(e^{j\frac{\pi}{2}})\rvert \cos\left(\tfrac{\pi}{2}n + \angle H(e^{j\frac{\pi}{2}})\right) =$$

$$= 0.5263 + 2.8475 \cos\left(\tfrac{\pi}{4}n + 0.3709\right) + 7.433 \cos\left(\tfrac{\pi}{2}n + 0.7328\right)$$

Magnitude Frequency Response Plot



at $\omega = 0$ $\quad \lvert H(e^{j0})\rvert = \frac{1}{\sqrt{1.81 + (1.8)(1)}} =$

$= 0.5263$

at $\omega = \pm\pi$ $\quad \lvert H(e^{j\pi})\rvert = \frac{1}{\sqrt{1.81 + (1.8)(-1)}} =$

$= 10$