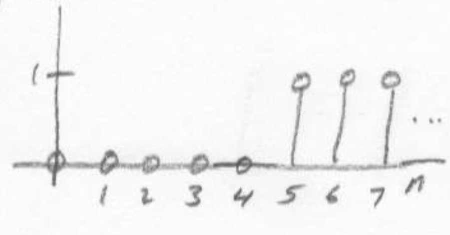
Sketch the following discrete-time signals.

a) $x[n] = u[n-5] - u[n-6]$

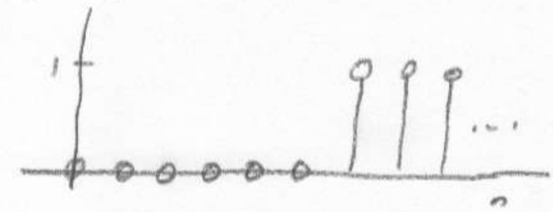
Plot the two portions of the signal separately, and then subtract

$u[n-5]$



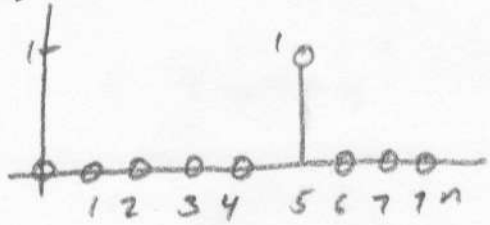
$-$

$u[n-6]$



Subtract

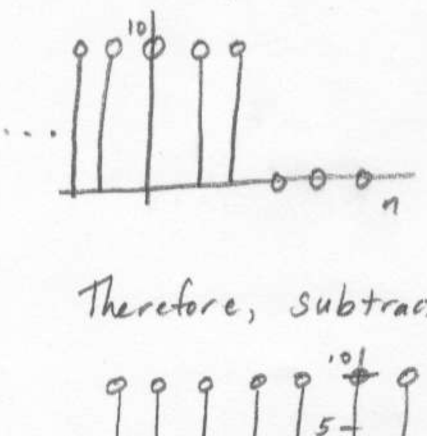
$x[n]$



b) $x[n] = 10\,\underbrace{u[-n+2]} - 5u[n-2]$

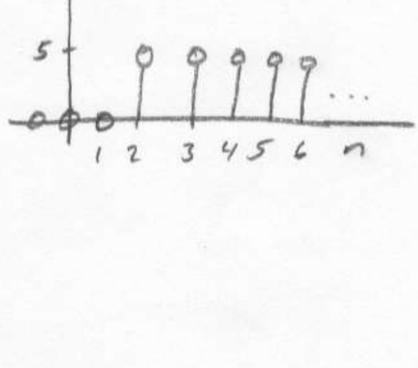
turns on for $-n+2 \geq 0 \Rightarrow n \leq 2$

$10u[-n+2]$

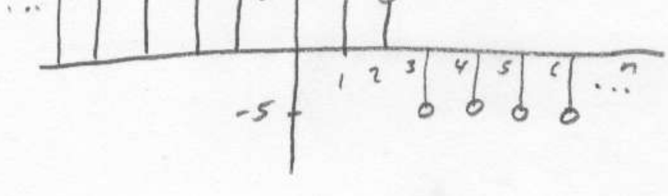


$-$

$5u[n-2]$



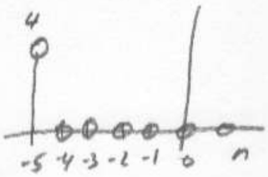
Therefore, subtracting



c) $x[n] = 4\delta[n+5] + (n+5)u[n+3] - nu[n]$

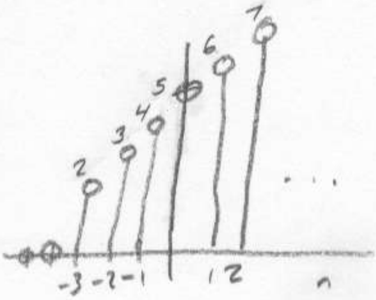
Plot each individual portion of this signal separately

$4\delta[n+5]$

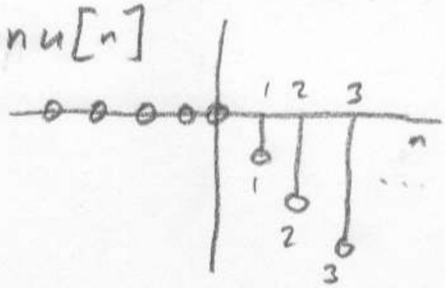


$(n+5)u[n+3]$

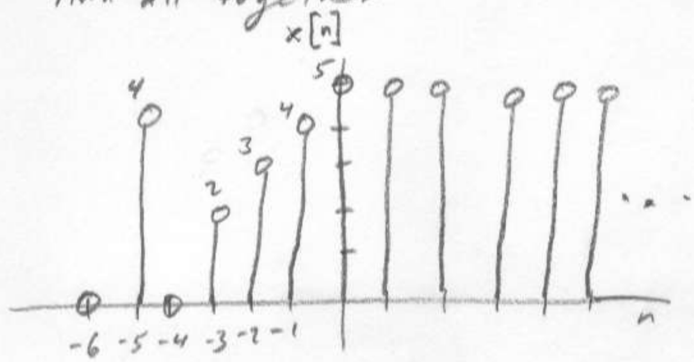
shifted ramp function that starts at $n=-3$



$-nu[n]$



Add all together



d) $x[n] = (0.1)^n \left(u[n] - u[n-5]\right)$

A decreasing exponential that is only turned on for samples 0 through 4

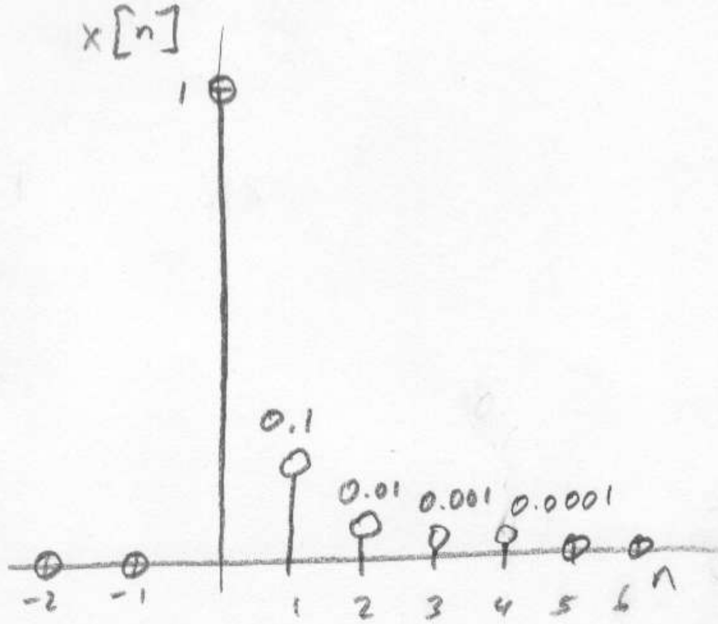
$x[0] = (0.1)^0 = 1$

$x[1] = (0.1)^1 = 0.1$

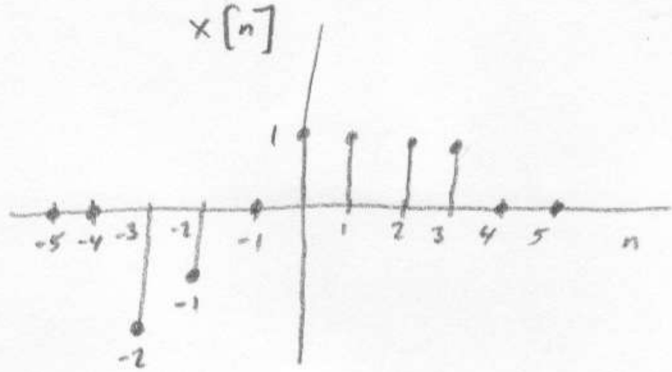
$x[2] = (0.1)^2 = 0.01$

$x[3] = (0.1)^3 = 0.001$

$x[4] = (0.1)^4 = 0.0001$

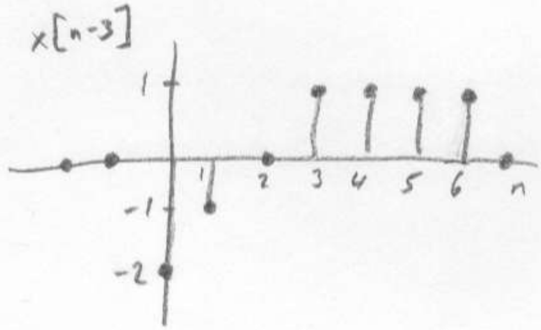


A discrete-time signal, $x[n]$, is shown below. Sketch the following signals



a) $y[n] = x[n-3]$

→ delayed by 3 samples



b) $y[n] = x[3-n]$

There is both time reversal and time shifting

$\therefore$ Let $v[n] = x[an]$

Then $y[n] = v[n+b] = x[a(n+b)] = x[an+ab]$

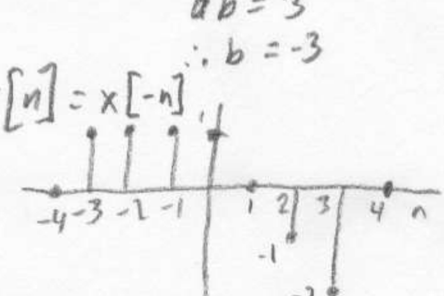
Matching terms

$a = -1$

$ab = 3$

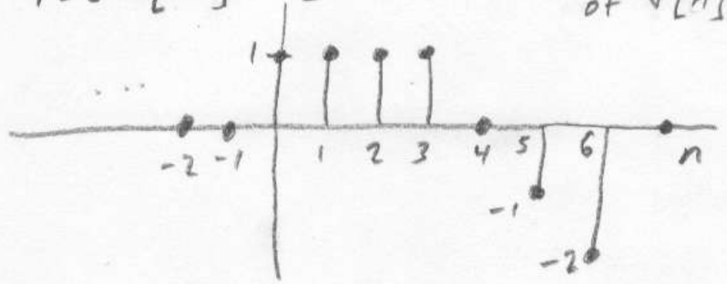
$\therefore b = -3$

$v[n] = x[-n]$



→

$y[n] = v[n-3] = x[-n+3]$ delayed version of $v[n]$



c) $y[n] = x[3n]$

This is time scaling, also known as subsampling in the discrete-time domain. This subsampling is valid because 3 is an integer, and we must ensure that $3n$ is an integer (the argument of x must be an integer). Therefore, we are subsampling at a rate of 3, meaning that the output, $y[n]$, only looks at every third value of $x[n]$.

Simply put, we can plug in values of n (integer values) into x to get the resulting values of $y[n]$

For example,

Let $n=0$

$$y[0] = x[3\cdot 0] = x[0] = 1$$

Let $n=1$

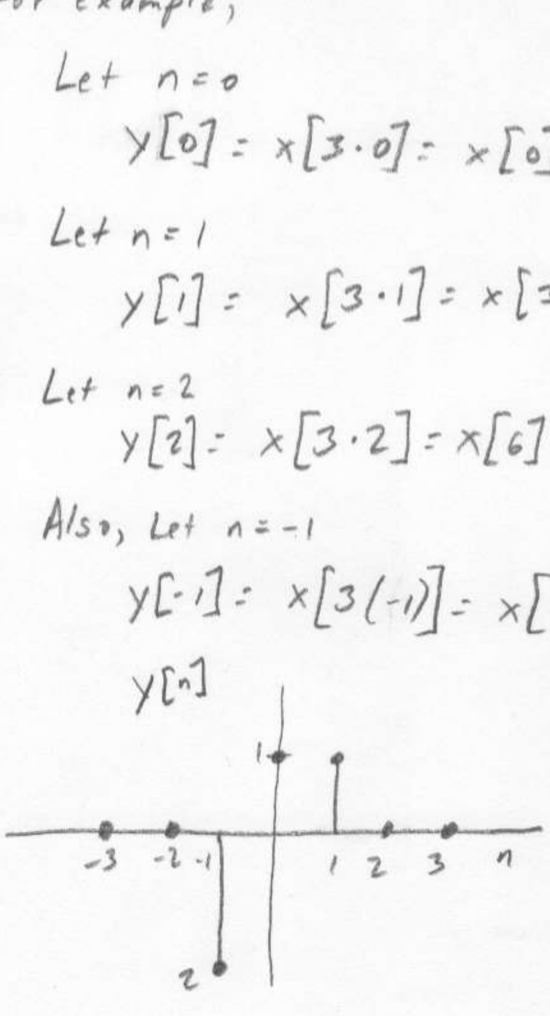
$$y[1] = x[3\cdot 1] = x[3] = 1$$

Let $n=2$

$$y[2] = x[3\cdot 2] = x[6] = 0$$

Also, Let $n=-1$

$$y[-1] = x[3(-1)] = x[-3] = -2$$



Again, we are simply taking every third sample value of x to determine the values of y

d) $y[n] = x[3n+1]$

This transformation includes both time scaling and time shifting.

Let $v[n] = x[n+b]$

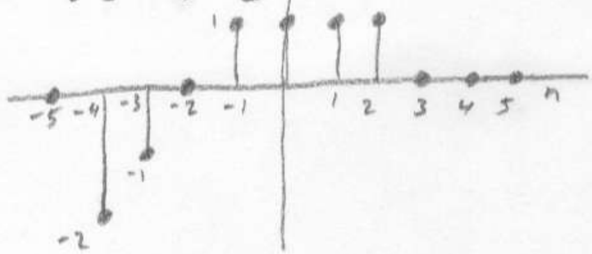
Then $y[n] = v[an] = x[an+b]$

Matching terms

$a = 3$

$b = 1$

$v[n] = x[n+1]$



Now, we subsample at a rate of 3

For example, plugging in values of n, we get

Let $n = 0$

$$y[0] = v[3(0)] = x[3(0)+1] = v[0] = x[1] = 1$$

Let $n = 1$

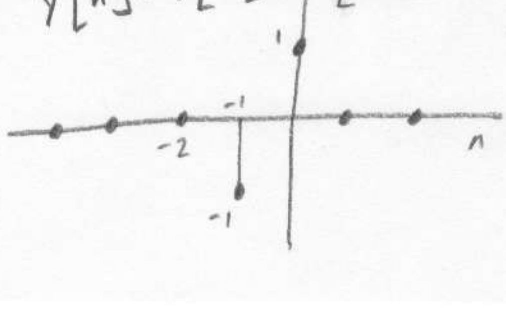
$$y[1] = v[3(1)] = x[3(1)+1] = v[3] = x[4] = 0$$

Let $n = -1$

$$y[-1] = v[3(-1)] = x[3(-1)+1] = v[-3] = x[-2] = -1$$

(or we could simply look at the plots and do this by inspection)

$y[n] = v[3n] = x[3n+1]$



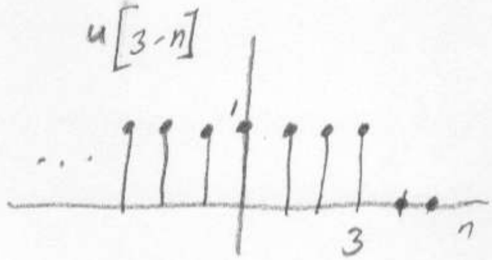
e) $y[n] = x[n]\, u[3-n]$

This is the multiplication of two signals.

First, let us find out what $u[3-n]$ is

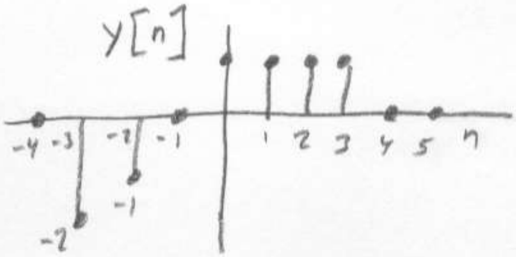
$$u[3-n] = \begin{cases} 1 & 3-n \geq 0 \rightarrow n \leq 3 \\ 0 & 3-n < 0 \end{cases}$$

$\therefore$ $u[3-n]$ is given by the following plot



We also notice from the plot of $x[n]$ that all the values of $x[n]$ for $n > 3$ are zero

$\therefore$ $y[n] = x[n]\, u[3-n] = x[n]$

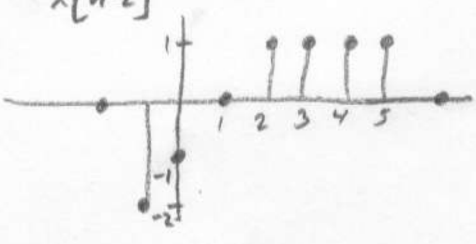


f) $y[n] = x[n-2] \; \delta[n-2]$

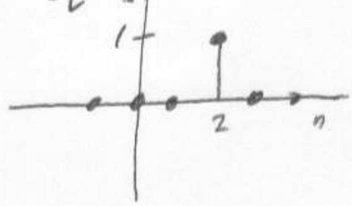
This is the multiplication of two signals – one is a delayed version of $x[n]$ and the other is a unit pulse function

(This is also the sifting property for the discrete-time domain.

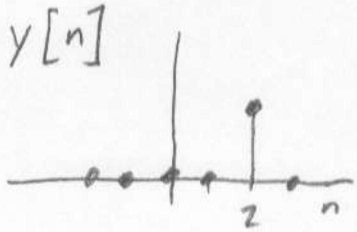
$x[n-2]$



$\delta[n-2]$



$\therefore \; y[n]$



Alternatively, plug in the sample value of the only nonzero value of the unit pulse function

Let $n = 2$

$$y[2] = x[2-2] \; \delta[2-2] = x[0] \, (1) = 1$$

All other values of n produce a 0 valued output from δ and, thus, $y[n]$

9) $y[n] = x[(n-1)^2]$

The simplest way to determine this output is to plug in values of n

Let $n=0$

$$y[0] = x[(0-1)^2] = x[1] = 1$$

Let $n=1$

$$y[1] = x[(1-1)^2] = x[0] = 1$$

Let $n=2$

$$y[2] = x[(2-1)^2] = x[1] = 1$$

Let $n=3$

$$y[3] = x[(3-1)^2] = x[4] = 0$$

Also for negative values of n

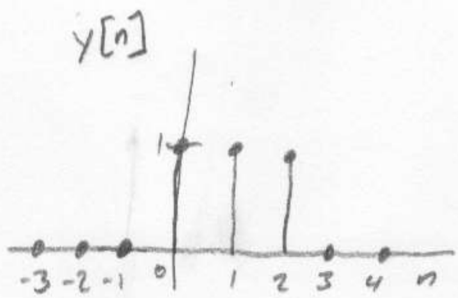
Let $n=-1$

$$y[-1] = x[(-1-1)^2] = x[4] = 0$$

Let $n=-2$

$$y[-2] = x[(-2-1)^2] = x[9] = 0$$

Therefore, $y[n]$ can be sketched as



The following continuous-time signal is to be discretized. What is the minimum sampling frequency that must be used in order to avoid aliasing?

$$x(t) = 1 + 5\cos\left((2\pi)(10)t\right) + 10\cos\left((2\pi)(100)t\right)$$

Solution

$x(t)$ contains frequency components a 0 Hz, 10 Hz, and 100 Hz

$\therefore$ Nyquist Rate $\Rightarrow f_{NS} = 2 f_{max} = 2(100\ Hz) = 200\ Hz$

$x(t)$ must be sampled at a frequency $> 200\ Hz$

(Good practice $\Rightarrow$ $f_s \geq 20 f_{NS} = 4\ kHz$)

Determine if the following system properties are valid

$y(t) = x(-t)$ Causal?

Let $y(-1) = x(1)$ $\rightarrow$ Input precedes output

Not Causal

$y(t) = (t+5)\,x(t)$ Memoryless?

Output only depends on "t" and current state of $x(t)$

Memoryless

$y(t) = x(5)$ Memoryless?

$\rightarrow$ Always depends on a particular value of $x(t) \rightarrow t=5$
$\rightarrow$ Could be looking to past, present, or future

Has Memory

$y(t) = 2x(t)$ stable (BIBO)?

if $|x(t)| \leq B_1$, $\sim$ some boundary B_1

then $|y(t)| \leq B_2$

where $B_2 = 2B_1$

$y(t)$ will always be $\leq B_2 = 2B_1$ for an input bounded by B_1

Stable

EE 327

Determine if the following system properties are valid

$y(t) = x(t) + a$ Linear ?

Homogeneity Test

$$S\{K\,x(t)\} = K x(t) + a$$

$$K\,y(t) = K\,(x(t) + a) = K x(t) + K a$$

$$S\{K\,x(t)\} \neq K\,y(t)$$

Nonlinear

$y(t) = t\,x(2t)$

Homogeneity Test

$$S\{K\,x(t)\} = K\,t\,x(2t)$$

$$K\,y(t) = K\,t\,x(2t)$$

$$S\{K\,x(t)\} = K\,y(t)$$ ⇒ Passes Homogeneity Test

Additivity Test

$$S\{x_1(t) + x_2(t)\} = t\left(x_1(2t) + x_2(2t)\right) =$$

$$= t\,x_1(2t) + t\,x_2(2t)$$

Let $y_1(t) = t\,x_1(2t)$

$y_2(t) = t\,x_2(2t)$

$$y(t) = y_1(t) + y_2(t) = t\,x_1(2t) + t\,x_2(2t)$$

$$S\{x_1(t) + x_2(t)\} = y_1(t) + y_2(t)$$

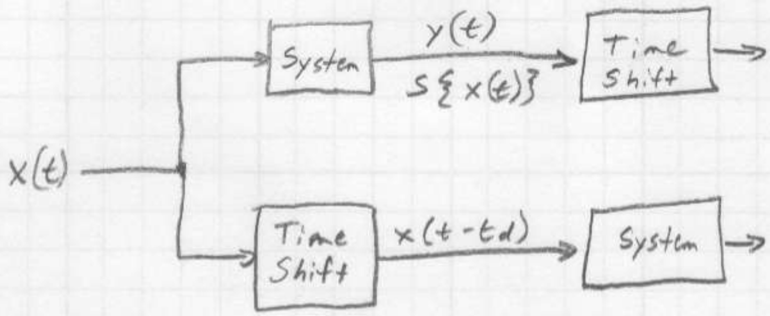
⇒ Passes Additivity Test

Linear

EE 321

Determine if the following system properties apply

$y(t) = \int_0^T x(t-\tau)\,d\tau$ Time Invariant ?



$y(t-t_d) = \int_0^T x(t-t_d-\tau)\,d\tau$

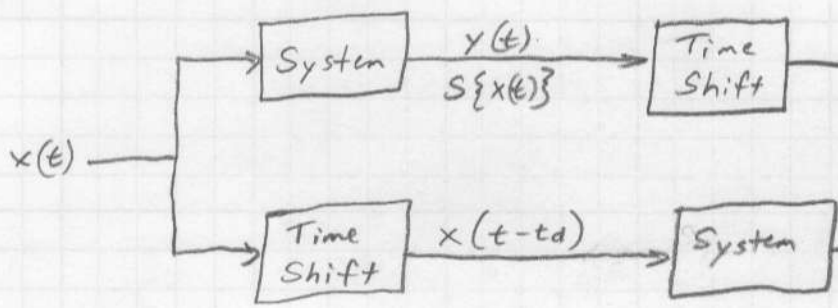
$S\{x(t-t_d)\} =$

$= \int_0^T x(t-t_d-\tau)\,d\tau$

⇒ They are equal

<u>Time Invariant</u>

$y(t) = x(2t)$ Time Invariant ?



$y(t-t_d) =$

$= x(2(t-t_d)) =$

$= x(2t-2t_d)$

$S\{x(t-t_d)\} =$

$= x(2t-t_d)$

Replace <u>only</u> the "t" with "2t"

⇒ They are not equal

<u>Time Varying</u>

Detemine the following properties of the given discrete-time system →

1. Causality 2. Memory 3. Stability 4. Linearity 5. Time Invariance 6. LTI

Let $y[n] = \left(\frac{n+0.5}{n-0.5}\right)^2 x[n]$

1. Causal → only depends on present value of n → $x[n]$

2. Memoryless → only depends on present value of n → $x[n]$

3. Stability

Let all inputs $|x[n]| < M$

$$\lim_{n\to\infty} y[n] = \lim_{n\to\infty} \left(\frac{n+0.5}{n-0.5}\right)^2 (M) \to M$$

maximum for $y[n]$ (for $n \geq 0$)

$$y[1] = \left(\frac{1.5}{0.5}\right)^2 M = 9M$$

$$|y[n]| \leq 9M = R \rightarrow \text{Bounded} \Rightarrow \text{Stable}$$

4. Linearity?

Additivity Test

Let $x_1[n] \mapsto y_1[n] = \left(\frac{n+0.5}{n-0.5}\right)^2 x_1[n]$

Let $x_2[n] \mapsto y_2[n] = \left(\frac{n+0.5}{n-0.5}\right)^2 x_2[n]$

$x_1[n] + x_2[n] \longmapsto \left(\frac{n+0.5}{n-0.5}\right)^2 \left[x_1[n] + x_2[n]\right] = y_1[n] + y_2[n]$

⇒ Additive

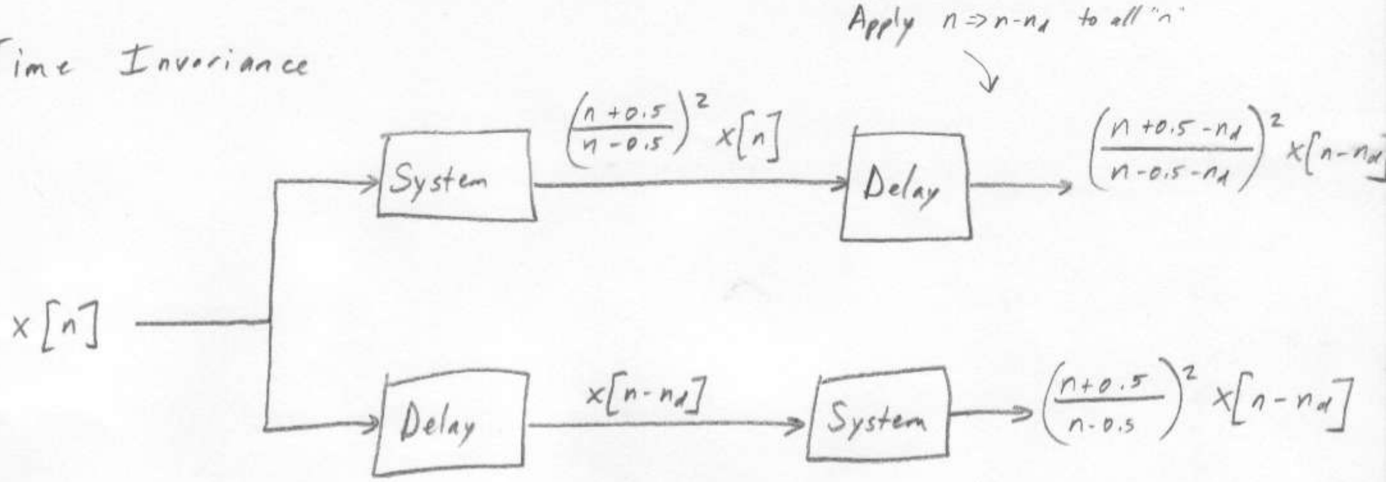
Homogeneity Test

Let $x[n] \longmapsto y[n] = \left(\frac{n+0.5}{n-0.5}\right)^2 x[n]$

Let $a x[n] \longmapsto a\left(\frac{n+0.5}{n-0.5}\right)^2 x[n] = a\, y[n]$ ⇒ Homogeneous

∴ Linear

5. Time Invariance



They do not agree

Time Varying

6. LTI ?

Not LTI. It is time varying.