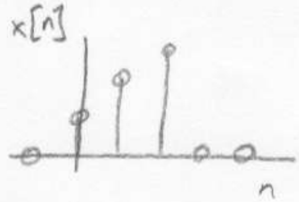
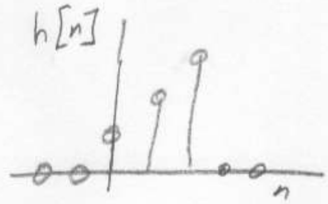
Find the output of the system to the following input

$x[n] = \{1, 2, 3\}$, $n = \{0, 1, 2\}$ $\qquad$ $h[n] = \{1, 2, 3\}$, $n = \{0, 1, 2\}$





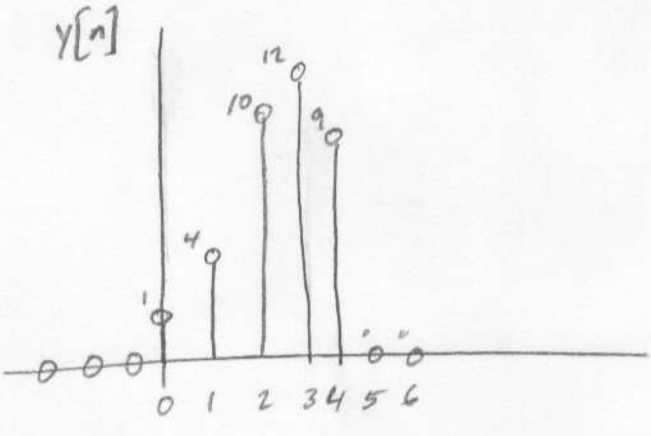
Convolution Sum

$N = 0 \rightarrow$ starting index for $x[n]$
$M = 0 \rightarrow$ starting index for $h[n]$

$x[n] \rightarrow$	1	2	3		
$h[n] \rightarrow$	1	2	3		
	1	2	3		
		2	4	6	
			3	6	9
	1	4	10	12	9

First output at $y[N+M] = y[0]$

$$y[n] = \delta[n] + 4\,\delta[n-1] + 10\,\delta[n-2] + 12\,\delta[n-3] + 9\,\delta[n-4]$$



Find the output, $y[n]$, of the system, $h[n]$, to the input $x[n]$

Let $x[n] = 10\delta[n+1] + 5\delta[n] - 5\delta[n-2] - 10\delta[n-3]$

Let $h[n] = -\delta[n-5] + \delta[n-7]$

$N = -1 \rightarrow$ first index of $x[n]$ for a nonzero value

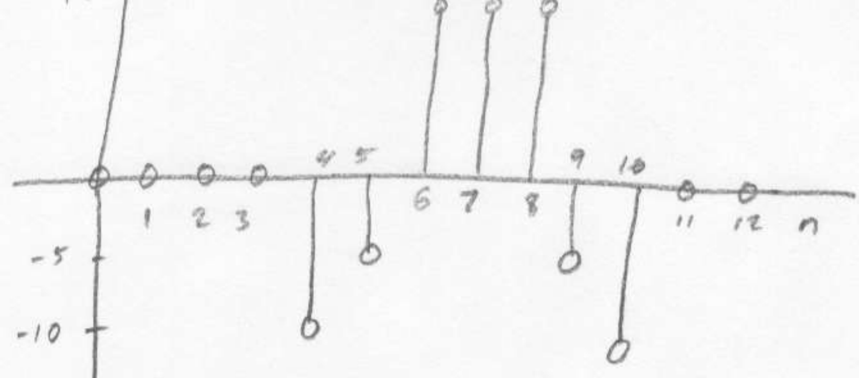
$M = 5 \rightarrow$ first index of $h[n]$ for a nonzero value

10	5	0	-5	-10		
-1	0	1				
-10	-5	0	5	10		
	0	0	0	0	0	
		10	5	0	-5	-10
-10	-5	10	10	10	-5	-10

First output at

$y[N+M] = y[-1+5] = y[4]$

$$y[n] = -10\delta[n-4] - 5\delta[n-5] + 10\delta[n-6] + 10\delta[n-7] + 10\delta[n-8] - 5\delta[n-9] - 10\delta[n-10]$$



Perform discrete-time convolution on the following signal and system.

$$x[n] = 5\delta[n] + 10\delta[n-1] + 15\delta[n-2] + 20\delta[n-3]$$
$$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$

$N = 0 \rightarrow$ First index of $x[n]$ for a nonzero value
$M = 0 \rightarrow$ First index of $h[n]$ for a nonzero value

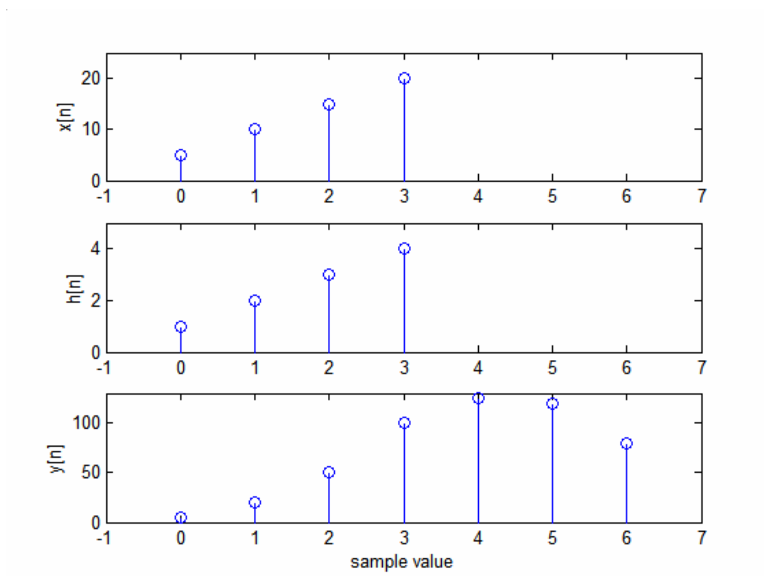
5	10	15	20			
1	2	3	4			
5	10	15	20			
	10	20	30	40		
		15	30	45	60	
			20	40	60	80
5	20	50	100	125	120	80

First output at $y[N+M] = y[0+0] = y[0]$

$$y[n] = 5\delta[n] + 20\delta[n-1] + 50\delta[n-2] + 100\delta[n-3] + 125\delta[n-4] + 120\delta[n-5] + 80\delta[n-6]$$

MATLAB Solution:

```
x=[5 10 15 20];
h=[1 2 3 4];
y=conv(x,h);
subplot(3,1,1),stem(0:length(x)-1,x);
axis([-1, 7, 0, 25]);
ylabel('x[n]');
subplot(3,1,2), stem(0:length(h)-1,h);
axis([-1, 7, 0, 5]);
ylabel('h[n]');
subplot(3,1,3), stem(0:length(y)-1,y);
axis([-1, 7, 0, 130]);
ylabel('y[n]');
xlabel('sample value');
```



Perform discrete-time convolution on the following signal and system.

$$x[n] = -\delta[n+5] - 3\delta[n+2] - 4\delta[n-1]$$
$$h[n] = 2\delta[n-100] + 4\delta[n-102]$$

$N = -5 \rightarrow$ First index of $x[n]$ for a nonzero value
$M = 100 \rightarrow$ First index of $h[n]$ for a nonzero value

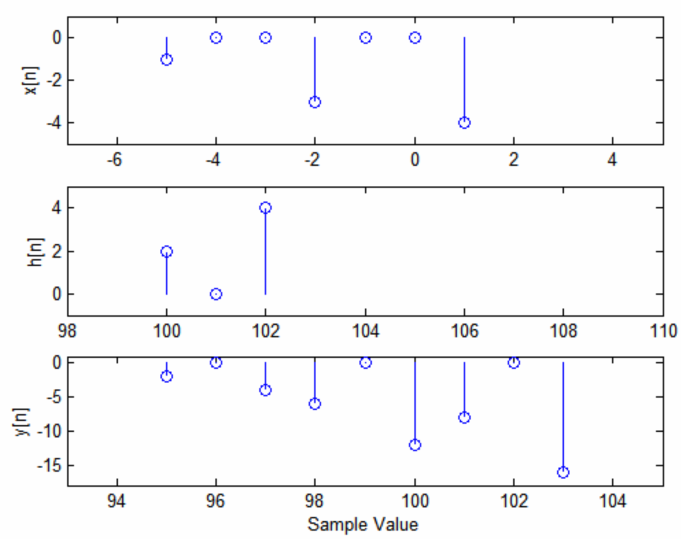
$$\begin{array}{ccccccccc}
-1 & 0 & 0 & -3 & 0 & 0 & -4 & & \\
2 & 0 & 4 & & & & & & \\
\hline
-2 & 0 & 0 & -6 & 0 & 0 & -8 & & \\
 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
 & & -4 & 0 & 0 & -12 & 0 & 0 & -16 \\
\hline
-2 & 0 & -4 & -6 & 0 & -12 & -8 & 0 & -16
\end{array}$$

First output at $y[N+M] = y[-5+100] = y[95]$

$$y[n] = -2\delta[n-95] - 4\delta[n-97] - 6\delta[n-98] - 12\delta[n-100] - 8\delta[n-101] - 16\delta[n-103]$$

MATLAB Solution:

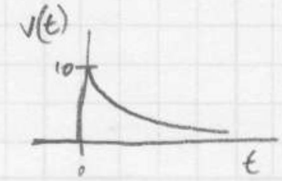
```
nn = -5:2;
mm = 100:105;
xn_start_index = -5;
hn_start_index = 100;
xn=[-1 0 0 -3 0 0 -4];
hn=[2 0 4];
yn = conv(xn,hn);
subplot(3,1,1);
stem(nn(1:length(xn)),xn);
axis([-7, 5, -5, 1]);
ylabel('x[n]');
subplot(3,1,2);
stem(mm(1:length(hn)),hn);
axis([98, 110, -1, 5]);
ylabel('h[n]');
subplot(3,1,3);
nn2 = (xn_start_index+hn_start_index):200;
stem(nn2(1:length(yn)),yn);
axis([93, 105, -18, 1]);
xlabel('Sample Value');
ylabel('y[n]');
```

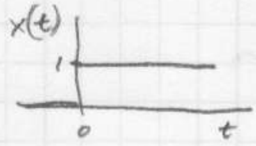


Convolution for

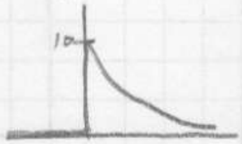
$v(t) = 10\, e^{-10t}\, u(t)$

$x(t) = u(t)$

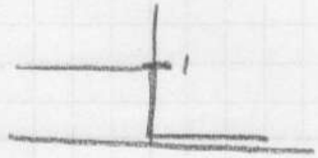




$v(\lambda)$



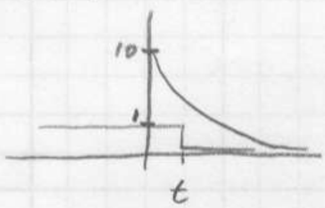
$x(-\lambda)$



$$v(t) * x(t) = \int_{-\infty}^{\infty} v(\lambda)\, x(t-\lambda)\, d\lambda$$

i) $t > 0$ $\quad v(t) * x(t) = 0$ (No over-lap)

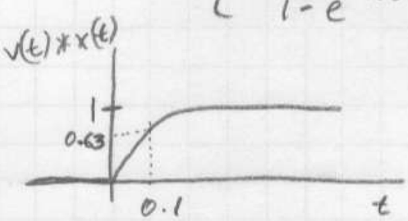
ii) $t \geq 0$



$$\int_0^t 10\, e^{-10\lambda}\, d\lambda = -e^{-10\lambda}\Big|_0^t =$$

$$= -e^{-10t} + 1 = 1 - e^{-10t}$$

$$v(t) * x(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-10t} & t \geq 0 \end{cases}$$



$\rightarrow$ time constant is 0.1

$e^{-t/\tau}$ $\quad \tau$ is the time constant

$\tau = 0.1$

Time Constant is the time it takes to decay to $\approx 37\%$ of its final value

EE 327

Convolution of $v(t) * x(t)$

$v(t) = 2e^{-2t}\,u(t)$

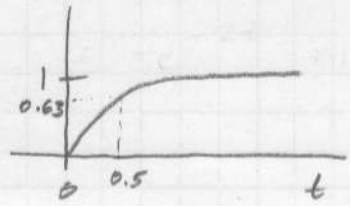
$x(t) = u(t)$

} This is essentially the same as the previous problem

This problem can be simplified by noticing that there will be no response until $t=0$

Therefore, we can procede directly to integration

$$v(t) * x(t) = \int_0^t 2e^{-2\lambda}\,d\lambda = -e^{-2\lambda}\Big|_0^t = \left(1-e^{-2t}\right)u(t)$$



This is a much slower system. Both systems exponentially rise to a value of "1." However, the first system rises to "1" much more quickly. Its time constant is smaller.

Convolution of $v(t) * x(t)$

$v(t) = 2e^{-2t}\,u(t)$

$x(t) = u(t+1)$

} This is the same as the previous convolution except that $x(t)$ is shifted to the left by 1 second

$\therefore$ The output of the convolution will also be shifted to the left by 1 second

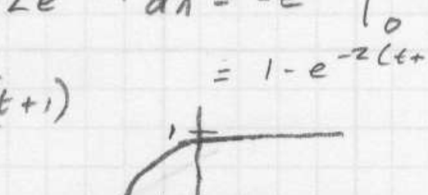
$$v(t) * x(t) = \left(1-e^{-2(t+1)}\right)u(t+1)$$

Or, by convolution

for $t \le -1$ $\quad v(t) * x(t) = 0$

for $t > -1$ $\quad v(t) * x(t) = \int_0^{t+1} 2e^{-2\lambda}\,d\lambda = -e^{-2\lambda}\Big|_0^{t+1} = 1-e^{-2(t+1)}$

$$\therefore\ v(t) * x(t) = \left(1-e^{-2(t+1)}\right)u(t+1)$$

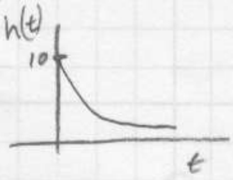


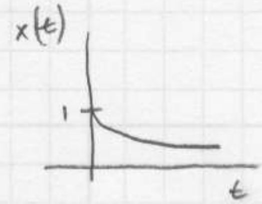
EE 327

Find the output signal, $y(t)$, given

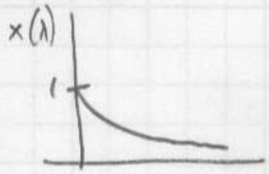
$h(t) = 10e^{-10t}\,u(t)$

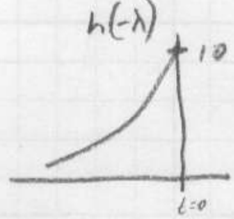
$x(t) = e^{-t}\,u(t)$





Draw $x(\lambda)$ and $h(-\lambda)$





for $t < 0$ $\quad x(t) * h(t) = 0$

for $t \geq 0$ $\quad x(t) * h(t) = \int_0^t \left(e^{-\lambda}\right)\left(10\,e^{-10(t-\lambda)}\right) d\lambda =$

$$= \int_0^t 10\,e^{-10t}\,e^{9\lambda}\,d\lambda = 10\,e^{-10t}\int_0^t e^{9\lambda}\,d\lambda =$$

$$= \frac{10}{9}\,e^{-10t}\,e^{9\lambda}\Big|_0^t = \frac{10}{9}\,e^{-10t}\left(e^{9t}-1\right) =$$

$$= \frac{10}{9}\left(e^{-t} - e^{-10t}\right)$$

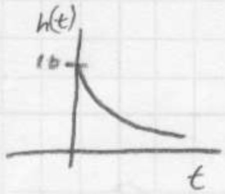
$$y(t) = \begin{cases} 0 & t < 0 \\ \frac{10}{9}\left(e^{-t} - e^{-10t}\right) & t \geq 0 \end{cases}$$

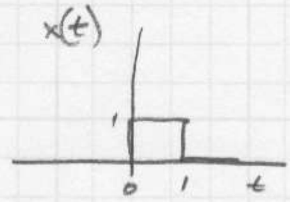
EE 327

Find the output signal, $y(t)$, given

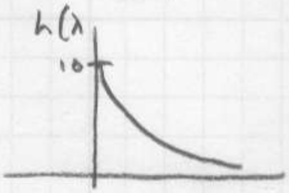
$h(t) = 10e^{-10t}\, u(t)$

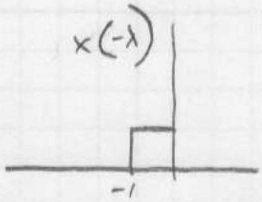
$x(t) = u(t) - u(t-1)$



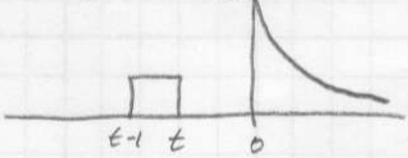


Draw $h(\lambda)$ and $x(-\lambda)$





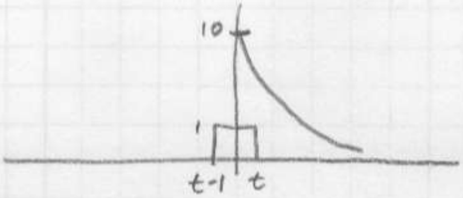
Case 1 $\quad t < 0$



$h(t) * x(t) = 0$

No overlap

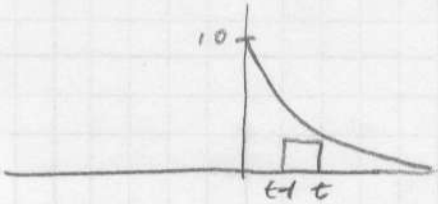
Case 2 $\quad 0 \le t < 1$



$$\int_0^t \left(10e^{-10\lambda}\right) d\lambda = -e^{-10\lambda}\Big|_0^t =$$

$$= -e^{-10t} + 1 = 1 - e^{-10t}$$

Case 3 $\quad t \ge 1$



$$\int_{t-1}^t \left(10e^{-10\lambda}\right) d\lambda = -e^{-10\lambda}\Big|_{t-1}^t =$$

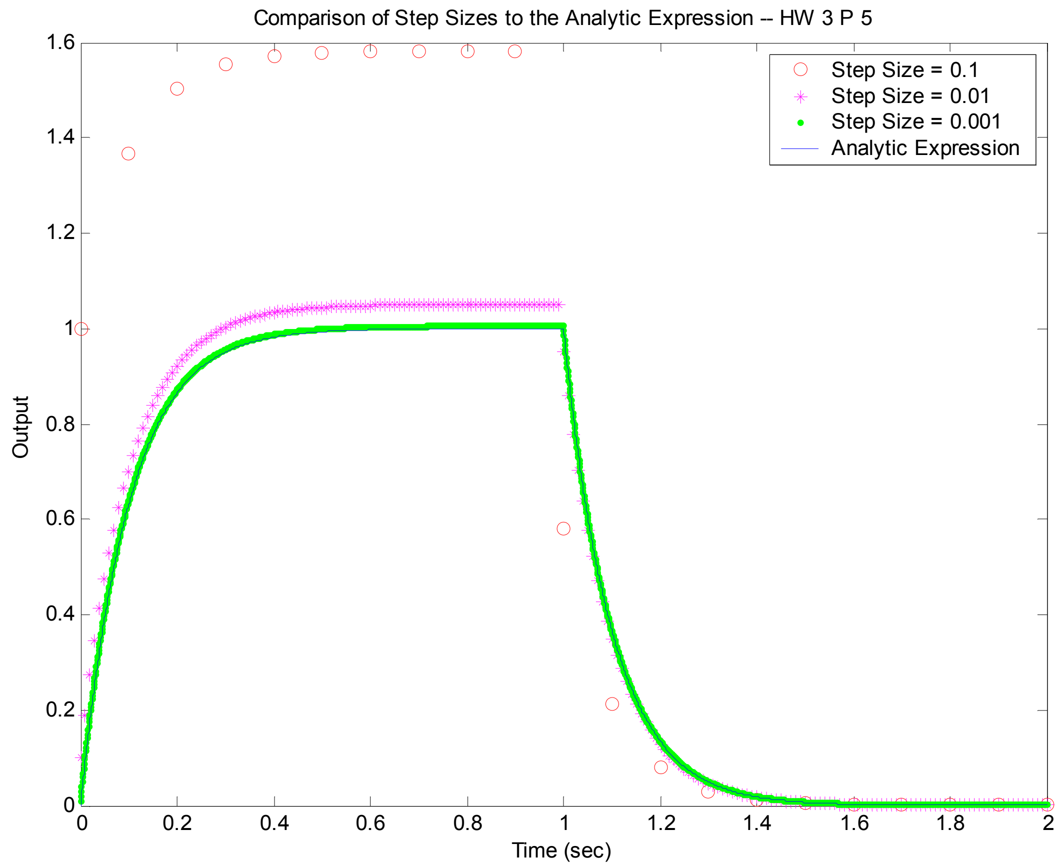
$$= -e^{-10t} + e^{-10(t-1)} =$$

$$= e^{-10t}\left(e^{10} - 1\right)$$

$$y(t) = h(t) * x(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-10t} & 0 \le t < 1 \\ e^{-10t}\left(e^{10} - 1\right) & t \ge 1 \end{cases}$$

Convolution of
$h(t) = 10e^{-10t}u(t)$
$x(t) = u(t) - u(t-1)$
using MATLAB and the “conv” function.

```
% Homework 3 Problem 5

% We will use several step sizes and compare the results to the analytic
% expression

step_size = 0.1;
tt = 0:step_size:2;
hh = 10*exp(-10.*tt).*stepfun(tt,0);
xx = stepfun(tt,0)-stepfun(tt,1);
y1 = conv(step_size*xx,hh);
figure;
plot(tt,y1(1:length(tt)),'ro')
hold on;

% Second Step Size (Smaller)
step_size = 0.01;
tt = 0:step_size:2;
hh = 10*exp(-10.*tt).*stepfun(tt,0);
xx = stepfun(tt,0)-stepfun(tt,1);
y2 = conv(step_size*xx,hh);
plot(tt,y2(1:length(tt)),'m*')

% Third Step Size (Smallest)
step_size = 0.001;
tt = 0:step_size:2;
hh = 10*exp(-10.*tt).*stepfun(tt,0);
xx = stepfun(tt,0)-stepfun(tt,1);
y3 = conv(step_size*xx,hh);
plot(tt,y3(1:length(tt)),'g.')

% Analytic Expression
yy = (1-exp(-10.*tt)).*(stepfun(tt,0)-stepfun(tt,1)) + exp(-10.*tt)*(exp(10)-
1).*stepfun(tt,1);
plot(tt,yy,'b-');

legend('Step Size = 0.1','Step Size = 0.01','Step Size = 0.001','Analytic
Expression')
xlabel('Time (sec)');
ylabel('Output')
title('Comparison of Step Sizes to the Analytic Expression -- HW 3 P 5')
```



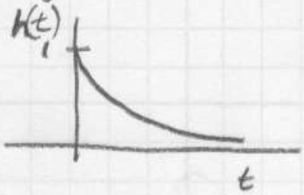
Increasing the step-size resolution increases the accuracy of the numerical convolution.

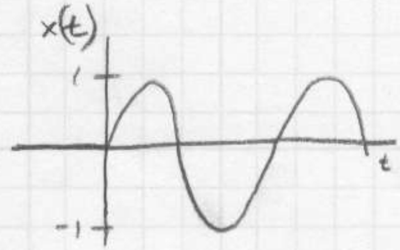
EE 327

Find the output signal, $y(t)$, given

$h(t) = e^{-t} u(t)$

$x(t) = \sin(t)\, u(t)$





Choose $h(t)$ to be flipped

for $t < 0$ $\quad y(t) = h(t) * x(t) = 0$

for $t \geq 0$

$$\int_0^t \left(\sin(\lambda)\, e^{-(t-\lambda)}\right) d\lambda = e^{-t} \int_0^t \sin(\lambda)\, e^{\lambda}\, d\lambda$$

Use Euler's Equation (Inverse)

$$\sin\lambda = \frac{e^{j\lambda} - e^{-j\lambda}}{2j}$$

$$\frac{e^{-t}}{2j} \int_0^t \left(e^{\lambda}\right)\left(e^{j\lambda} - e^{-j\lambda}\right) d\lambda = \frac{e^{-t}}{2j} \int_0^t \left(e^{\lambda(1+j)} - e^{\lambda(1-j)}\right) d\lambda =$$

$$= \frac{e^{-t}}{2j} \left[\frac{1}{1+j} e^{\lambda(1+j)} - \frac{1}{1-j} e^{\lambda(1-j)} \right]\Bigg|_0^t =$$

$$= \frac{e^{-t}}{2j} \left[\frac{1}{1+j} e^{t(1+j)} - \frac{1}{1-j} e^{t(1-j)} - \frac{1}{1+j} + \frac{1}{1-j} \right] =$$

$$= \frac{e^{-t}}{2j} \left[e^{t} \left(\frac{1}{1+j} e^{jt} - \frac{1}{1-j} e^{-jt} \right) - \frac{1}{1+j} + \frac{1}{1-j} \right] =$$

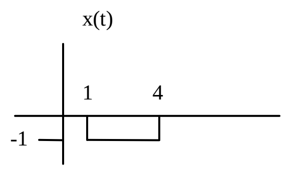
$$= \frac{1}{2j} \left[\frac{(1-j)e^{jt} - (1+j)e^{-jt}}{\underbrace{(1-j)(1+j)}_{1+j-j-j^2=2}} \right] + \frac{e^{-t}}{2j} \left[\frac{-(1-j) + (1+j)}{(1-j)(1+j)} \right] =$$

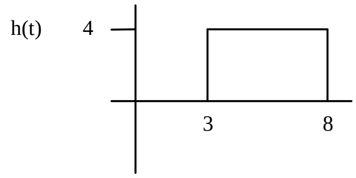
$$= \frac{1}{4j} \left[e^{jt} - e^{-jt} \right] - \frac{1}{4} \left[e^{jt} + e^{-jt} \right] + \left(\frac{e^{-t}}{\cancel{2j}} \right) \left(\frac{\cancel{2j}}{2} \right) =$$

$$= \frac{1}{2} \sin(t) - \frac{1}{2} \cos(t) + \frac{1}{2} e^{-t}$$

$$y(t) = x(t) * h(t) = \frac{1}{2} \sin(t)\, u(t) - \frac{1}{2} \cos(t)\, u(t) + \frac{1}{2} e^{-t} u(t)$$

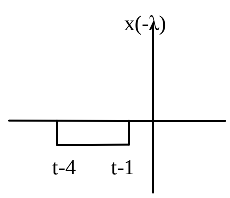
Find the output, y(t), given the input, x(t), and the impulse response, h(t), using convolution.

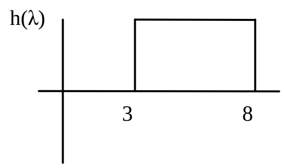




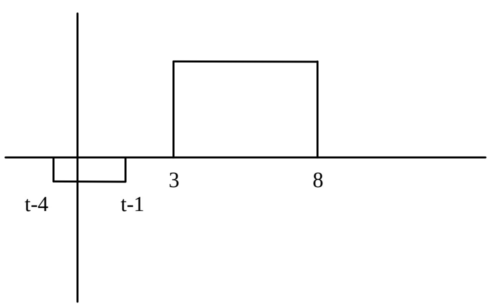
<u>Solution</u>

First, flip x(t), and redraw both x(t) and h(t) with respect to λ.



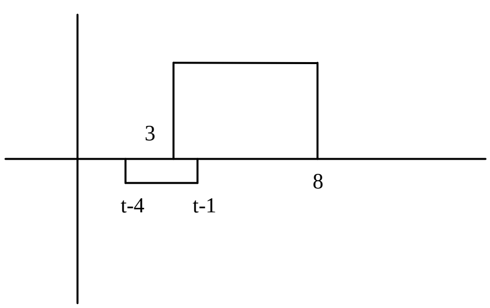


for t<4



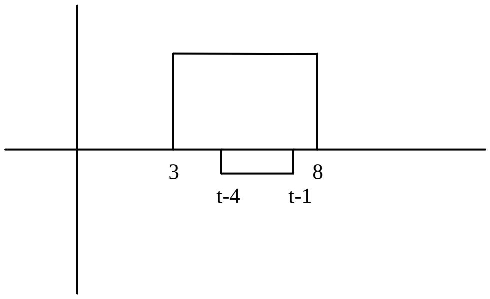
y(t) = **0**

for 4 <= t < 7



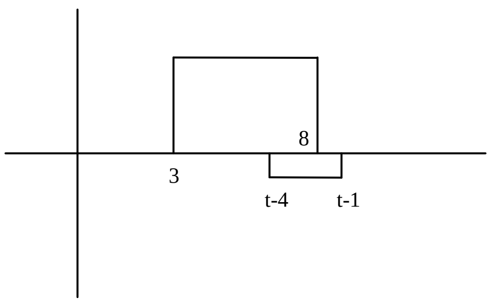
$y(t) = \int_{3}^{t-1} (-1)(4)\,d\lambda$ **= -4t + 16**

for 7 <= t < 9



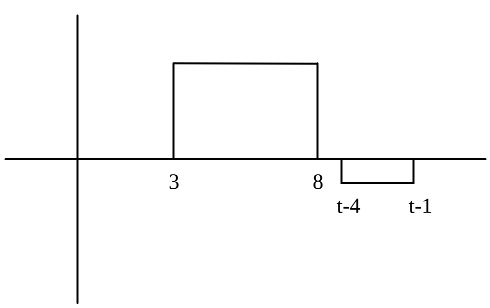
$$y(t) = \int_{t-4}^{t-1} (-1)(4)\,d\lambda \ \mathbf{= 12}$$

for 9 <= t < 12

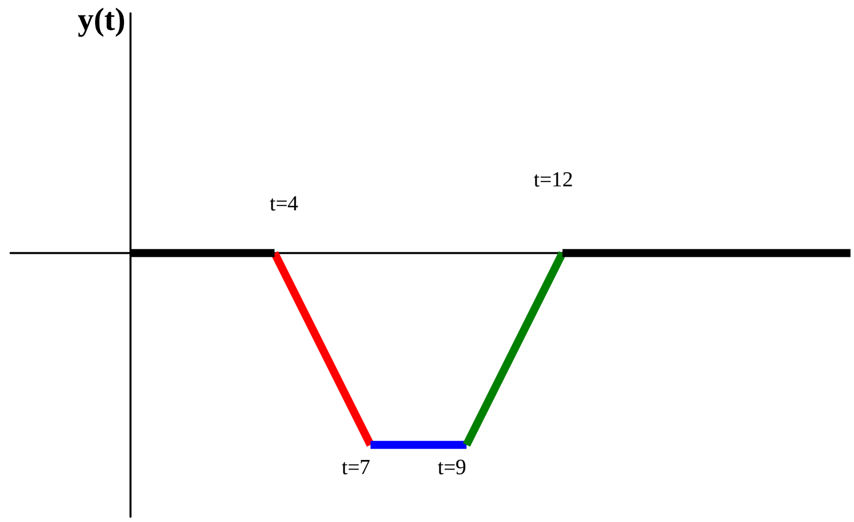


$$y(t) = \int_{t-4}^{8} (-1)(4)\,d\lambda \ \mathbf{= 4t - 48}$$

for t >= 12



y(t) = **0**



$$
y(t) = \begin{cases} 0 & t<4 \\ -4t - 16 & 4<=t<7 \\ -12 & 7<=t<9 \\ 4t - 48 & 9<=t<12 \\ 0 & t>=12 \end{cases}
$$