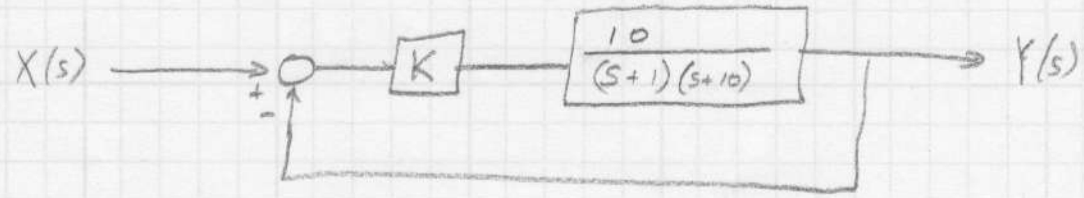
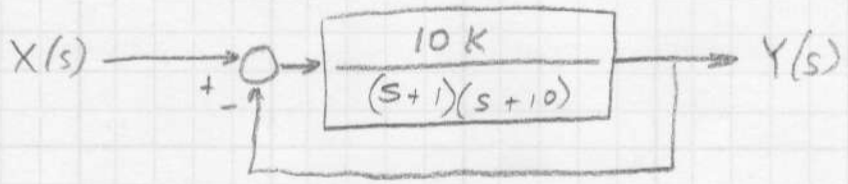
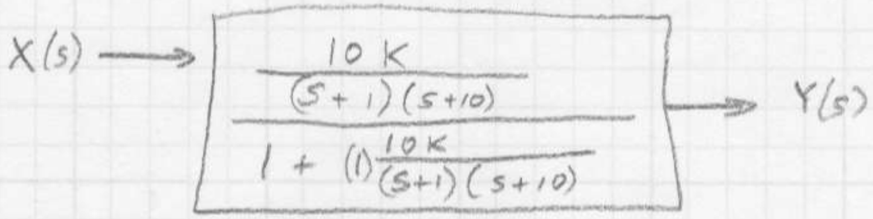
Perform the following block diagram reduction



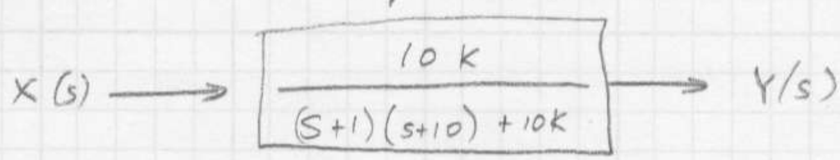
First, perform reduction on the series elements

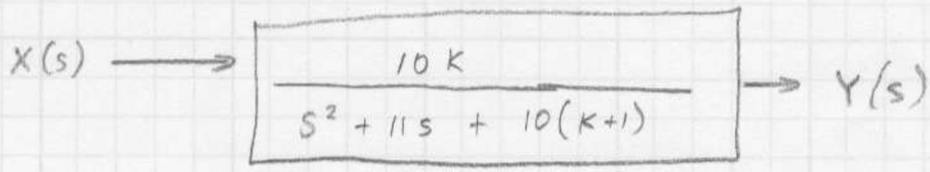


Next, reduce the feedback connection, with the feedback gain of 1

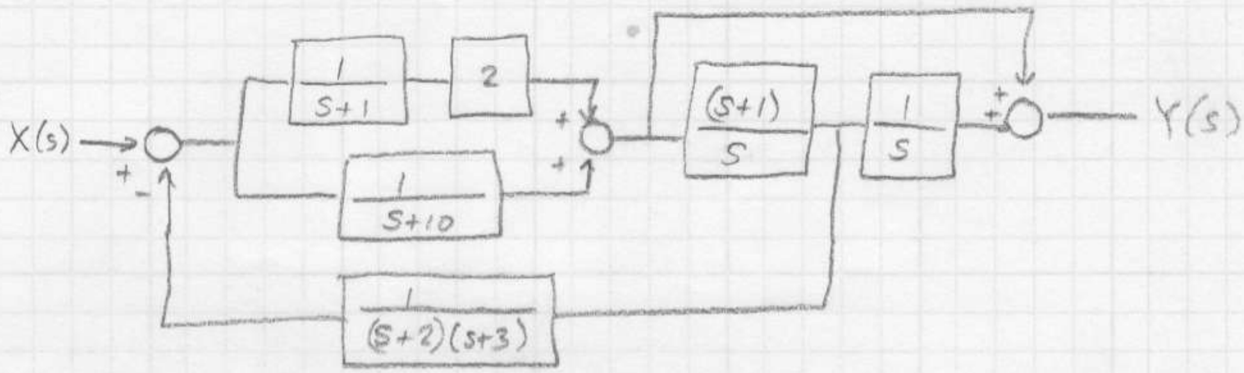


Reduce to a Rational Expression of "s"

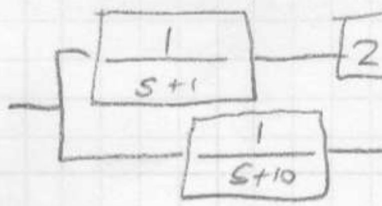




Perform the following block diagram reduction



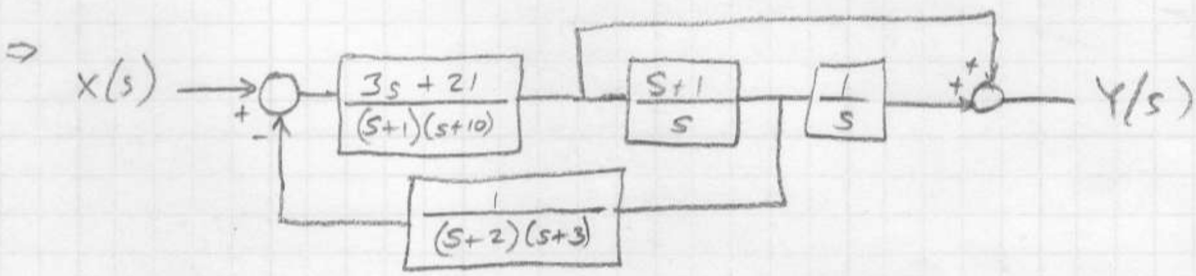
First, reduce the parallel/series combination



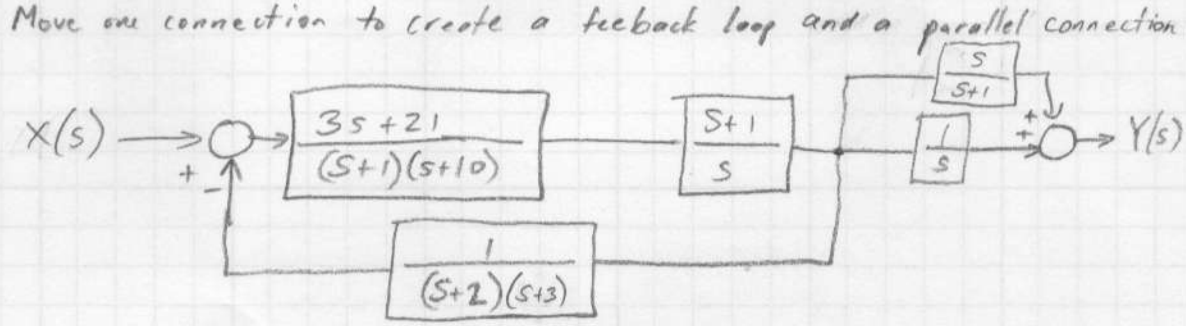
$$= \frac{2}{s+1} + \frac{1}{s+10} =$$

$$= \frac{2(s+10)+(s+1)}{(s+1)(s+10)} = \frac{3s+21}{(s+1)(s+10)}$$

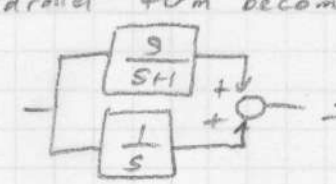
⇒



Move one connection to create a feeback loop and a parallel connection

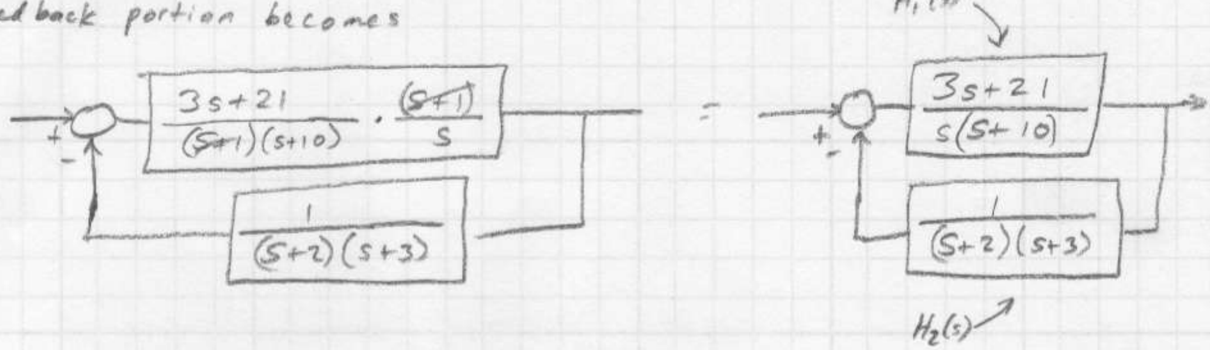


Parallel term becomes



$$= \frac{s}{s+1} + \frac{1}{s} = \frac{s^2+s+1}{s(s+1)}$$

Feedback portion becomes



$$F.B. = \frac{H_1(s)}{1 + H_1(s)H_2(s)} = \frac{\frac{3s+21}{s(s+10)}}{1 + \frac{3s+21}{s(s+2)(s+3)(s+10)}} =$$

$$= \frac{3(s+2)(s+3)(s+7)}{s(s+2)(s+3)(s+10) + 3(s+7)}$$

$$\therefore \quad X(s) \longrightarrow \boxed{\frac{3(s+2)(s+3)(s+7)}{s(s+2)(s+3)(s+10) + 3(s+7)}} \longrightarrow \boxed{\frac{s^2+s+1}{s(s+1)}} \longrightarrow Y(s)$$

$$X(s) \longrightarrow \boxed{\frac{3(s+2)(s+3)(s+7)(s^2+s+1)}{[(s+2)(s+3)(s+10) + 3(s+7)](s+1)\, s^2}} \longrightarrow Y(s)$$

Given the system $H(s) = \frac{s+1}{(s+2)(s+3)}$

Find the output for a unit step input

$\Rightarrow x(t) = u(t) \Leftrightarrow X(s) = \frac{1}{s}$

$$Y(s) = H(s)X(s) = \frac{s+1}{(s+2)(s+3)\,s} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+3}$$

Use the final value theorem for k_1

$$k_1 = \lim_{s \to 0} s\,Y(s) = \left.\frac{s+1}{(s+2)(s+3)}\right|_{s=0} = \frac{1}{6}$$

$$k_2 = Y(s)(s+2)\Big|_{s=-2} = \left.\frac{s+1}{s(s+3)}\right|_{s=-2} = \frac{-1}{(-2)(1)} = \frac{1}{2}$$

$$k_3 = Y(s)(s+3)\Big|_{s=-3} = \left.\frac{s+1}{s(s+2)}\right|_{s=-3} = \frac{-2}{(-3)(-1)} = \frac{-2}{3}$$

$$Y(s) = \frac{1}{6}\frac{1}{s} + \frac{1}{2}\frac{1}{s+2} - \frac{2}{3}\frac{1}{s+3} = \frac{\frac{1}{6}(s+2)(s+3) + \frac{1}{2}(s)(s+3) - \frac{2}{3}(s)(s+2)}{(s)(s+2)(s+3)}$$

$$= \frac{\frac{1}{6}(s^2+5s+6) + \frac{1}{2}(s^2+3s) - \frac{2}{3}(s^2+2s)}{(s)(s+2)(s+3)} = \frac{s+1}{(s)(s+2)(s+3)}$$

Agrees

$$\therefore\ y(t) = \frac{1}{6} + \underbrace{\frac{1}{2}e^{-2t} - \frac{2}{3}e^{-3t}}_{\text{transient part of the solution}}, \quad t \ge 0$$

$\frac{1}{6}$: Steady-state part of the solution

Find the output for a unit ramp input

$\Rightarrow x(t) = t\,u(t) \Leftrightarrow X(s) = \frac{1}{s^2}$

$$Y(s) = H(s)X(s) = \frac{s+1}{s^2(s+2)(s+3)} = \frac{k_1}{s} + \frac{k_2}{s^2} + \frac{k_3}{s+2} + \frac{k_4}{s+3}$$

general form of solution

$$y(t) = \underbrace{k_1 + k_2 t}_{\text{steady state}} + \underbrace{k_3 e^{-2t} + k_4 e^{-3t}}_{\text{transient}}, \quad t \ge 0$$

(Continued on next page)

$$k_2 = Y(s)\, s^2 \Big|_{s=0} = \frac{s+1}{(s+2)(s+3)} \Big|_{s=0} = \frac{1}{6}$$

$$k_3 = Y(s)(s+2) \Big|_{s=-2} = \frac{s+1}{s^2(s+3)} \Big|_{s=-2} = \frac{-1}{(4)(1)} = -\frac{1}{4}$$

$$k_4 = Y(s)(s+3) \Big|_{s=-3} = \frac{s+1}{s^2(s+2)} \Big|_{s=-3} = \frac{-2}{(9)(-1)} = \frac{2}{9}$$

Use brute-force method for k_1

$$\frac{s+1}{s^2(s+2)(s+3)} = \frac{k_1(s)(s+2)(s+3) + \frac{1}{6}(s+2)(s+3) - \frac{1}{4}(s^2)(s+3) + \frac{2}{9}(s^2)(s+2)}{s^2(s+2)(s+3)}$$

$$= \frac{k_1(s^3+5s^2+6s) + \frac{1}{6}(s^2+5s+6) - \frac{1}{4}(s^3+3s^2) + \frac{2}{9}(s^3+2s^2)}{s^2(s+2)(s+3)}$$

Equate like powers of "s"

$$s^3 \rightarrow 0 = k_1 - \frac{1}{4} + \frac{2}{9}$$

$$k_1 = \frac{9-8}{36} = \frac{1}{36}$$

$$Y(s) = \frac{1}{36} + \frac{1}{6}t - \frac{1}{4}e^{-2t} + \frac{2}{9}e^{-3t}, \quad t \geq 0$$

Determine the poles of the following systems and use the poles to determine the system's stability

$$H(s) = \frac{1}{s^2+s-6} = \frac{1}{(s-2)(s+3)} \Rightarrow \text{poles} = 2, -3$$

↑ in the Right Half Plane (RHP)

Unstable

$$H(s) = \frac{2(s-5)}{s^3+3s^2+3s+1} = \frac{2(s-5)}{(s+1)^3} \Rightarrow \text{poles} = -1, -1, -1$$

all in the Left Half Plane (LHP)

Stable

$$H(s) = \frac{(s+2)}{s(s+2)(s+3)} = \frac{1}{s(s+3)} \Rightarrow \text{poles} = 0, -3$$

Only one pole at 0. The rest of the poles are > 0

Marginally Stable

$$H(s) = \frac{(s+1)}{(s+5)((s-2)^2+9)} \qquad \text{poles} = 2 \pm j3, -5$$

↑ Real part in the right half plane

Unstable

$$H(s) = \frac{(s+1)}{(s+5)((s+2)^2-9)} = \frac{(s+1)}{(s+5)(s^2+4s-5)} = \frac{s+1}{(s+5)(s-1)(s+5)}$$

$$= \frac{s+1}{(s-1)(s+5)^2} \qquad \text{poles} = 1, -5, -5$$

Unstable

Determine if the following systems are stable. Use the Routh-Hurwitz Test.

$$H(s) = \frac{1}{s^4 + 2s^3 + s^2 + 10s + 4}$$

s^4	1	1	4 ⟸ All known from the denominator
s^3 (+)	2	10	
s^2 (−)	$\frac{2-10}{2} = -4$	4	
s^1 (+)	$\frac{-40-8}{-4} = 12$		
s^0	4		

Unstable with two unstable poles (because of two sign changes)

$$H(s) = \frac{2(s-5)}{s^3 + 4s^2 + 9s + 10}$$

s^3	1	9
s^2	4	10
s^1	$\frac{26}{4}$	0
s^0	10	

Stable → No sign changes

$$H(s) = \frac{s+10}{s^4 + 3s^3 + 6s^2 + 12s + 8}$$

s^4	1	6	8
s^3	3	12	
s^2	2	8	
s^1	$0 \approx \varepsilon$		
s^0	8		

No sign changes

Marginally Stable → There is a value of zero

Determine the values of K for which the system remains stable

$$H(s) = \frac{2(s+10)}{s^3 + 3s^2 + 4s + K}$$

s^3	1	4
s^2	3	K
s^1	$\frac{12-K}{3}$	0
s^0	K	

Stable if $\begin{cases} \frac{12-K}{3} > 0 \rightarrow K < 12 \\ K > 0 \end{cases}$

$\therefore$ Stable if $0 < K < 12$

$$H(s) = \frac{K(s+10)}{s^2 + (K+2)s + 2 - K}$$

s^2	1	$(2-K)$
s^1	$(K+2)$	
s^0	$2-K$	

Stable if $\begin{cases} K+2 > 0 \rightarrow K > -2 \\ 2-K > 0 \rightarrow K < 2 \end{cases}$

$\therefore$ Stable if $-2 < K < 2$