EE 327 Signals and Systems 1
Homework 11

1. Find the state-space equations for the following differential equations.
   a. $\ddot{y} + 2\dot{y} + 5y = 7v$
   b. $\ddot{y} - 2y = v$
   c. $\dddot{y} + 2\ddot{y} + 3\dot{y} + y = 5v$

2. Determine the transfer function of the following state-space models.
   a.
   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v$$
   $$y = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
   b.
   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v$$
   $$y = \begin{bmatrix} 5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

3. Find the matrix exponential, e^{At}, for the following state-space equations. Also, solve the state-space model by finding *x(t)* and *y(t)* for a step input and any arbitrary initial values for the states.
   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v$$
   $$y = \begin{bmatrix} 5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

4. Determine a state-space model for the discrete-time system given by the following difference equation.
   a. $y[n+3] + 5y[n+2] + 7y[n+1] + 9y[n] = 10v[n]$
   b. $y[n] + 2y[n-1] + 3y[n-2] = 10v[n-2]$

5. Determine the transfer function of the following discrete-time state-space model.
   a.
   $$x[n+1] = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v[n]$$
   $$y[n] = \begin{bmatrix} 1 & 0 \end{bmatrix} x[n]$$

b.
$$x[n+1] = \begin{bmatrix} 0 & 1 \\ -8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v$$
$$y = \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

6. Determine the characteristic equation, the eigenvalues, and the stability of the following *discrete-time* state-space models.
   a. $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$
   b. $A = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$
   c. $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

7. Determine the first five values of the states. Also, determine the first five values of the output for the following discrete-time state-space systems.
   a.
$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v[n]$$
$$y[n] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + [0] v[n]$$

   b.
$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v[n]$$
$$y[n] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + [0] v[n]$$