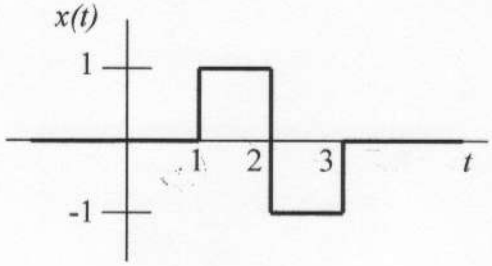
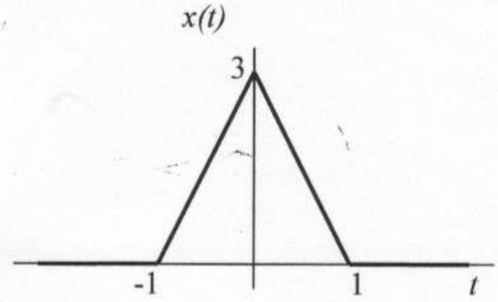
1. Determine an expression for the following signals. Simplify your answer.

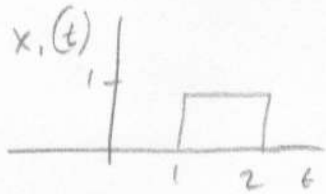


(a)

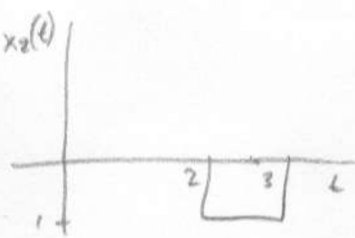


(b)

a) Split $x(t)$ into two portions



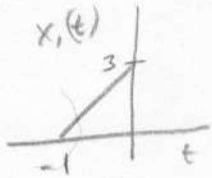
and



$x_1(t) = u(t-1) - u(t-2)$ $\qquad$ $x_2(t) = -u(t-2) + u(t-3)$

$$x(t) = x_1(t) + x_2(t) =$$
$$= u(t-1) - u(t-2) - u(t-2) + u(t-3) =$$
$$= u(t-1) - 2u(t-2) + u(t-3)$$

b) Split $x(t)$ into two portions



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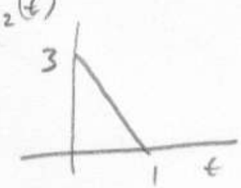


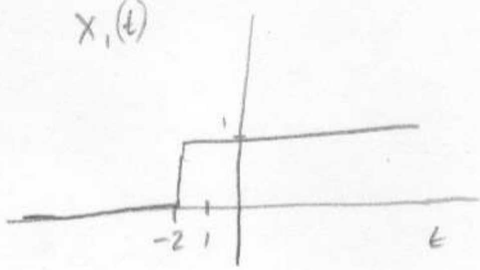
$$x_1(t) = \underbrace{(3t+3)}_{y=mx+b}\big(\underbrace{u(t+1)}_{\text{signal turns on}} - \underbrace{u(t)}_{\text{signal turns off}}\big)$$

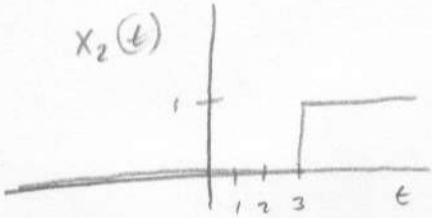
$$x_2(t) = (-3t+3)\big(u(t) - u(t-1)\big)$$

$$x(t) = x_1(t) + x_2(t) = (3t+3)\big(u(t+1) - u(t)\big) + (-3t+3)\big(u(t) - u(t-1)\big)$$
$$= (3t+3)\,u(t+1) - 6t\,u(t) - (-3t+3)\,u(t-1)$$

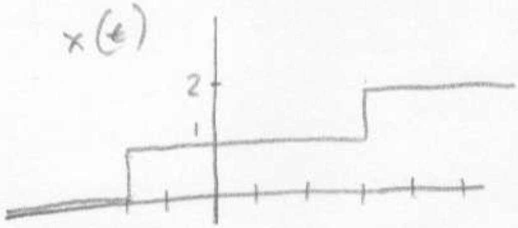
Sketch the following continuous-time signals

a) $x(t) = \underbrace{u(t+2)}_{x_1(t)} + \underbrace{u(t-3)}_{x_2(t)}$





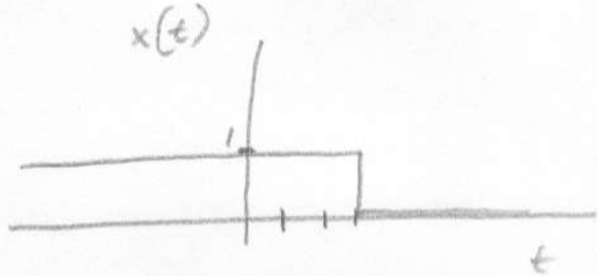
Add together



b) $x(t) = 5u(-2t+6) = \begin{cases} 5 & -2t+6 \geq 0 \\ 0 & -2t+6 < 0 \end{cases}$

Switch occurs at

$$-2t + 6 \geq 0$$
$$2t \leq 6$$
$$t \leq 3$$

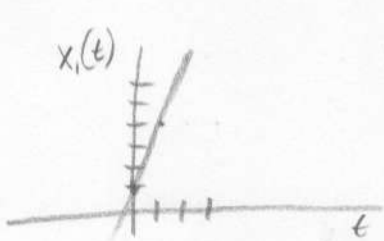


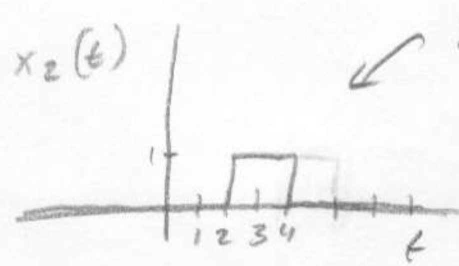
c) $x(t) = (3t+1)\left(u(t-2) - u(t-4)\right)$

Define two signals (multiplication of two signals)

$x_1(t) = 3t+1$ Ramp plus a constant (straight line)

$x_2(t) = u(t-2) - u(t-4)$





$x_2(t)$ effectively determines the range of times $x_1(t)$ is "turned on"

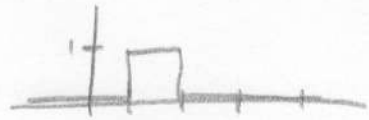
The line $3t+1$ only over the range for which $x_2(t) = 1$

$y(2) = 3(2) + 1 = 7$

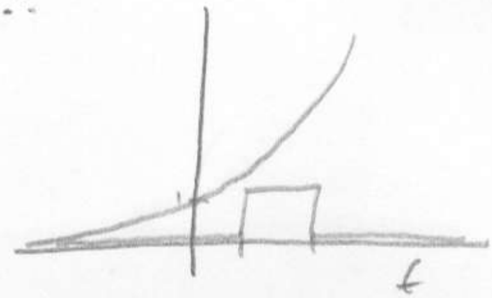
$y(4) = 3(4) + 1 = 13$

d) $x(t) = e^t \underbrace{\left(u(t-1) - u(t-2)\right)}$

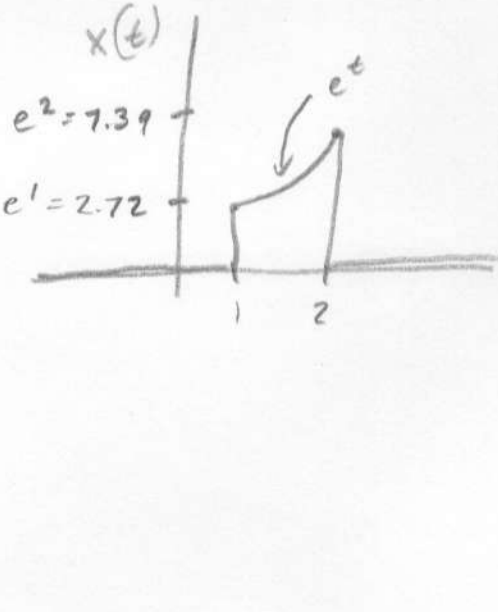
Effectively limits the signal's non zero values to fall between $t=1$ and $t=2$



$\therefore$

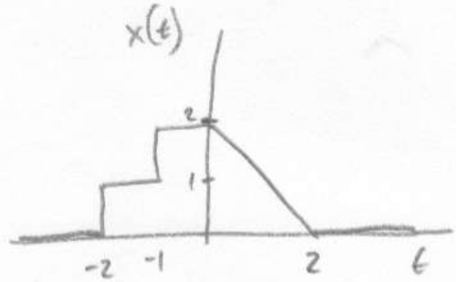


Multiply together



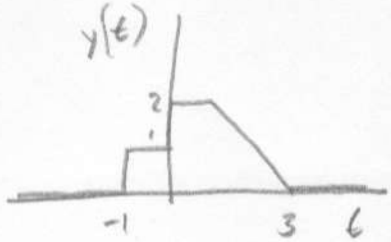
Given the following continuous-time signal, sketch the required signals.

Let $x(t)$ be



a) $y(t) = x(t-1)$

$\Rightarrow$ Delayed by 1 second



b) $y(t) = x(2-t)$

Solve in steps

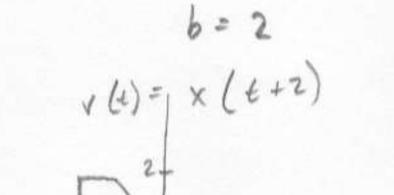
Let $v(t) = x(t+b)$

Then $y(t) = v(at) = x(at+b)$

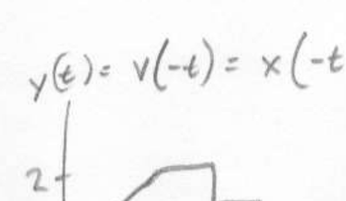
$\therefore \ a = -1$

$b = 2$

$v(t) = x(t+2)$



$y(t) = v(-t) = x(-t+2)$



Time reversal of $v(t)$

c) $y(t) = x(2t+1)$

Let $v(t) = x(t+b)$

Then $y(t) = v(at) = x(at+b)$

Match terms

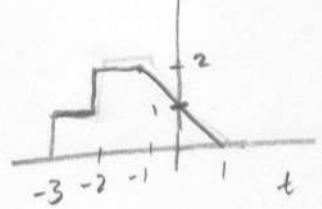
$a = 2$

$b = 1$

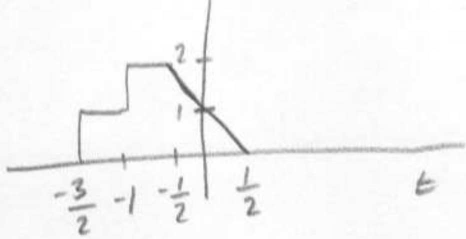
$\therefore$ Shift $x(t)$ by 1 to create $v(t)$

Then scale $v(t)$ by 2 to create $y(t)$

$v(t) = x(t+1)$



$y(t) = v(2t) = x(2t+1)$



d) $y(t) = x\left(4 - \frac{t}{2}\right)$

Let $v(t) = x(t+b)$

Then $y(t) = v(at) = x(at+b)$

Match terms

$a = -\frac{1}{2}$

$b = 4$

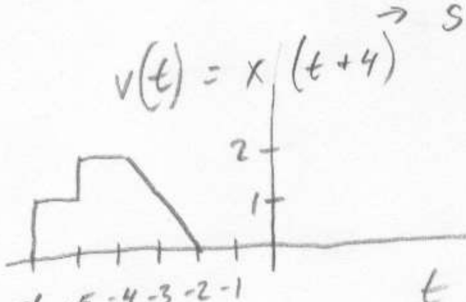
$\therefore$ Shift $x(t)$ by 4 to create $v(t)$

Then scale $v(t)$ by $-\frac{1}{2}$ to create $y(t)$

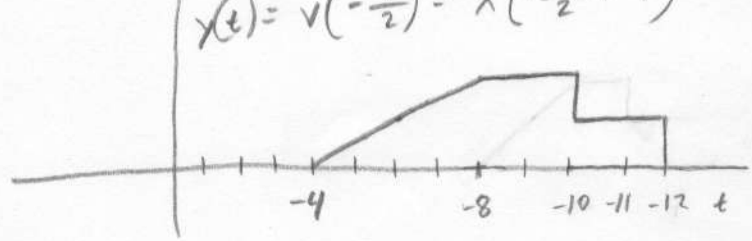
→ This includes time reversal

→ Slows the signal down

$v(t) = x(t+4)$

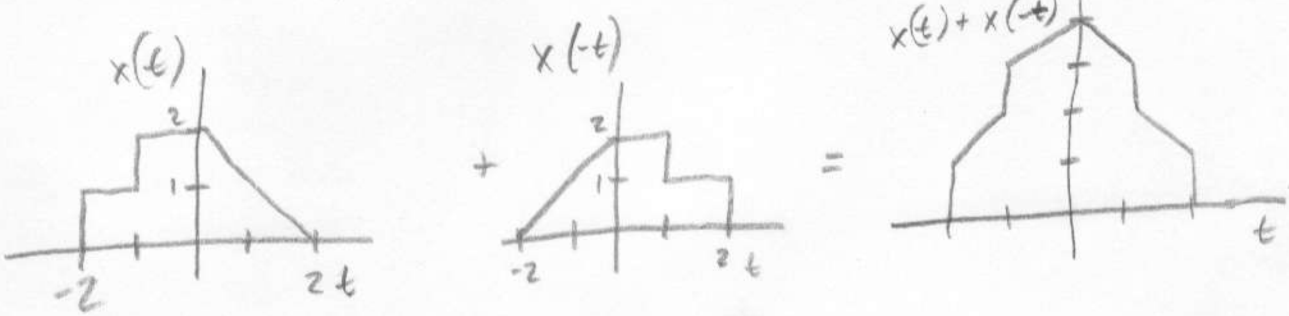


$y(t) = v\left(-\frac{t}{2}\right) = x\left(-\frac{t}{2} + 4\right)$

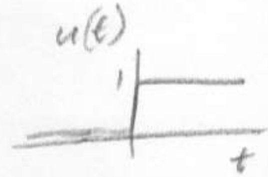


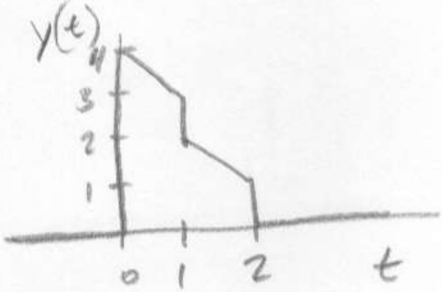
e) $y(t) = \left(x(t) + x(-t)\right) u(t)$

This is effectively the addition of the signal with its time-reversed version, but only valid for $t \geq 0$ (because of the multiplication with the step function)



Now, multiply this by $u(t)$

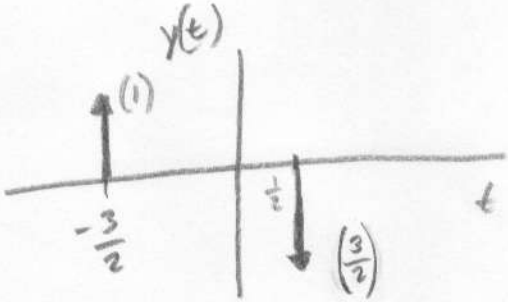




f) $y(t) = x(t)\left(\delta\left(t + \frac{3}{2}\right) - \delta\left(t - \frac{1}{2}\right)\right)$

Effectively, sample $x(t)$ at times $t = -\frac{3}{2}, \frac{1}{2}$

(Negative of the sample value at $t = \frac{1}{2}$



Determine if the following signals are periodic. If they are periodic, then determine the fundamental frequency.

a. $x(t) = 5\sin\left(4t - \frac{\pi}{6}\right)$

If the signal is periodic, then

$$x(t) = x(t+T) = x(t+nT)$$

where T is the period and n is an integer

$$x(t+T) = 5\sin\left(4(t+T) - \frac{\pi}{6}\right) = 5\sin\left(4t + 4T - \frac{\pi}{6}\right)$$

$$x(t+nT) = 5\sin\left(4(t+nT) - \frac{\pi}{6}\right) = 5\sin\left(4t + 4nT - \frac{\pi}{6}\right)$$

periodic if $\quad 4nT = n2\pi$

$\therefore$ Periodic with a fundamental period of $T = \frac{2\pi}{4} = \frac{\pi}{2}$

b) $x(t) = e^{\cos(t)}$

$$x(t+T) = e^{\cos(t+T)} = e^{\cos(t)} = x(t) \text{ if } T = 2\pi$$

$$x(t+nT) = e^{\cos(t+nT)} = e^{\cos(t)} = x(t) \text{ if } nT = 2\pi n$$

$$T = 2\pi$$

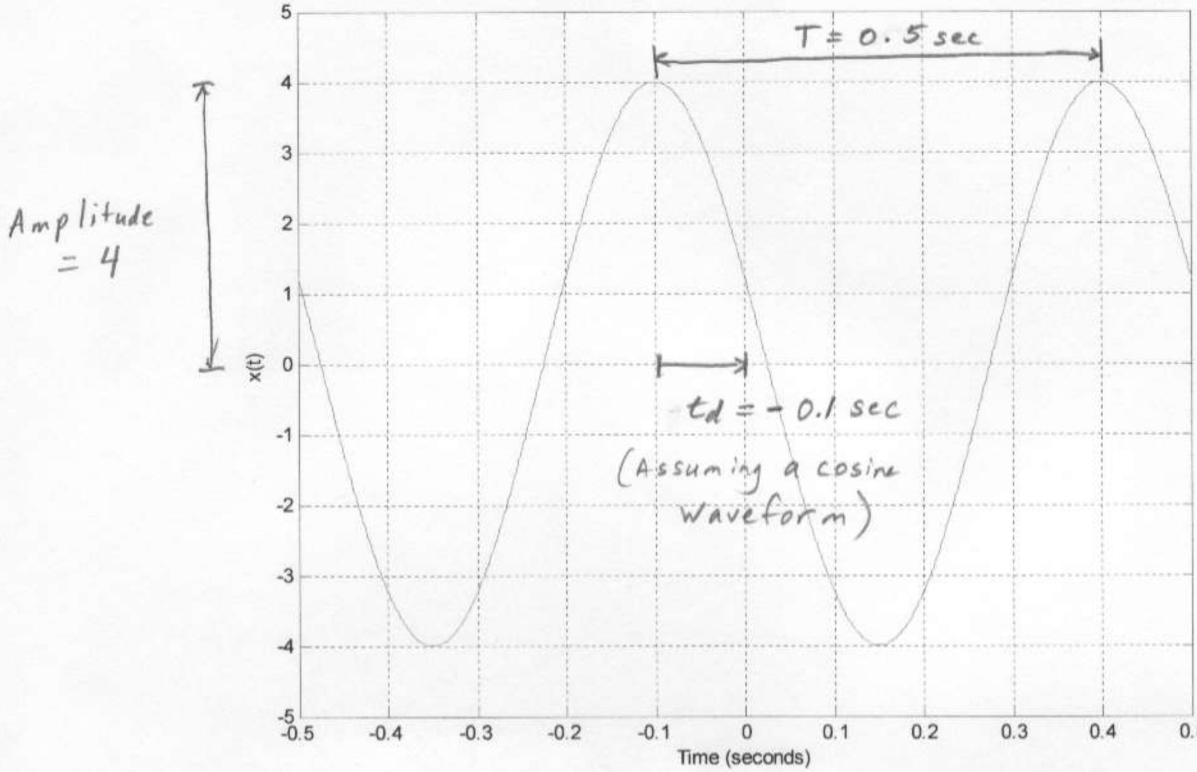
$\therefore$ Periodic with a fundamental period of $T = 2\pi$

c) $x(t) = te^{\cos(t)}$

$$x(t+T) = (t+T)e^{\cos(t+T)} \neq x(t) \text{ for } T > 0$$

Not periodic

For the following waveform, determine the amplitude, period, frequency, time shift, and phase delay. Write an expression for the waveform.



Amplitude = 4

Period = 0.5 sec

$f = \frac{1}{0.5} = 2\ Hz$

$\omega = 2\pi (2\ Hz) = 4\pi\ rad/sec$

Time Shift $\rightarrow t_d = -0.1\ sec$

Phase Delay $\rightarrow \phi = -2\pi f t_d = -2\pi (2\ Hz)(-0.1\ sec)$

$= 0.4\pi$

$\therefore\ x(t) = 4\cos(4\pi t + 0.4\pi)$