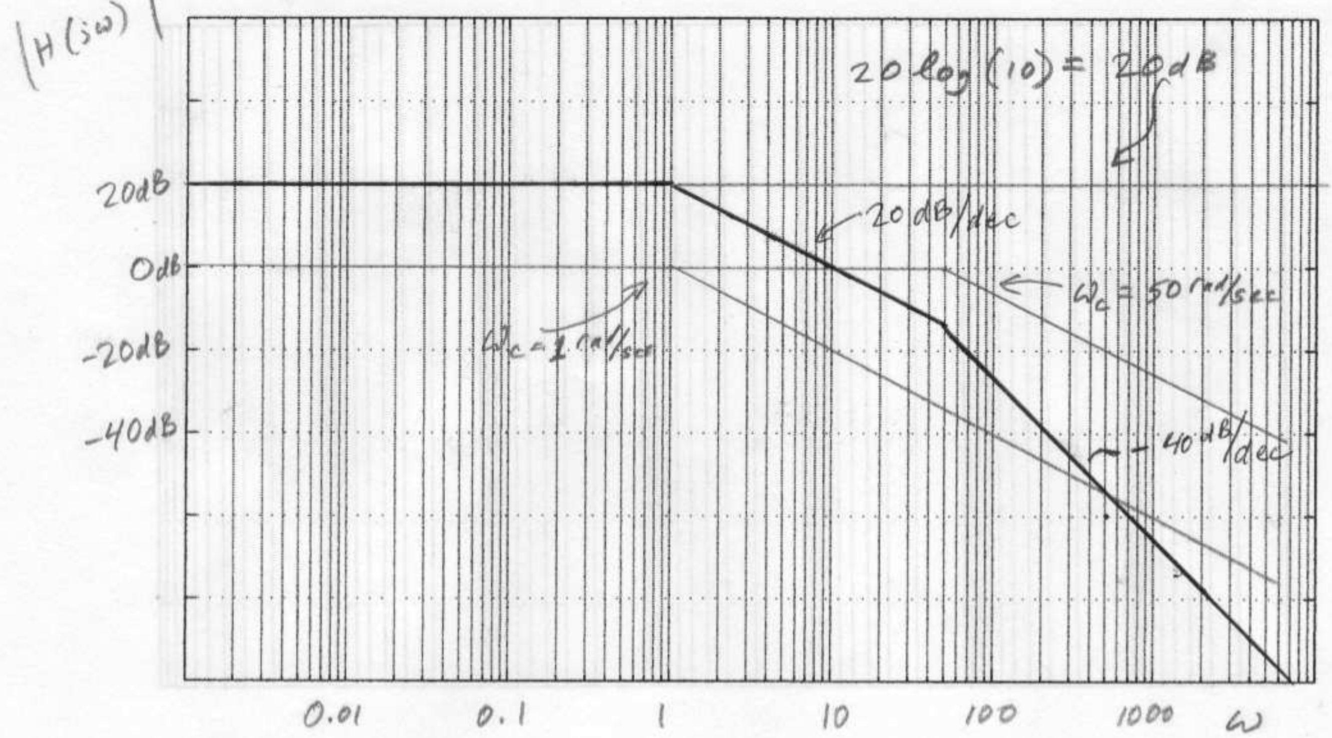
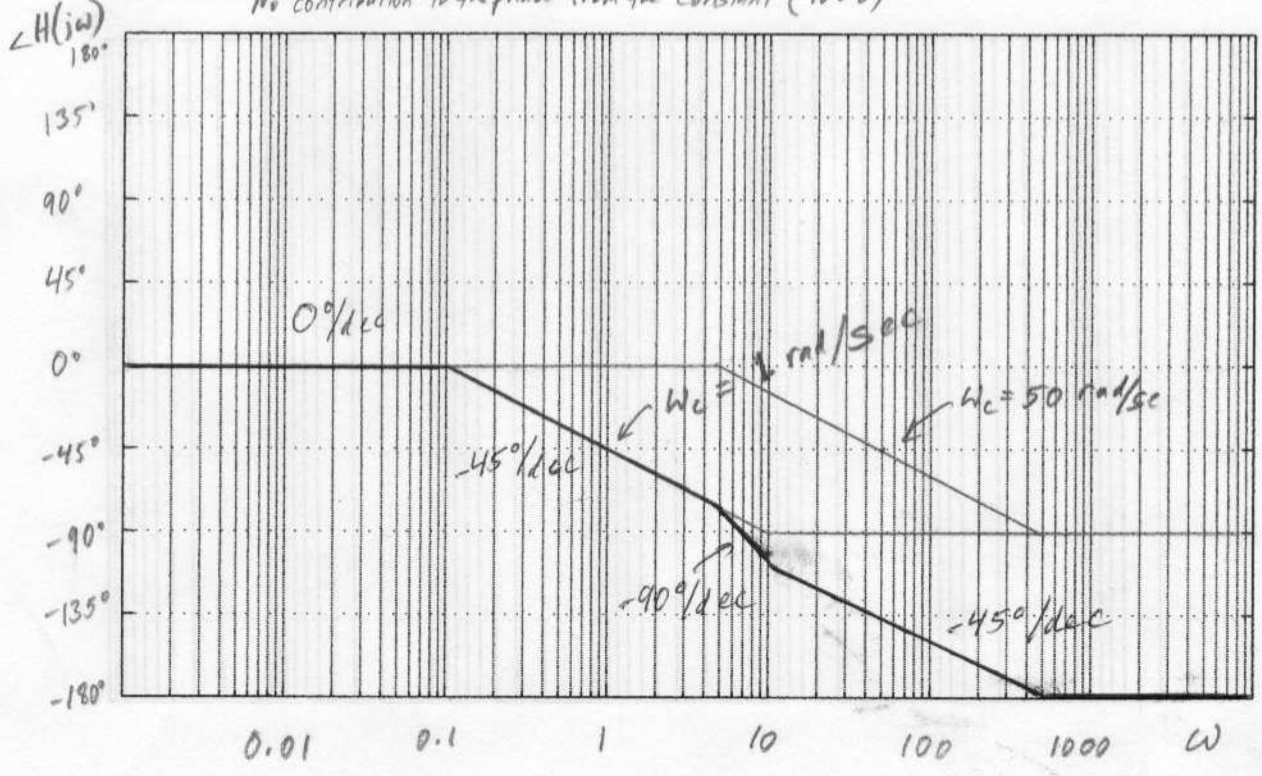
$$H(s) = \frac{500}{(s+1)(s+50)} \qquad H(j\omega) = \frac{10}{(j\omega+1)\left(\frac{j\omega}{50}+1\right)}$$

corner frequencies at $\omega = 1, 50$



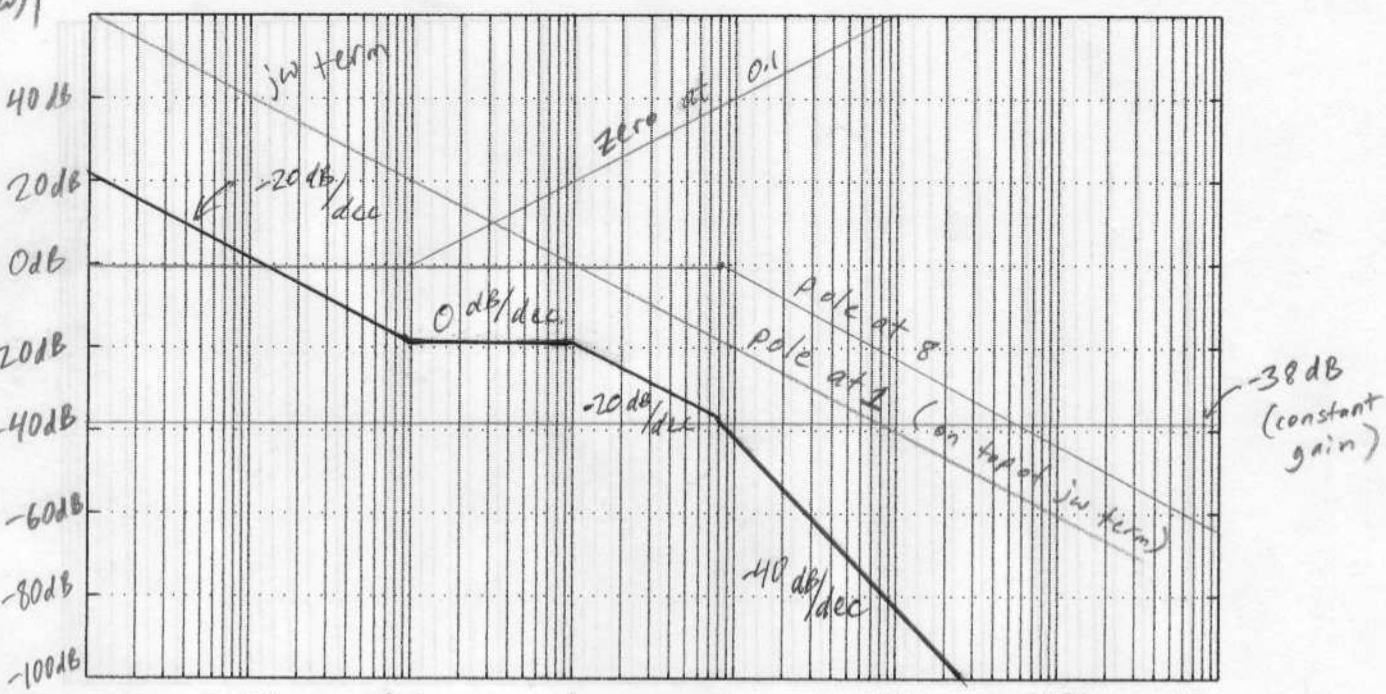
No contribution to the phase from the constant (10 > 0)



Overall contribution to the phase (as $\omega \to \infty$) is $-180°$ because there are two poles (each adds $-90°$ of phase)

$$H(s) = \frac{(s+0.1)}{(s)(s+1)(s+8)} \qquad H(j\omega) = \frac{j\omega + 0.1}{(j\omega)(j\omega+1)(j\omega+8)} = \frac{0.0125\left(\frac{j\omega}{0.1}+1\right)}{(j\omega)(j\omega+1)\left(\frac{j\omega}{8}+1\right)}$$

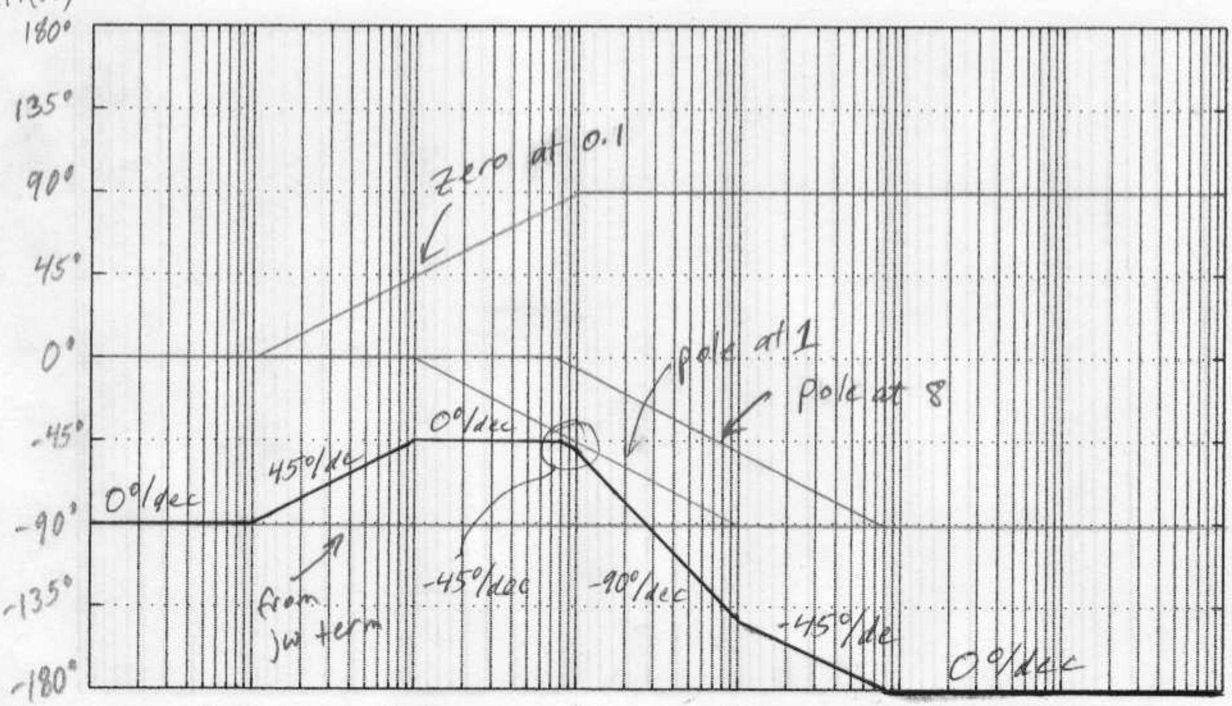
$|H(j\omega)|$



Zero at $\omega = 0.1$
poles at $\omega = 1, 8, 0$ ($j\omega$ term)

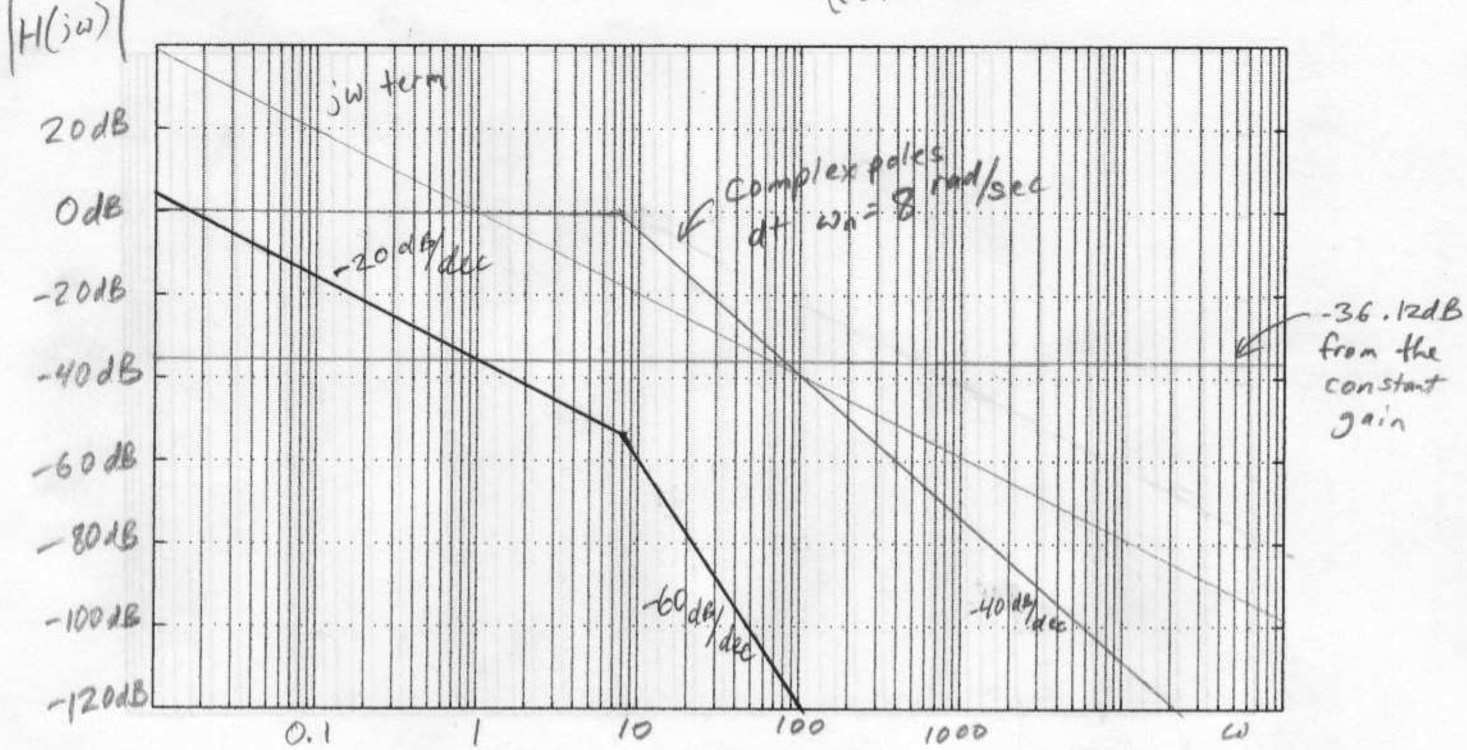
$20 \log_{10} 0.0125 = -38\,dB$

$\angle H(j\omega)$



Three poles and one zero → as $\omega \to \infty$, $\angle H(j\omega) = -180°$

$$H(s) = \frac{1}{(s)(s^2 + 8s + 64)} \qquad H(j\omega) = \frac{\frac{1}{64}}{(j\omega)\left(\left(\frac{j\omega}{8}\right)^2 + \frac{1}{8}j\omega + 1\right)}$$

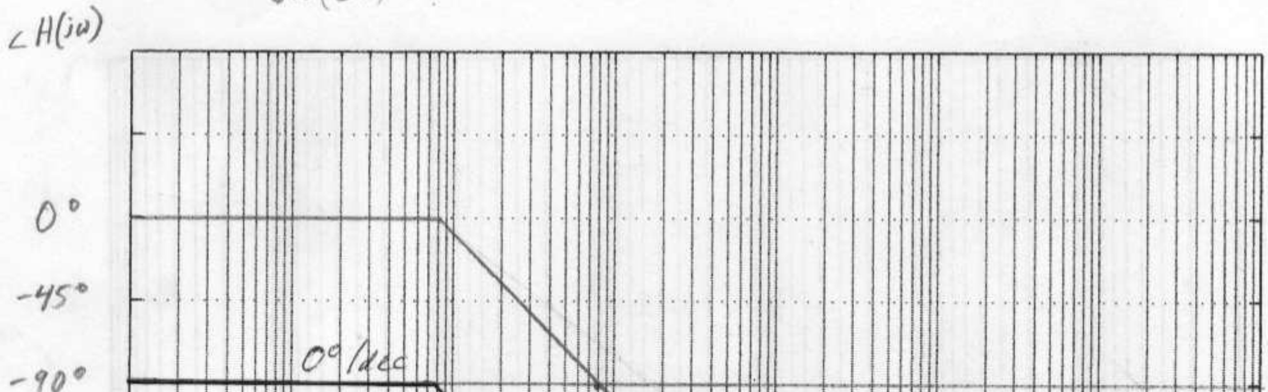


Two poles → $\omega = 0$ ($j\omega$ term)
$\omega = 8$ rad/sec

$2\zeta\omega_n = 8$

$\zeta = \frac{1}{2}$ complex poles

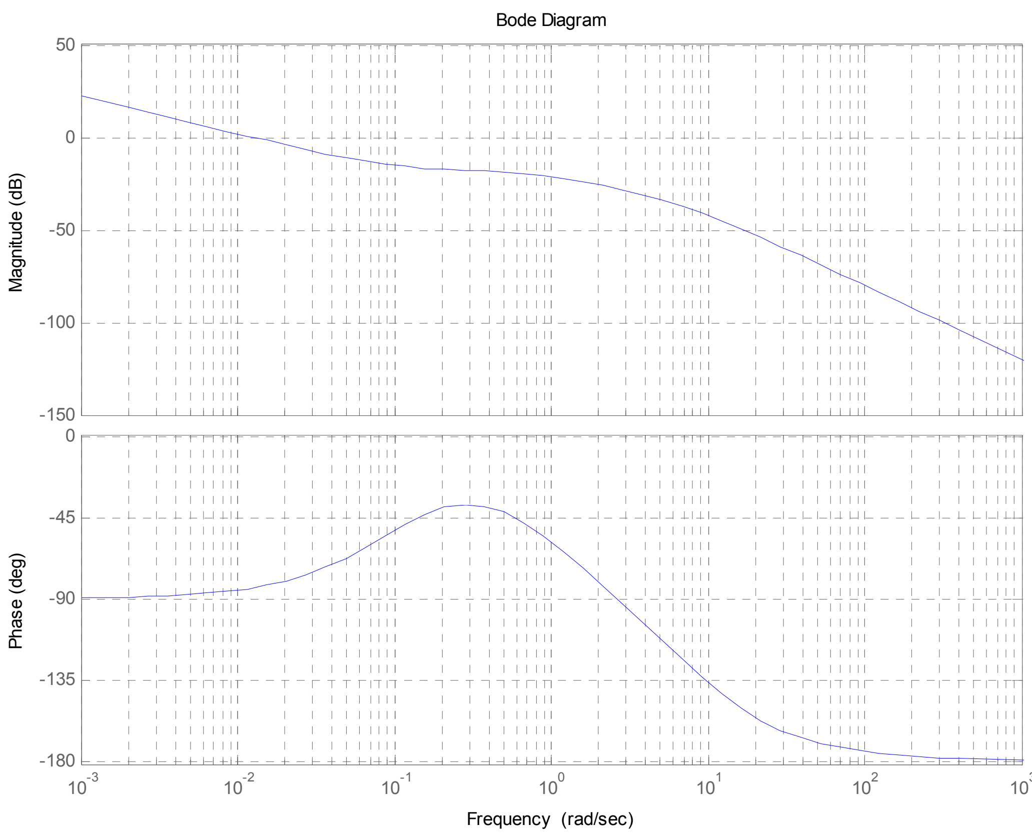
$20 \log_{10}\left(\frac{1}{64}\right) = -36.12\,dB$



Use MATLAB to generate the Bode plots of the following system.

$$H(s) = \frac{(s+0.1)}{s(s+1)(s+8)}$$

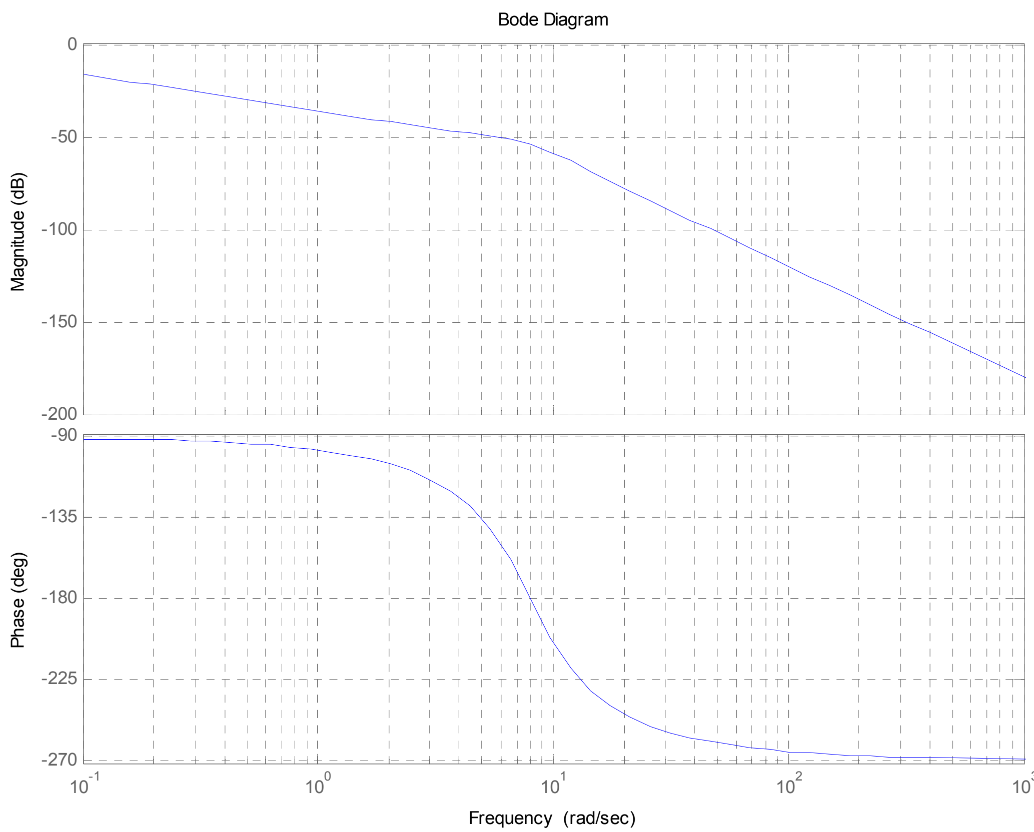
```
num = [1 0.1];
den = [1 9 8 0];
figure;
bode(num,den);
grid on;
```



Use MATLAB to generate the Bode plots of the following system.

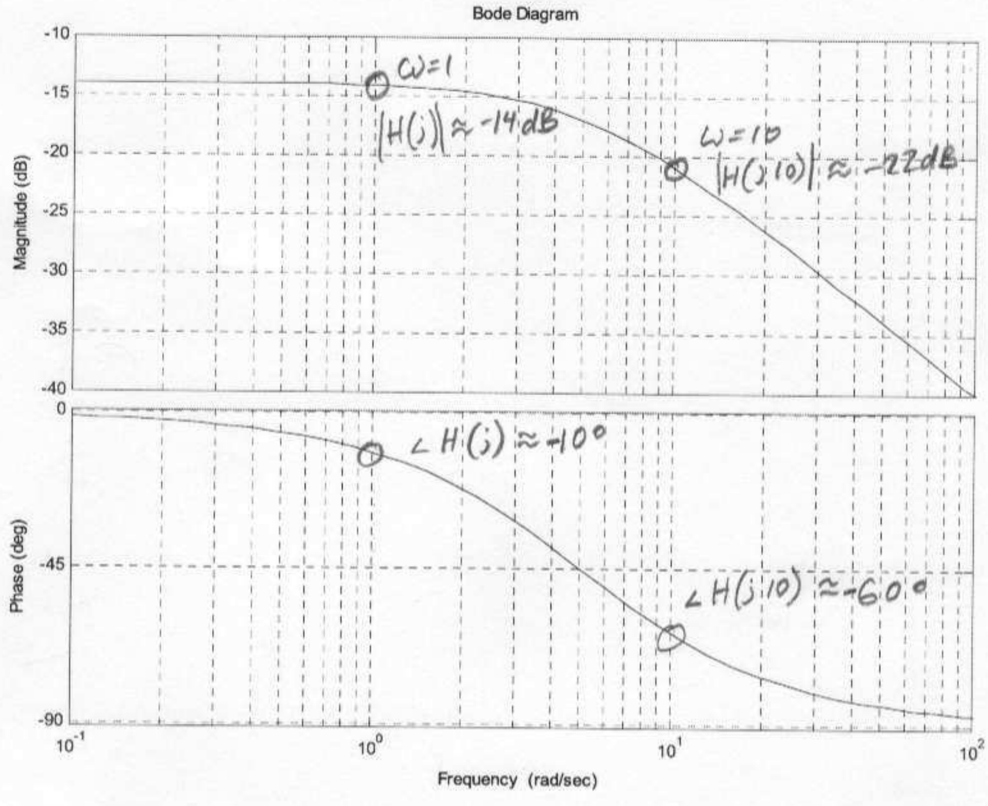
$$H(s) = \frac{1}{s(s^2 + 8s + 64)}$$

```
num = [1];
den = [1 8 64 0];
figure;
bode(num,den);
grid on;
```



7. Let a system have the following frequency response. Find the output of the system to the following input.

$$x(t) = 10 + 5\cos(t) + 100\cos(10t + 45^\circ)$$



Three frequencies of interest

$\omega = 0, 1, 10$

$|H(j0)| \approx -14\,dB$

$\angle H(j0) \approx 0^\circ$

See above for $\omega = 1, 10$

Convert to scalar numbers for multiplication

$\omega=0 \quad A_0 = 10^{-14dB/20} = 0.1995 \approx 0.2$

$\omega=1 \quad A_1 = 10^{-14dB/20} = 0.1995 \approx 0.2$

$\omega=10 \quad A_{10} = 10^{-22dB/20} = 0.0794 \approx 0.08$

Steady-state output

$$y(t) = (1)\,|H(j0)| + (5)\,|H(j1)|\cos(t + \angle H(j1)) + (100)\,|H(j10)|\cos(10t + 45^\circ + \angle H(j10)) =$$

$$= 0.2 + (5)(0.2)\cos(t - 10^\circ) + (100)(0.08)\cos(10t + 45^\circ - 60^\circ) =$$

$$= 0.2 + \cos(t - 10^\circ) + 8\cos(10t - 15^\circ)$$

A continuous-time filter is given by

$$H(s) = \frac{20}{\frac{s^2}{100} + \frac{s}{10} + 1}$$

Determine the filter type (what type of filtering operation), ω_0, Q, A (gain), and sketch the Bode plot.

Answer

No zeros

$\therefore$ Lowpass Filter (second order)

Compare to the cannonical form $\rightarrow A \dfrac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$

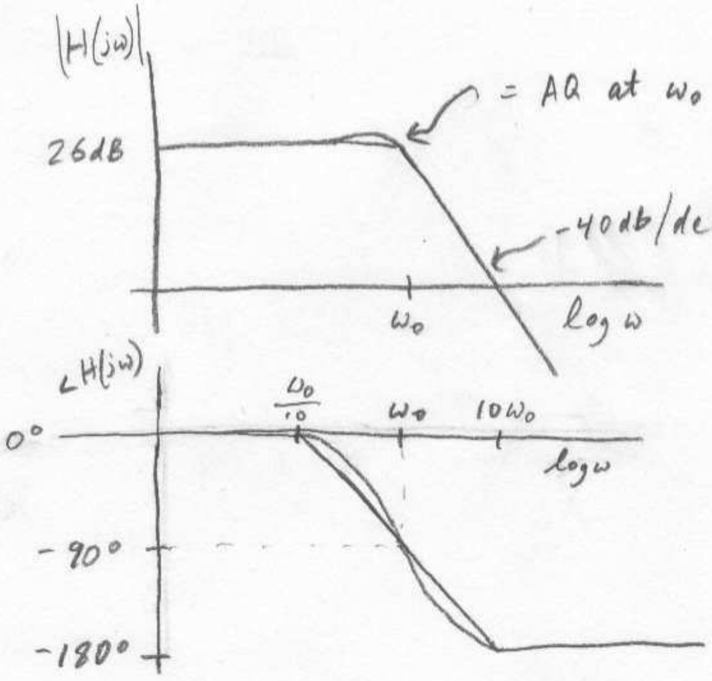
$\rightarrow \omega_0 = \sqrt{100} = 10$

$Q\omega_0 = 10$

$\Rightarrow Q = 1$

$A = 20 \rightarrow 20\log(20) \approx 26\,dB$

at $\omega_0 \rightarrow |H(j\omega_0)| = AQ = (20)(1) = 26dB$



Asymptotes and actual responses are shown

A continuous-time filter is defined by

$$H(s) = \frac{\frac{s^2}{10}}{\frac{s^2}{100} + \frac{s}{20} + 1}$$

Determine the filter type, ω_0, Q, A, and sketch the Bode plot.

Answer

Two zeros → Highpass filter

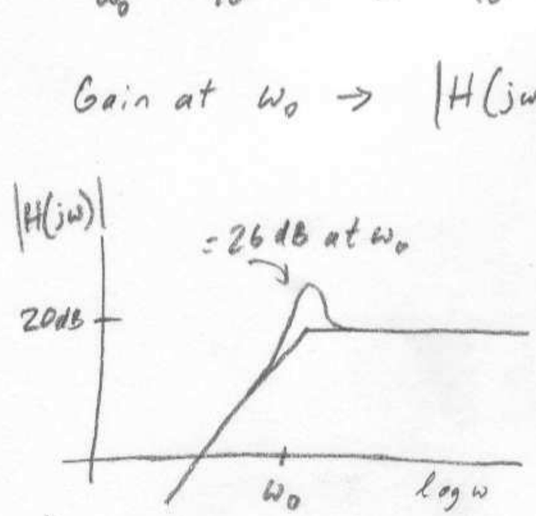
Compare to the cannonical form → $A \dfrac{\frac{s^2}{\omega_0^2}}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$

$\rightarrow \omega_0 = 10$ rad/sec

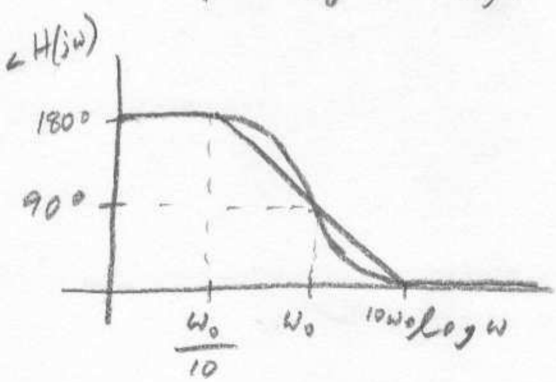
$Q\omega_0 = 20 \Rightarrow Q = 2$

$\frac{A}{\omega_0^2} = \frac{1}{10} \Rightarrow A = \frac{1}{10}\omega_0^2 = 10 \rightarrow 20\log(10) = 20\,dB$

Gain at ω_0 → $|H(j\omega_0)| = AQ = 20 \rightarrow 20\log(20) \approx 26\,dB$



Asymptotes and actual responses are shown



A continuous-time filter is defined by

$$H(s) = \frac{\frac{s}{20}}{\frac{s^2}{100} + \frac{s}{20} + 1}$$

What type of filter is this?

What is A, ω_0, Q, and the bandwidth?

Sketch the frequency response.

Answer

One pole → Bandpass filter

Compare to the cannonical form → $A \dfrac{\frac{s}{Q\omega_0}}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$

$\rightarrow \omega_0 = 10$

$Q\omega_0 = 20 \Rightarrow Q = 2$

$A = 1 \rightarrow 20 \log(1) = 0\,dB$

$BW = \frac{\omega_0}{Q} = \frac{10}{2} = 5$ rad/sec