1. Use the definition of the Laplace transform to prove that

$$L\{\sin(\omega t)\} = \frac{\omega}{s^2 + \omega^2}$$

2. Use the definition of the Laplace transform to prove that
    a. $L\{e^{-at} f(t)\} = F(s+a)$
    b. $L\{tf(t)\} = -\dfrac{dF(s)}{ds}$

3. Find the Laplace transform of the following continuous-time signals.
    a. $x(t) = 5$
    b. $x(t) = 5te^{-3t}$
    c. $x(t) = 5e^{-3t} \cos(5t)u(t)$

4. Find the following Laplace transforms. Assume all initial conditions are zero.
    a. $L\left\{\dfrac{d}{dt} t^2 e^{-3t}\right\}$

    b. $L\left\{\left(\dfrac{d}{dt}\right)^3 t^2\right\}$

    c. $L\{3t^3(t-1) + e^{-5t}\}$

5. Find the inverse Laplace transform of

$$X(s) = \frac{5s + 13}{s^2 + 6s + 5}$$

6. Find the inverse Laplace transform of

$$F(s) = \frac{1}{(s^2 + 4)(s^2 - 4)}$$

7. Find the inverse Laplace transform of

$$G(s) = \frac{1}{(s+1)(s+2)^2}$$