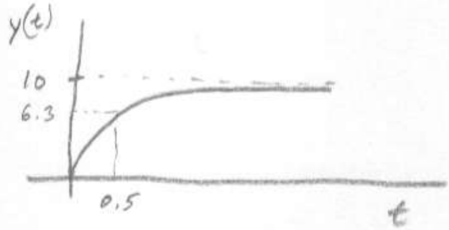
Sketch the step response for $H(s) = \frac{20}{s+2}$

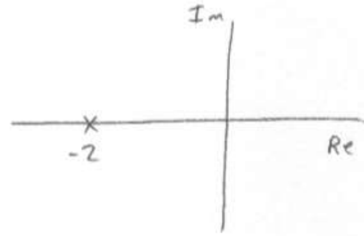
pole $= -2$

$\tau = -\frac{1}{p} = \frac{1}{2}$

Final Value (steady-state value) $\Rightarrow$ use Final Value theorem

$$y_{ss} = \underbrace{\lim_{s \to 0} s}_{\text{from F.V.T.}} \cdot \underbrace{\frac{20}{s+2}}_{H(s)} \cdot \underbrace{\frac{1}{s}}_{\text{step input}} = \lim_{s \to 0} H(s) = \frac{20}{2} = 10$$



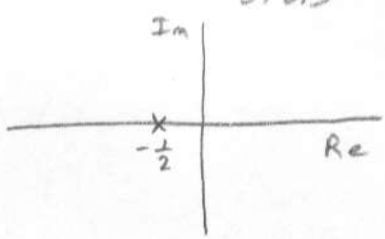


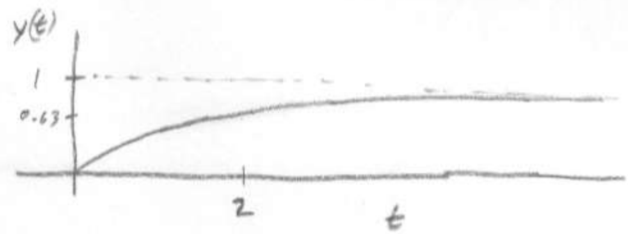
Sketch the step response for $H(s) = \frac{0.5}{s+0.5}$

pole $= -0.5$

$\tau = 2$

$y_{ss} = \lim_{s \to 0} H(s) = 1$

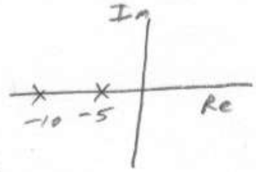




Slower exponential rise than the previous system because τ is larger

Plot the step response of the following systems

$$H(s) = \frac{10}{s^2+15s+50} = \frac{10}{(s+5)(s+10)}$$



poles $= -5, -10 \rightarrow \zeta > 1$ (Overdamped)

dominant time constant is the slower τ (farther right)

$$\tau \approx \frac{1}{5}$$

Find ζ & ω_n

$$\underbrace{\frac{k}{s^2 + 2\zeta\omega_n s + \omega_n}}_{\text{general form}} = \frac{10}{s^2+15s+50}$$

$\Rightarrow \omega_n = \sqrt{50} = 7.07$

$2\zeta\omega_n = 15$

$\zeta = \frac{15}{2}\frac{1}{\sqrt{50}} = 1.06$

Again $\rightarrow \zeta > 1 \rightarrow$ real, distinct poles $\rightarrow$ over damped

Settling time

$5\% \rightarrow 3\tau = \frac{3}{5}$ second

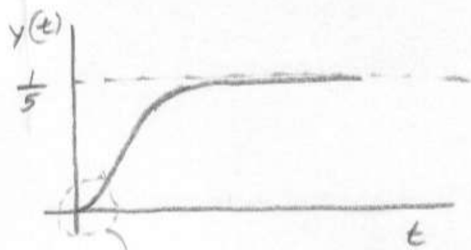
$2\% \rightarrow 4\tau = \frac{4}{5}$ second

Really only applies to under damped systems.

Percent Overshoot $\rightarrow$ No overshoot $\rightarrow$ Critically Damped

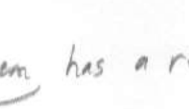
Steady-state value

$$Y_{ss} = \lim_{s \to 0} H(s) = \frac{1}{5}$$



This part due to the fact that there are two poles

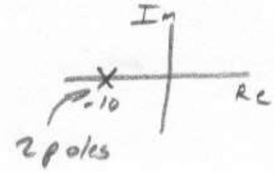
A first-order system has a response that is perfectly exponential



Step Response for $H(s) = \frac{10}{s^2+20s+100} = \frac{10}{(s+10)(s+10)}$

poles = $-10, -10$ $\rightarrow$ Real, Repeated Poles

$\therefore\ \zeta = 1$ Critically Damped



$\omega_n = \sqrt{100} = 10$

$\tau \approx \frac{1}{10}$

$y_{ss} = \lim_{s \to 0} H(s) = \frac{10}{100} = \frac{1}{10}$

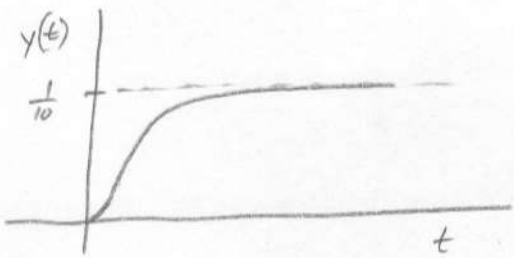
No overshoot because the system is critically damped

Settling time

$2\%\ \ t_s \approx 3\tau = \frac{3}{10}$ second

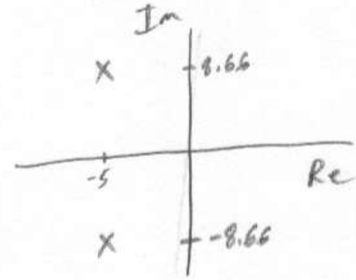
$5\%\ \ t_s \approx 4\tau = \frac{4}{10} = \frac{2}{5}$ second

} Really only Applies to Underdamped systems



Step response for $H(s) = \dfrac{10}{s^2 + 10s + 100}$

$$\text{poles} = \underbrace{\frac{-10 \pm \sqrt{100 - 400}}{2}}_{\text{quadratic equation}} = \frac{-10 \pm \sqrt{-300}}{2} = -5 \pm j5\sqrt{3} = -5 \pm j\,8.66$$



$\therefore 0 < \zeta < 1 \rightarrow$ Complex Poles

$\rightarrow$ Under Damped

$$\frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{10}{s^2 + 10s + 100}$$

$\Rightarrow \omega_n = \sqrt{100} = 10$

$2\zeta\omega_n = 10$

$\zeta = \frac{1}{2} = 0.5 \rightarrow$ Underdamped

$\tau = \dfrac{1}{\zeta\omega_n} = \dfrac{1}{5} = 0.2$

Settling Time, t_s

$2\% \rightarrow t_s \approx 3\tau = \frac{3}{5}$ second

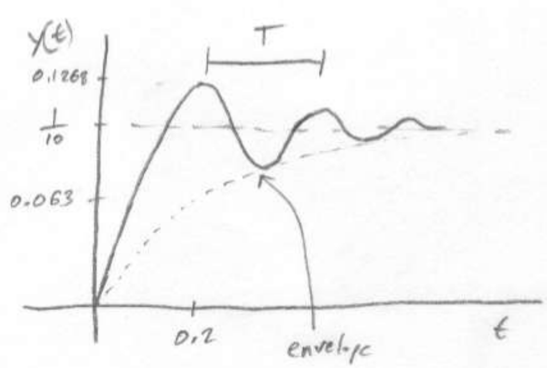
$5\% \rightarrow t_s \approx 4\tau = \frac{4}{5}$ second

Percent Overshoot

$$P.O. \approx 100\% \times e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100\% \times e^{-\frac{\pi}{2}/\sqrt{1-0.25}} = 26.76\%$$

Final Value

$y_{ss} = \lim_{s \to 0} H(s) = \frac{1}{10}$



Peak value $= \dfrac{P.O.}{100\%} \times y_{ss} + y_{ss}$

$= \dfrac{26.76}{100\%} \times 0.1 + 0.1 = 0.1268$

$T = \dfrac{2\pi}{\omega_d}$

$\omega_d = 8.66$

$T = \dfrac{2\pi}{8.66} = 0.7255$

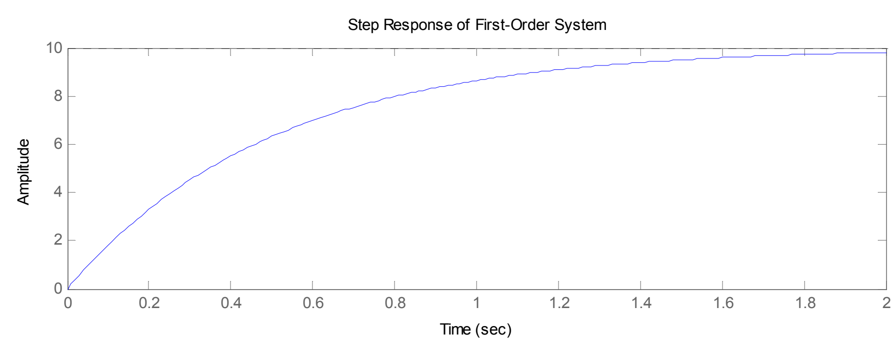
Use MATLAB to generate the step response of the following systems. Find the rise time and percent overshoot (if applicable).

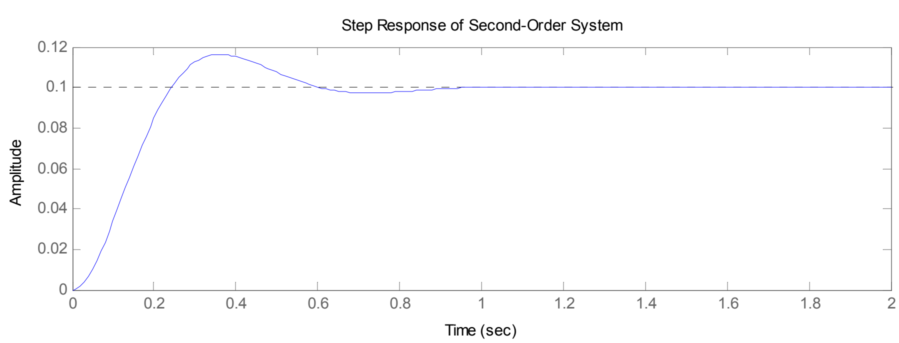
a. $H(s) = \dfrac{20}{(s+2)}$

b. $H(s) = \dfrac{10}{s^2 + 10s + 100}$

```
% find the step response of the following two systems

% first-order system
num = [20];
den = [1 2];
tt = 0:0.01:2;
figure;
subplot(2,1,1);
step(num,den,tt);
title('Step Response of First-Order System')


% second-order system
num = [10];
den = [1 10 100];
tt = 0:0.01:2;
subplot(2,1,2);
step(num,den,tt);
title('Step Response of Second-Order System')
axis([0 2 0 0.12]);
```

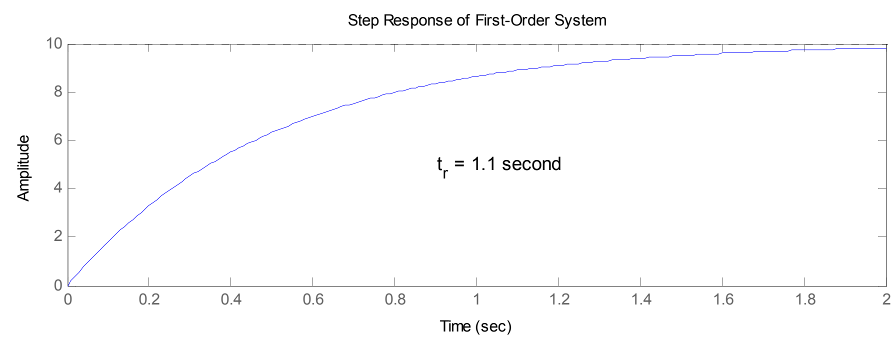
The rise time and the percent overshoot can be found from the following plots.

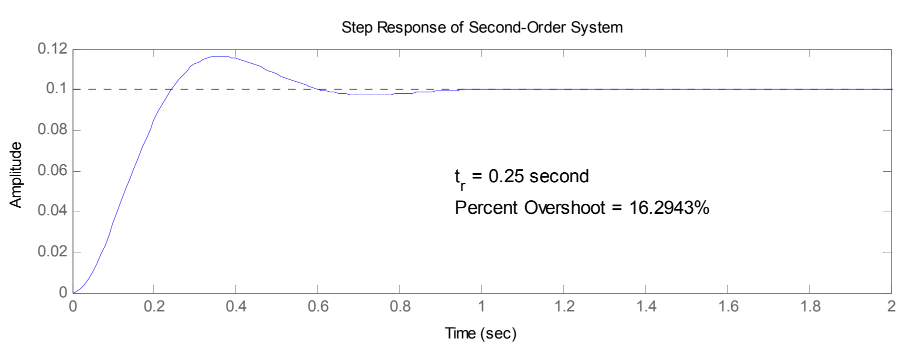




Alternatively, we can use more sophisticated MATLAB code to find out the rise times and percent overshoot for us. Type “help function_name” to learn more about MATLAB functions.

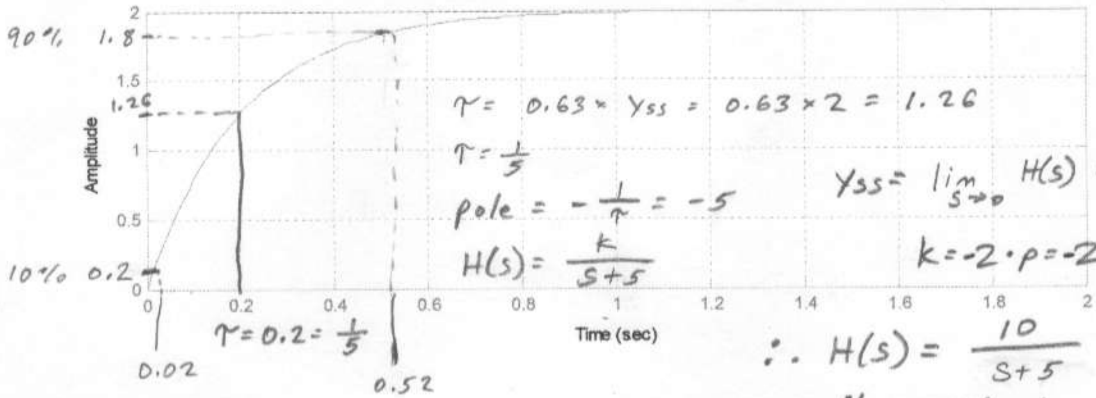
```
% first-order system
num = [20];
den = [1 2];
tt = 0:0.01:2;

figure;
subplot(2,1,1);
step(num,den,tt);
[y] = step(num,den,tt);
title('Step Response of First-Order System')
[I1,J1] = find(y >= 0.1*10);
[I2,J2] = find(y >= 0.9*10);
tr = tt(I2(1))-tt(I1(1))

% second-order system
num = [10];
den = [1 10 100];
tt = 0:0.01:2;

subplot(2,1,2);
step(num,den,tt);
[y] = step(num,den,tt);
title('Step Response of Second-Order System')
axis([0 2 0 0.12]);
[I,J] = find(y >= 0.1);
tr = tt(I(1))
y_max = max(y);
y_ss = y(end);
percent_overshoot = ((y_max - y_ss)/y_ss)*100
```





From the following step response plots, determine the transfer function. Also, determine the rise times.

First-Order System $\rightarrow H(s) = \dfrac{k}{S+P}$



$\tau = 0.63 \times Y_{ss} = 0.63 \times 2 = 1.26$

$\tau = \frac{1}{5}$

$\text{pole} = -\frac{1}{\tau} = -5$

$H(s) = \dfrac{k}{s+5}$

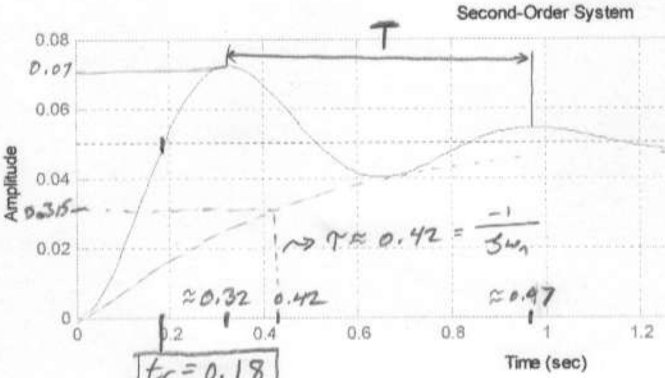
$Y_{ss} = \lim_{s \to 0} H(s) = \dfrac{k}{-p} = 2$

$k = -2 \cdot p = -2 \cdot (-5) = 10$

$\therefore H(s) = \dfrac{10}{S+5}$

No overshoot

Second-Order System



Alternate Method
Use Percent Overshoot to find ζ
Calculate $\omega_n = \dfrac{\omega_d}{\sqrt{1-\zeta^2}}$
Then use final value theorem for k

First-Order System $\rightarrow$ Rise time, t_r $\rightarrow$ Never exceeds final value

$H(s) = \dfrac{10}{S+5}$

$\therefore$ use t_r for 10% to 90% of final value

$t_r \approx 0.52 - 0.02 = 0.5$

Second-Order System

$T \approx 0.97 - 0.32 = 0.65 = \dfrac{2\pi}{\omega_d}$

$\therefore \omega_d = \dfrac{2\pi}{0.65} = 9.67$

$Y_{ss} = 0.05$

$H(s) = \dfrac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \dfrac{k}{(s+\zeta\omega_n)^2 + \omega_d^2}$

$P.O. \approx \dfrac{0.07 - 0.05}{0.05} \times 100\% = 40\%$

$P.O. \approx 100\%\, e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

(see Alternate Method above)

$Y_{ss} = \lim_{s \to 0} H(s) = \dfrac{k}{\underbrace{(\zeta\omega_n)^2}_{= -\frac{1}{\tau}} + \omega_d^2} = 0.05$

$k = Y_{ss}\left((\zeta\omega_n)^2 + \omega_d^2\right) = Y_{ss}\left(-\frac{1}{\tau} + \omega_d^2\right)$

$\therefore k = (0.05)\left(\left(\frac{1}{0.42}\right)^2 + (9.67)^2\right) = 4.96$

$H(s) = \dfrac{4.96}{\left(S + \frac{1}{0.42}\right)^2 + (9.67)^2} = \dfrac{4.96}{S^2 + 4.76s + 99.18}$

Actual (what was plotted) $\rightarrow H(s) = \dfrac{5}{S^2 + 5s + 100}$

Close to analytic expression